A constitutive framework for double porosity materials with evolving internal structure

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IV International workshop
Modern Trends in Geomechanics
Assisi, May 16 -18 2016

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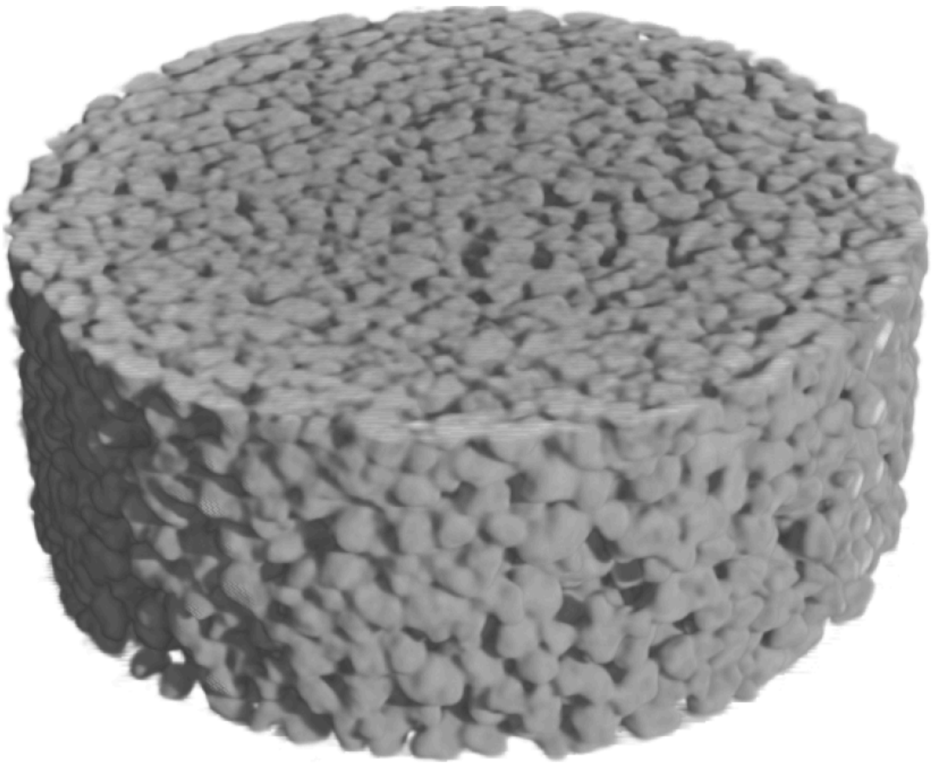
Reference

Borja, R.I. & Choo, J. (2016). Cam-Clay plasticity, Part VIII: A constitutive framework for porous materials with evolving internal structure. *Computer Methods in Applied Mechanics and Engineering*, doi: 10.1016/j.cma.2016.06.016

Double porosity

Aggregated Bioley silt (Koliji, Vulliet & Laloui 2010)



neutron tomography

Double porosity

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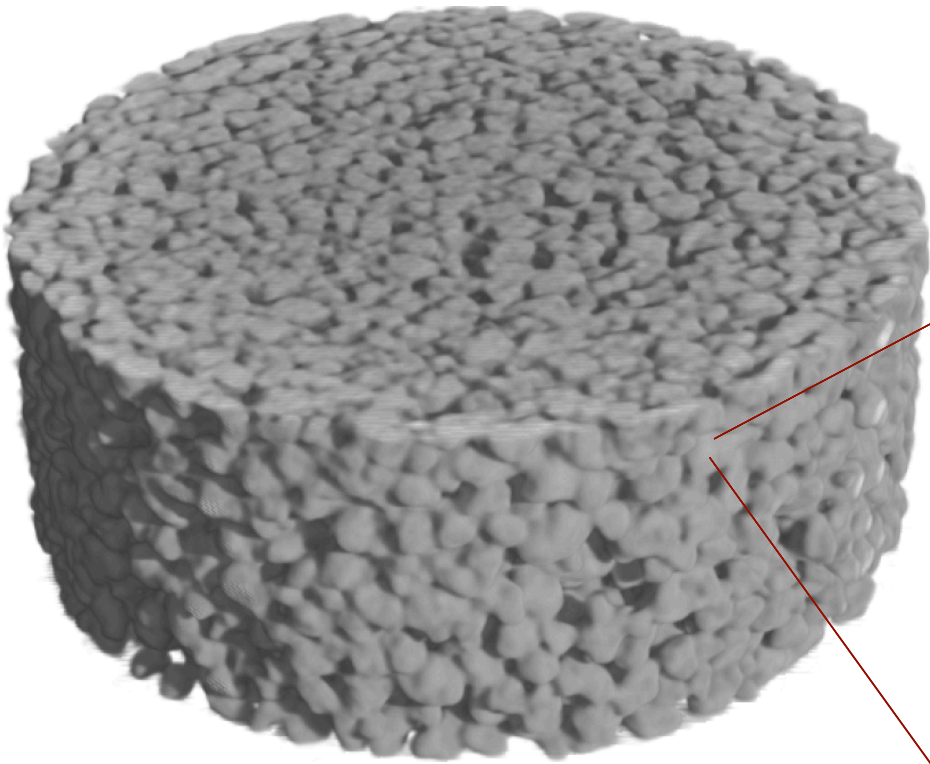
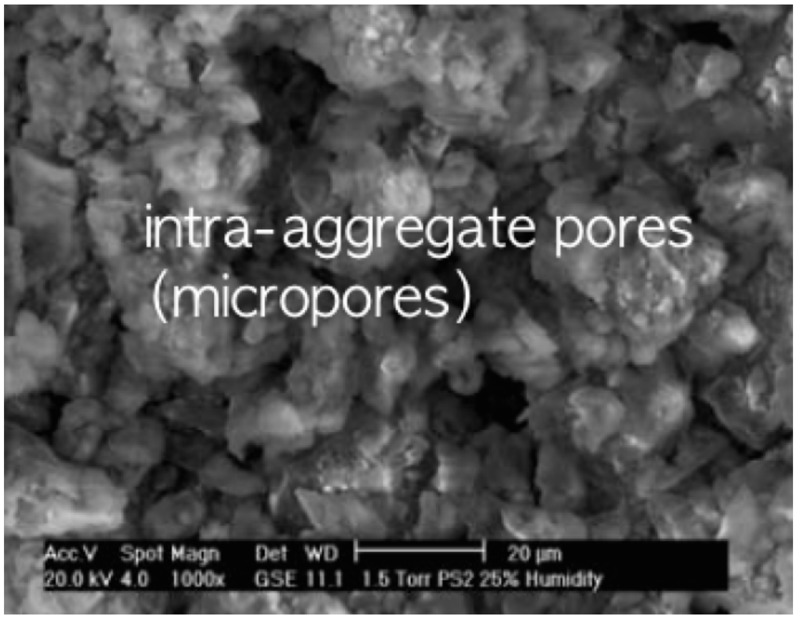


neutron tomography



ESEM

Double porosity

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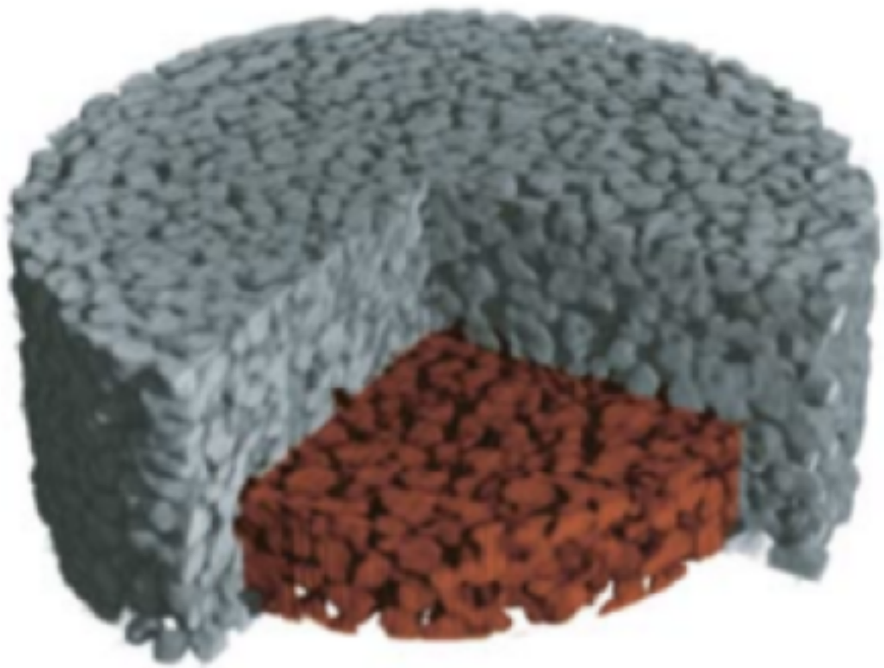
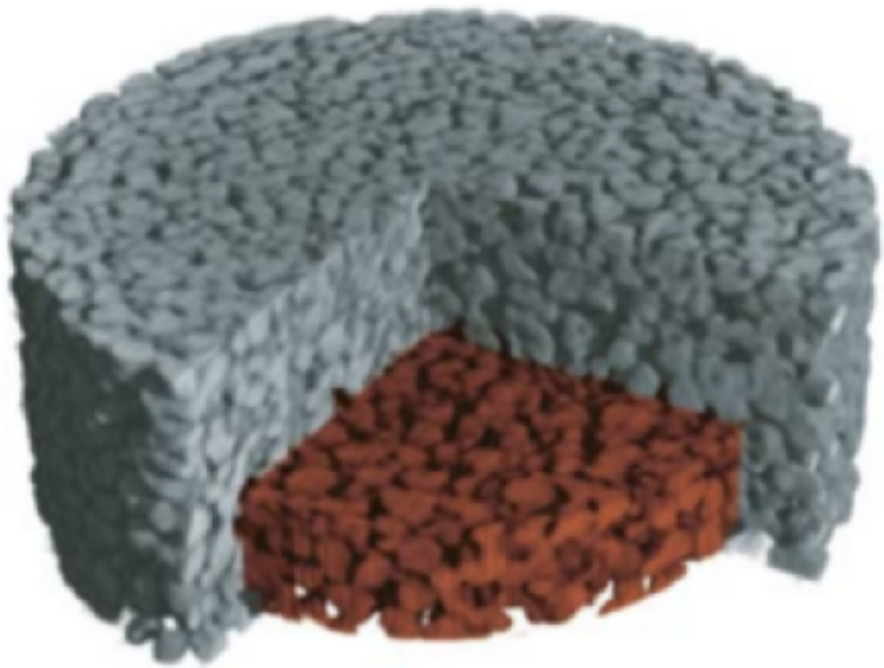


- macropores
- micropores

- dual porosity
- dual permeability

Double porosity

Aggregated Bioley silt (Koliji, Vulliet & Laloui 2010)



- macropores
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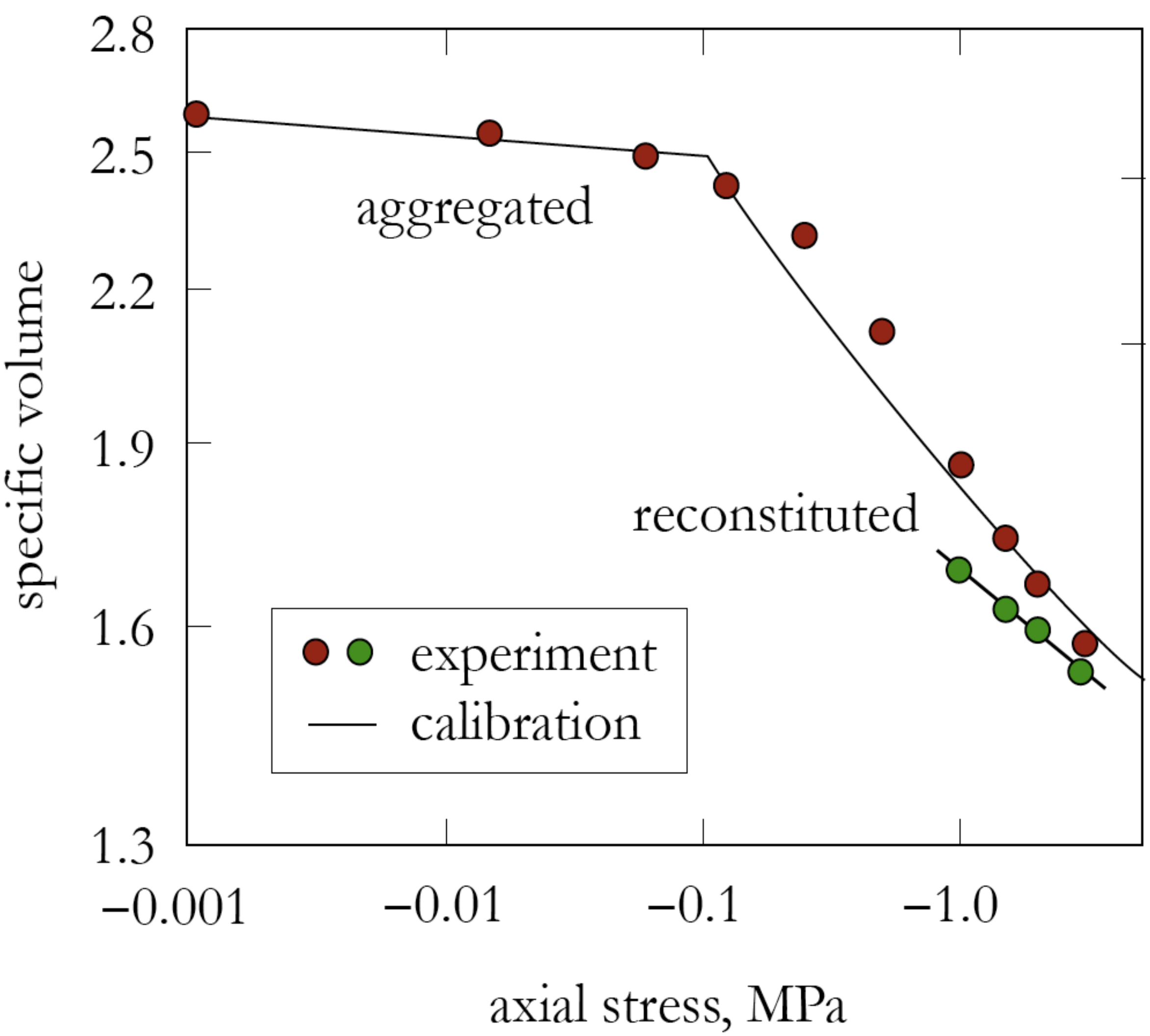
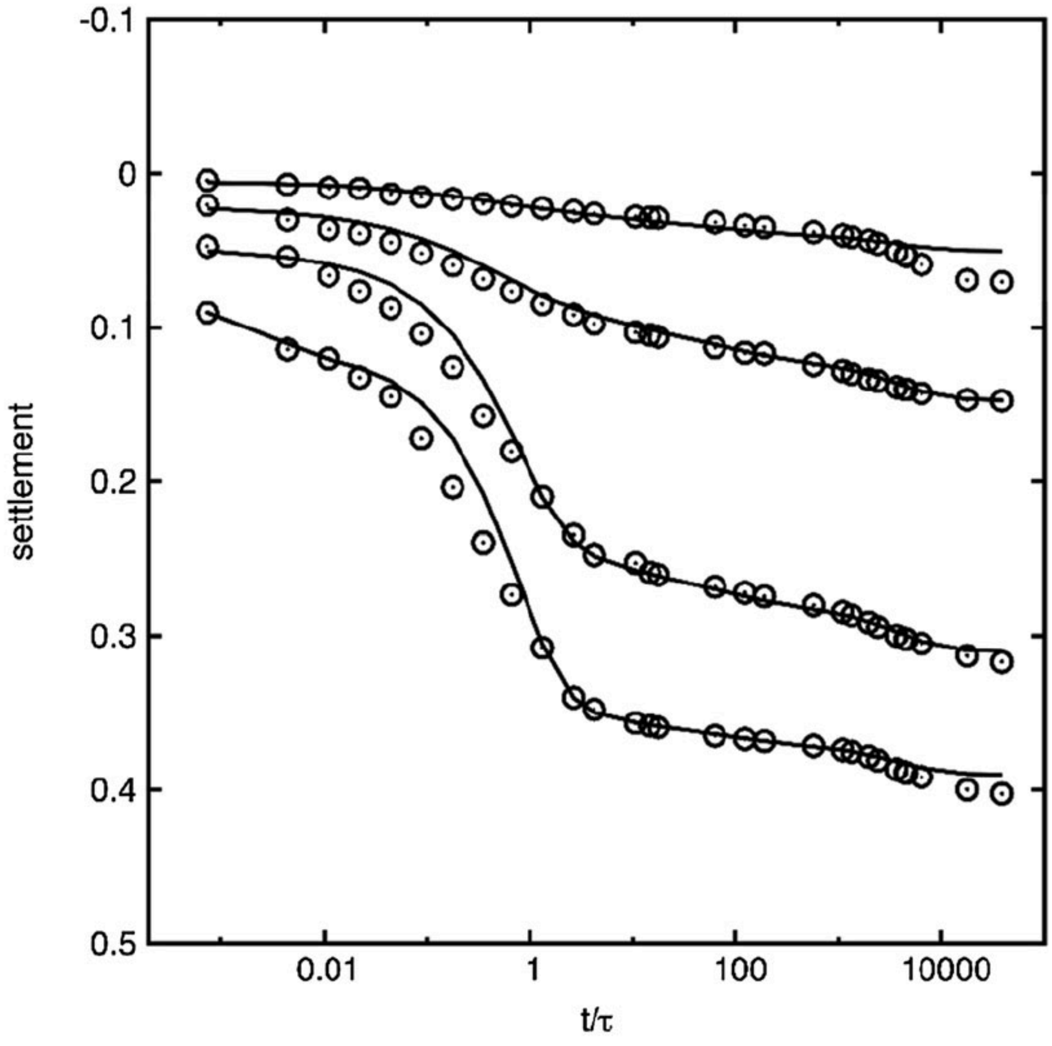


Secondary compression

Cubzak-Les-Ponts clay (Cosenza & Korosak 2014)



Objectives

- Develop a 3D constitutive framework for porous materials with evolving internal structure (i.e. pore fraction)
- Framework must accommodate changes in the preconsolidation stresses at each pore scale
- Framework must be amenable to finite element implementation.

Theoretical basis

Effective stress equation (Borja & Koliji, JMPS, 2009)

$$\boldsymbol{\sigma}' = \boldsymbol{\sigma} + B\bar{p}\mathbf{1}$$

$$\bar{p} = \sum_{i=M,m} \left[\psi^i S^i p_i + \psi^i (1 - S^i) p_{ia}\right]$$

Overall mean pore pressure:

- For each pore scale, take the weighted sum of the pore air and pore water pressures with the local saturations taken as the weights.
- Take the sum of the mean pore pressures at each pore scale with the pore fractions taken as the weights.

Theoretical basis

Effective stress equation (Borja & Koliji, JMPS, 2009)

$$\boldsymbol{\sigma}' = \boldsymbol{\sigma} + B\bar{p}\mathbf{1}$$

1.0 0

$$\bar{p} = \sum_{i=M,m} \left[\psi^i S^i p_i + \psi^i (1 - S^i) p_{ia} \right]$$

Overall mean pore pressure:

- For fully saturated media, take the weighted sum of the pore water pressures with the pore fractions as the weights.
- We want a finite deformation formulation to accommodate the evolution of the pore fractions.

Theoretical basis

* Limit scope to fully saturated double-porosity media.

Void ratios

$$e_m(\boldsymbol{X}, t) = \frac{\mathrm{d}V_{vm}}{\mathrm{d}V_s}\,, \qquad e_M(\boldsymbol{X}, t) = \frac{\mathrm{d}V_{vM}}{\mathrm{d}V_s}$$

Volume fractions

$$\phi^s(\boldsymbol{X}, t) = \frac{\mathrm{d}V_s}{\mathrm{d}V}\,, \qquad \psi(\boldsymbol{X}, t) = \frac{\mathrm{d}V_{vm}}{\mathrm{d}V_v}$$

Specific volumes

$$v_m(\boldsymbol{X}, t) = 1 + e_m(\boldsymbol{X}, t)$$

$$v(\boldsymbol{X}, t) = v_m(\boldsymbol{X}, t) + e_M(\boldsymbol{X}, t)$$

Internal energy equation

$$J\rho\dot{e} = \langle \bar{\boldsymbol{\tau}}, \boldsymbol{d} \rangle + \sum_{i=M,m} \langle \tilde{\boldsymbol{v}}_i, \phi^i, p_i \rangle - \sum_{i=M,m} \langle c^i, p_i, \tilde{\boldsymbol{v}}_i \rangle - \langle (1-\phi^s), \pi, \dot{\psi} \rangle$$

Theoretical basis

Internal energy equation

$$J\rho\dot{e} = \langle \bar{\boldsymbol{\tau}}, \boldsymbol{d} \rangle + \sum_{i=M,m} \langle \tilde{\boldsymbol{v}}_i, \phi^i, p_i \rangle - \sum_{i=M,m} \langle c^i, p_i, \tilde{\boldsymbol{v}}_i \rangle - \langle (1-\phi^s), \pi, \dot{\psi} \rangle$$

- mechanical constitutive law in terms of effective stress

Theoretical basis

Internal energy equation

$$J\rho\dot{e} = \langle \bar{\boldsymbol{\tau}}, \boldsymbol{d} \rangle + \sum_{i=M,m} \langle \tilde{\boldsymbol{v}}_i, \phi^i, p_i \rangle - \sum_{i=M,m} \langle c^i, p_i, \tilde{\boldsymbol{v}}_i \rangle - \langle (1-\phi^s), \pi, \dot{\psi} \rangle$$

- Darcy's law (or non-Darcy's law) at each pore scale

Theoretical basis

Internal energy equation

$$J\rho\dot{e} = \langle \bar{\boldsymbol{\tau}}, \boldsymbol{d} \rangle + \sum_{i=M,m} \langle \tilde{\boldsymbol{v}}_i, \phi^i, p_i \rangle$$

$$- \sum_{i=M,m} \langle c^i, p_i, \tilde{\boldsymbol{v}}_i \rangle - \langle (1 - \phi^s), \pi, \dot{\psi} \rangle$$

- mass transfer constitutive law (Gerke & Van Genuchten 1993)

Theoretical basis

Internal energy equation

$$J\rho\dot{e} = \langle \bar{\boldsymbol{\tau}}, \boldsymbol{d} \rangle + \sum_{i=M,m} \langle \tilde{\boldsymbol{v}}_i, \phi^i, p_i \rangle$$

$$\pi = p_M - p_m$$

$$- \sum_{i=M,m} \langle c^i, p_i, \tilde{\boldsymbol{v}}_i \rangle - \boxed{\langle (1-\phi^s), \pi, \dot{\psi} \rangle}$$

- compressibility laws determine the evolution of the micropore fraction

Solid-water mixture

Effective stress with B=1

$$\bar{\boldsymbol{\tau}} = \boldsymbol{\tau} + \bar{p}\mathbf{1}$$

$$\bar{p} = \psi p_m + (1 - \psi) p_M$$

Double porosity – fully saturated

Effective stress with B=1

$$\bar{\boldsymbol{\tau}} = \boldsymbol{\tau} + \bar{p}\mathbf{1}$$

$$\bar{p} = \psi p_m + (1 - \psi) p_M$$

$$\boldsymbol{\tau} = \psi \boldsymbol{\tau} + (1 - \psi) \boldsymbol{\tau}$$

Double porosity – fully saturated

Effective stress with *B*=1

$$\bar{\boldsymbol{\tau}} = \boldsymbol{\tau} + \bar{p}\mathbf{1}$$

$$\bar{p} = \psi p_m + (1-\psi) p_M$$

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$$\bar{\boldsymbol{\tau}} = \psi \bar{\boldsymbol{\tau}}_m + (1-\psi)\bar{\boldsymbol{\tau}}_M$$

$$\bar{\boldsymbol{\tau}}_m = \boldsymbol{\tau} + p_m\mathbf{1}\,, \qquad \bar{\boldsymbol{\tau}}_M = \boldsymbol{\tau} + p_M\mathbf{1}$$

Double porosity – fully saturated

Effective stress with *B*=1

$$\bar{\boldsymbol{\tau}} = \boldsymbol{\tau} + \bar{p}\mathbf{1}$$

$$\bar{p} = \psi p_m + (1-\psi) p_M$$

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$$\bar{\boldsymbol{\tau}} = \psi \bar{\boldsymbol{\tau}}_m + (1-\psi)\bar{\boldsymbol{\tau}}_M$$

$$\bar{\boldsymbol{\tau}}_m = \boldsymbol{\tau} + p_m \mathbf{1}\,, \qquad \bar{\boldsymbol{\tau}}_M = \boldsymbol{\tau} + p_M \mathbf{1}$$

- The effective stress in a double-porosity medium is the weighted sum of the single-porosity effective stresses with the pore fractions taken as the weights.

Double porosity – fully saturated

Preconsolidation pressures

$$\pi = p_M - p_m$$

$$\pi = \bar{\tau}_M - \bar{\tau}_m$$

Double porosity – fully saturated

Preconsolidation pressures

$$\pi = p_M - p_m$$

$$\pi = \bar{\tau}_M - \bar{\tau}_m$$

$$\pi = p_{cM} - p_{cm}$$

$$\bar{p}_c = \psi p_{cm} + (1 - \psi) p_{cM}$$

Double porosity – fully saturated

Preconsolidation pressures

$$\pi = p_M - p_m$$

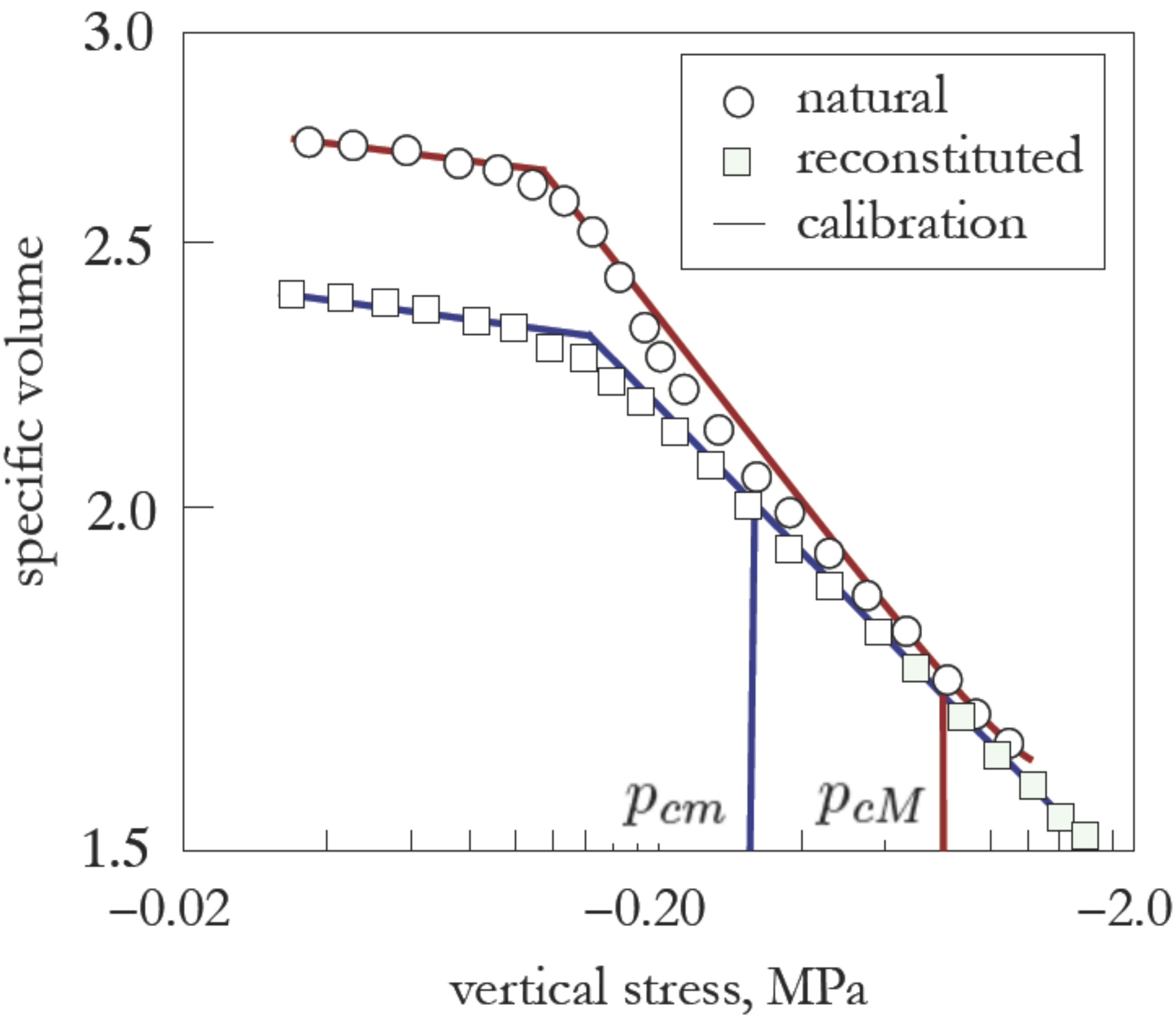
$$\pi = \bar{\tau}_M - \bar{\tau}_m$$

$$\pi = p_{cM} - p_{cm}$$

$$\bar{p}_c = \psi p_{cm} + (1 - \psi) p_{cM}$$

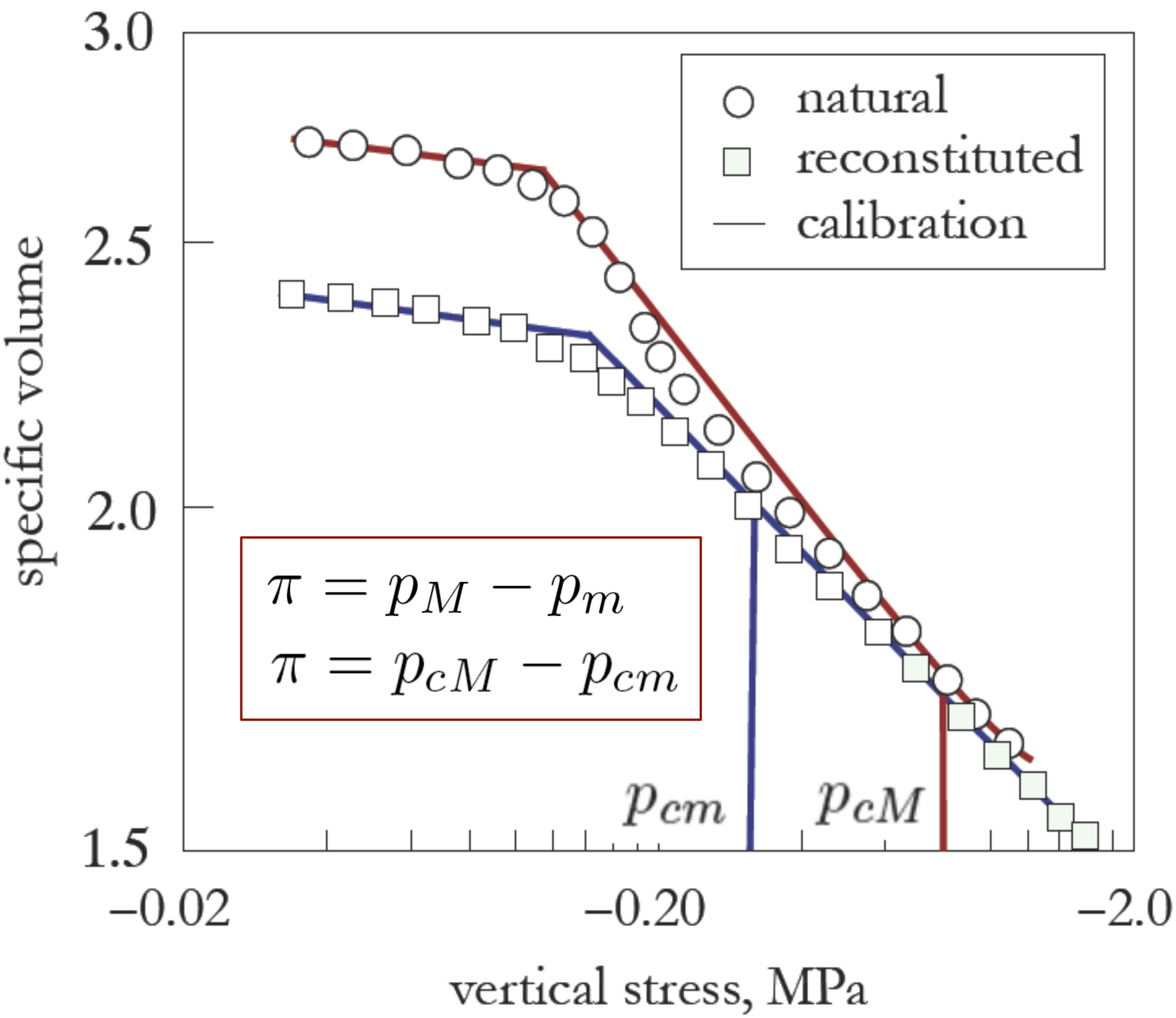
- The preconsolidation stress for a double-porosity medium is the weighted sum of single-porosity preconsolidation stresses with the pore fractions taken as the weights.

Pancone clay



Ref: Callisto & Calabresi (1998)

Pancone clay



Ref: Callisto & Calabresi (1998)

Double porosity

- Specific volumes

$$v = \frac{1}{\phi^s} \qquad v_m = 1 + \psi \frac{1-\phi^s}{\phi^s}$$

$$e_M = v - v_m$$

- Compressibility laws

$$\frac{\dot{v}_m}{v_m} = -c_c \frac{\dot{p}_{cm}}{p_{cm}} \qquad \dot{\varepsilon}_v^e = -c_r \frac{\dot{p}_{cm}}{p_{cm}}$$

$$\frac{\dot{e}_M}{e_M} = -c_M \frac{\dot{p}_{cM}}{p_{cM}}$$

Conservation laws

Balance of linear momentum

$$\mathrm{DIV}(\boldsymbol{P}) + \rho_0 \boldsymbol{G} = c_0(\tilde{\boldsymbol{v}}_m - \tilde{\boldsymbol{v}}_M)$$

Balance of fluid mass

$$\dot{\rho}_0^M + \mathrm{DIV}(\boldsymbol{Q}_M) = -c_0$$
$$\dot{\rho}_0^m + \mathrm{DIV}(\boldsymbol{Q}_m) = c_0$$

- subject to appropriate boundary and initial conditions

Constitutive laws – 4 sets

- Solid phase constitutive law

$$\bar{\boldsymbol{\tau}} = \bar{\boldsymbol{\tau}}(\boldsymbol{u}, p_M, p_m)$$

- Darcy's law

$$J\tilde{\boldsymbol{v}}_M = -\boldsymbol{K}_M \cdot \boldsymbol{\nabla}\mathcal{U}_M\,, \qquad J\tilde{\boldsymbol{v}}_m = -\boldsymbol{K}_m \cdot \boldsymbol{\nabla}\mathcal{U}_m$$

- Fluid mass transfer law

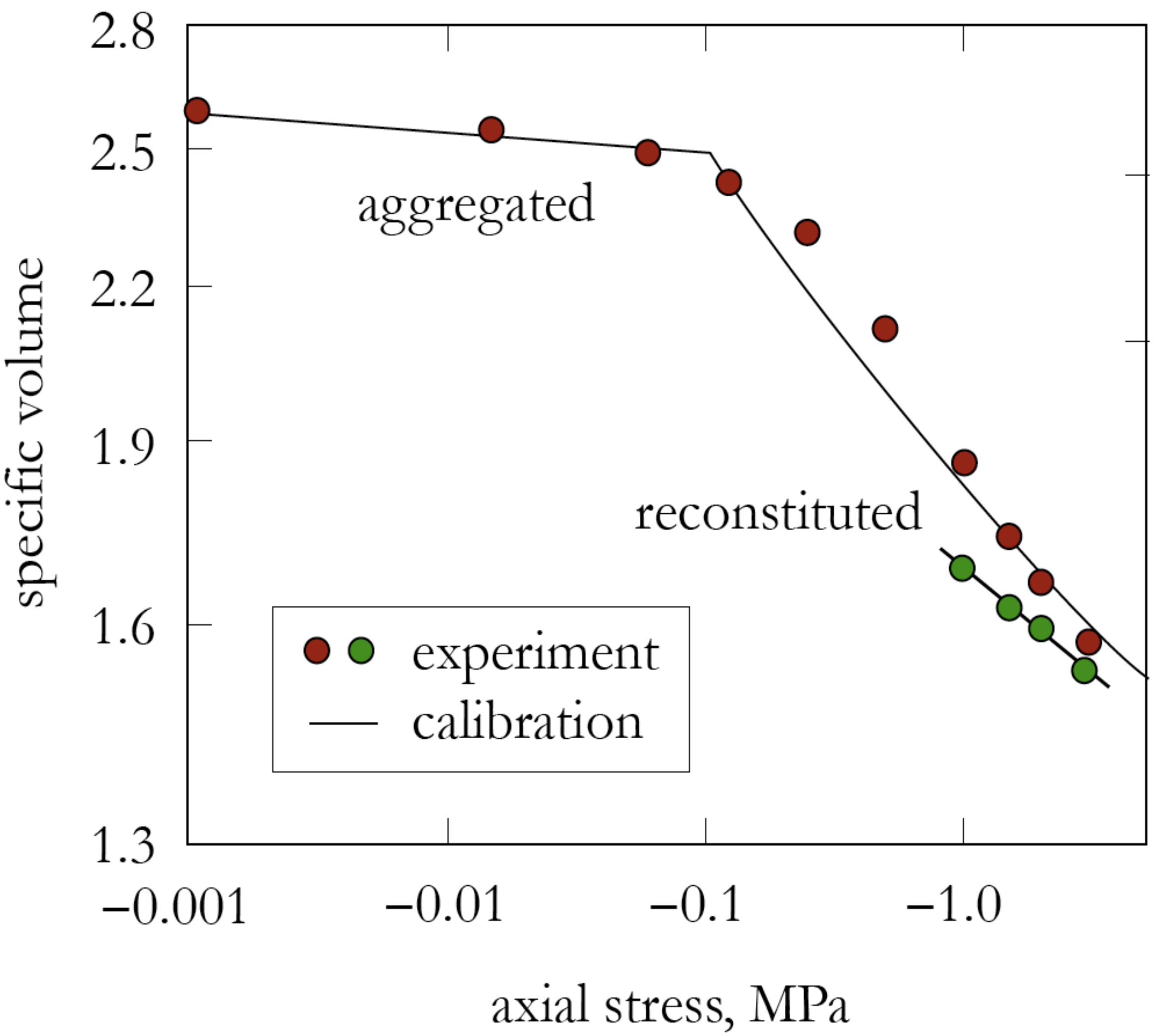
$$c = \frac{\bar{\alpha}}{\mu_w}(p_M - p_m)$$

- Compressibility-pressure jump law

$$\pi = p_{cM} - p_{cm}$$

Double porosity

Bioley silt (Koliji, Vulliet & Laloui 2008)



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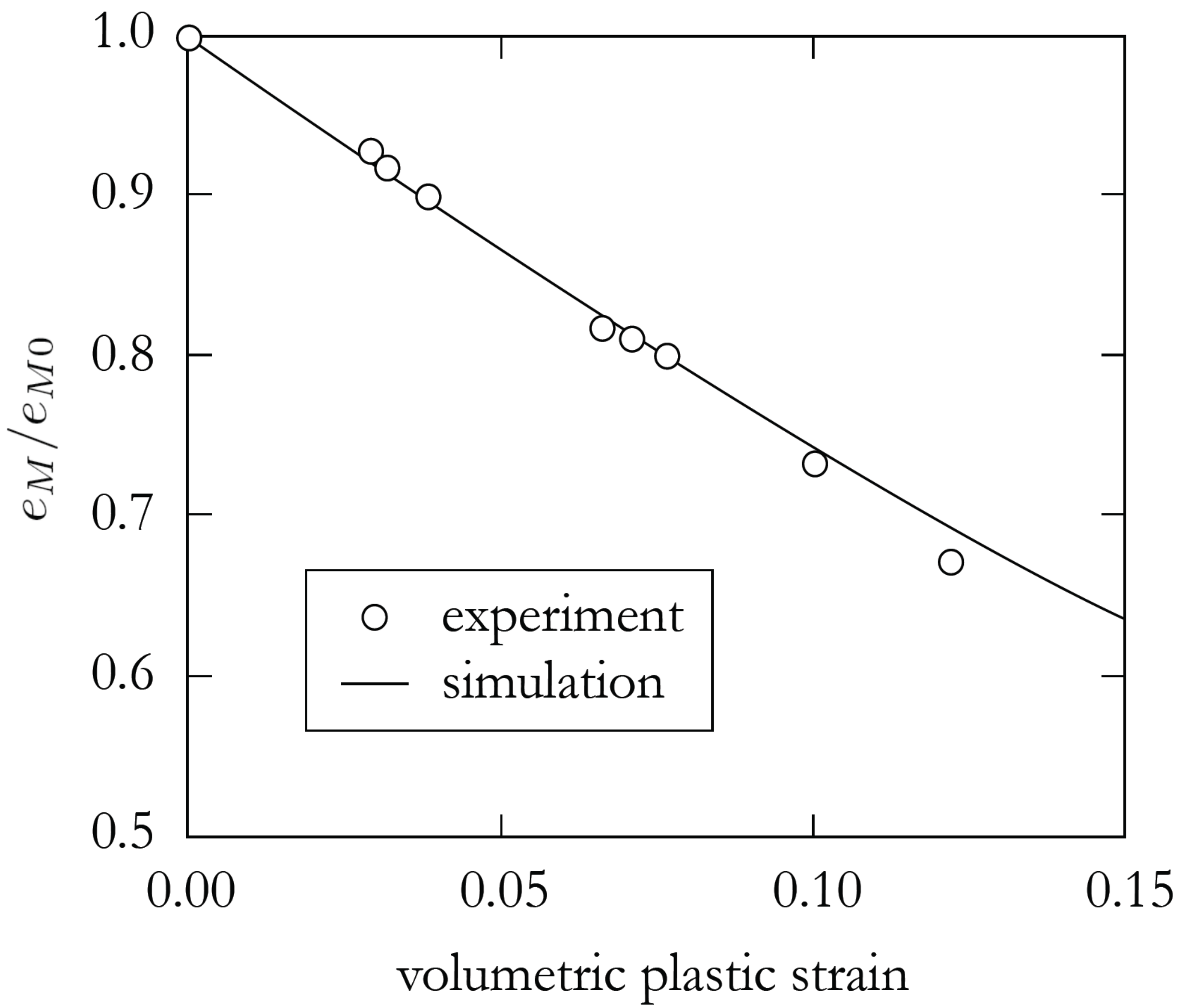
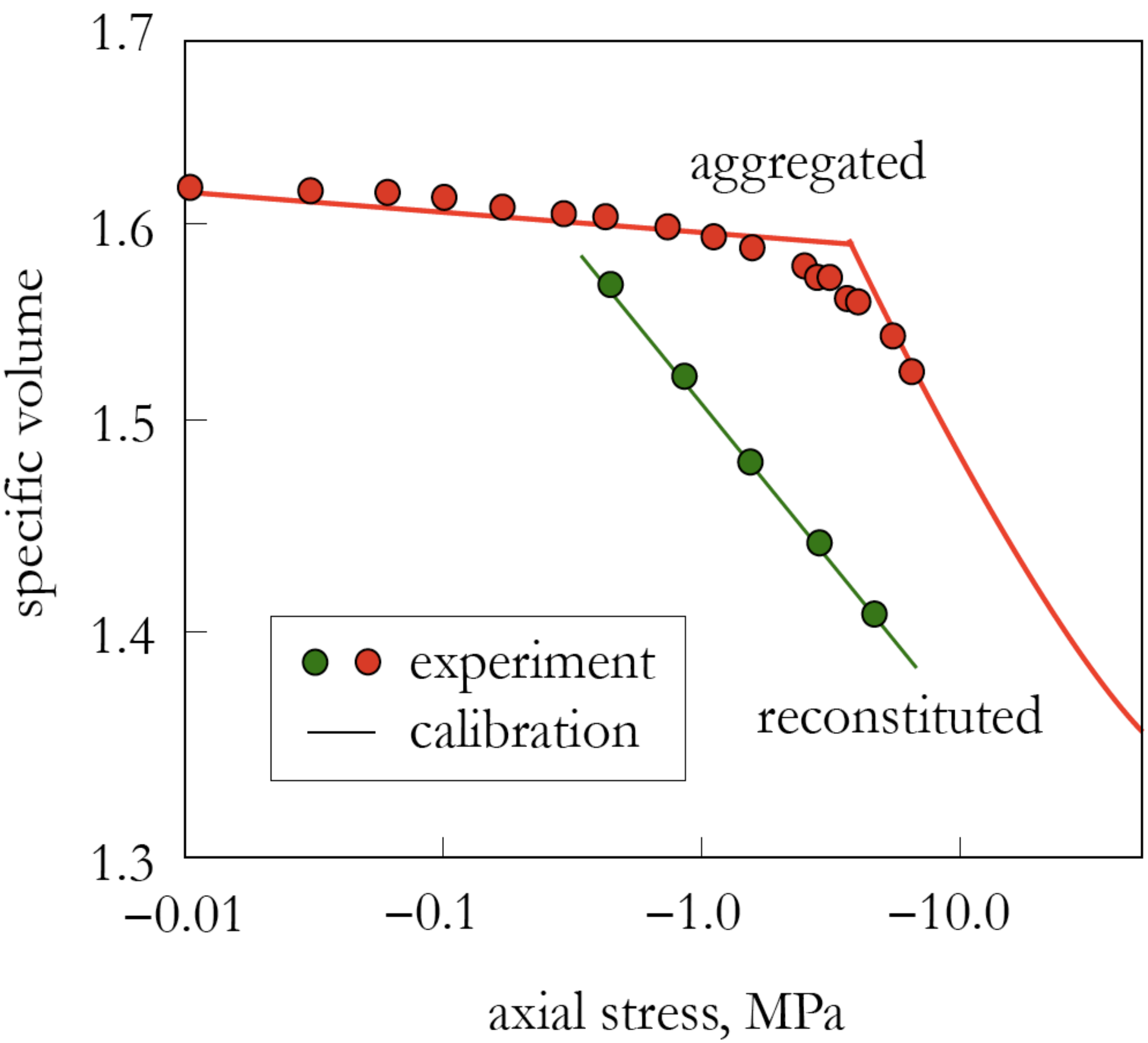


Corinth marl (Anagnostopolous et al. 1991)



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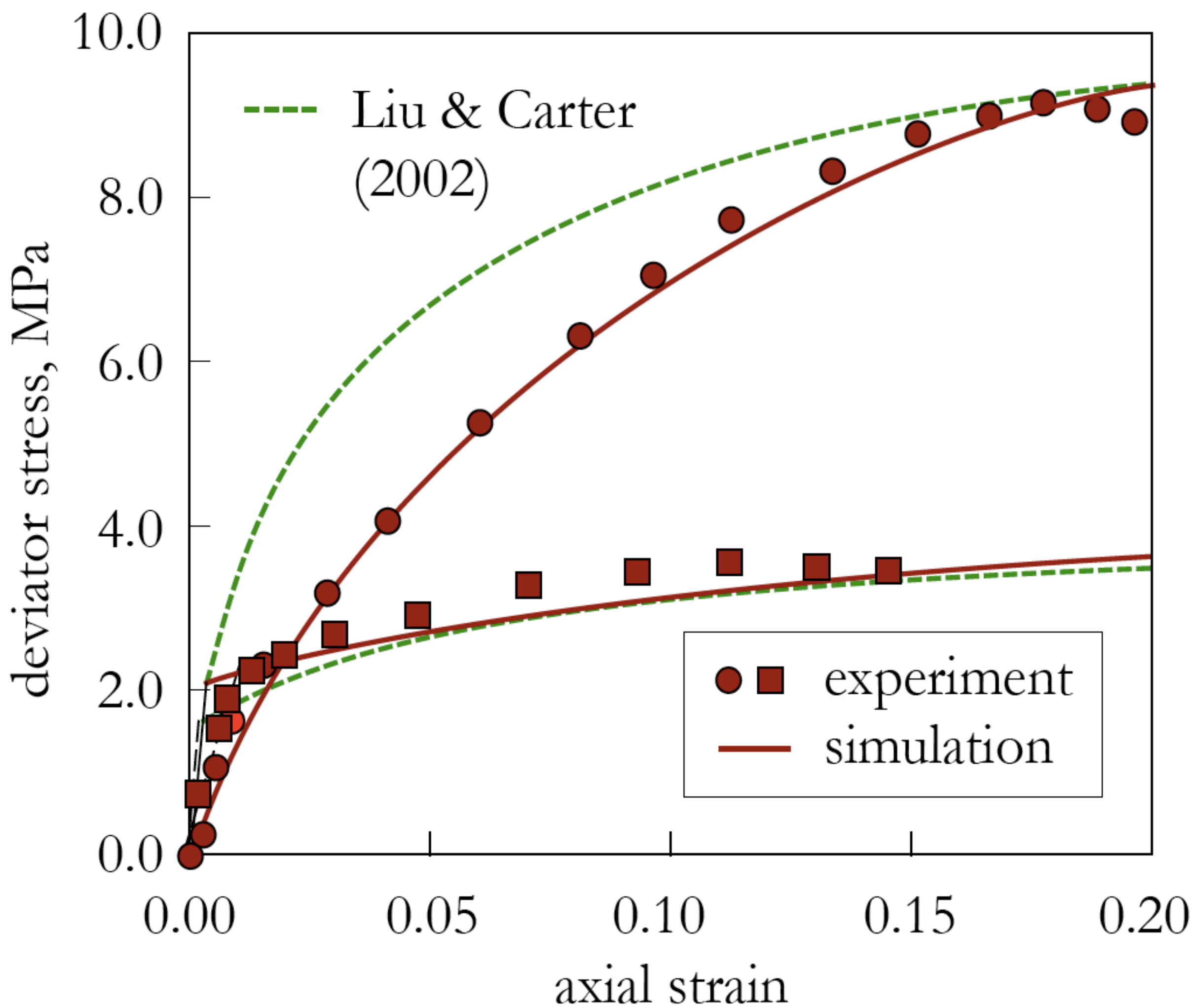

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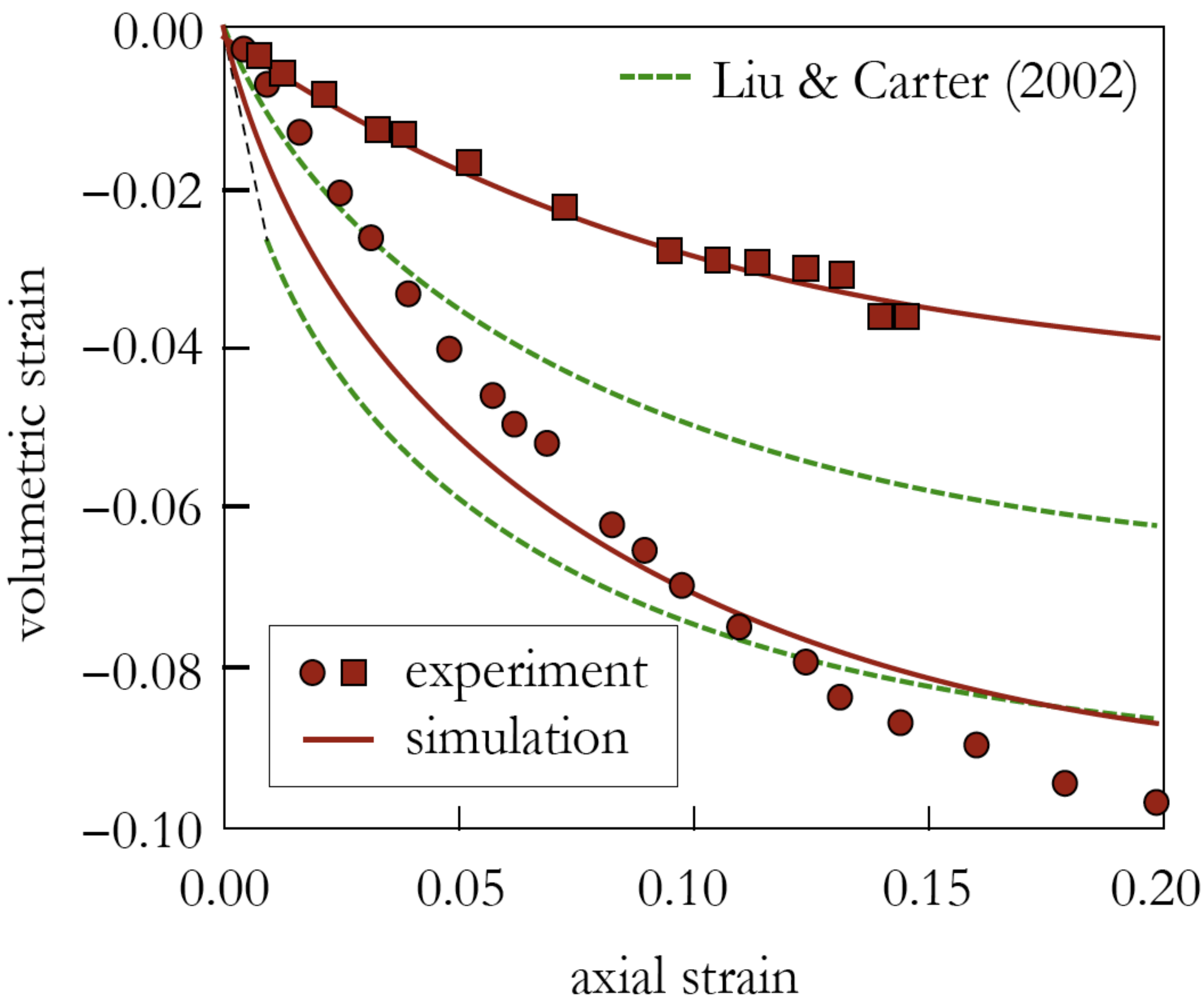

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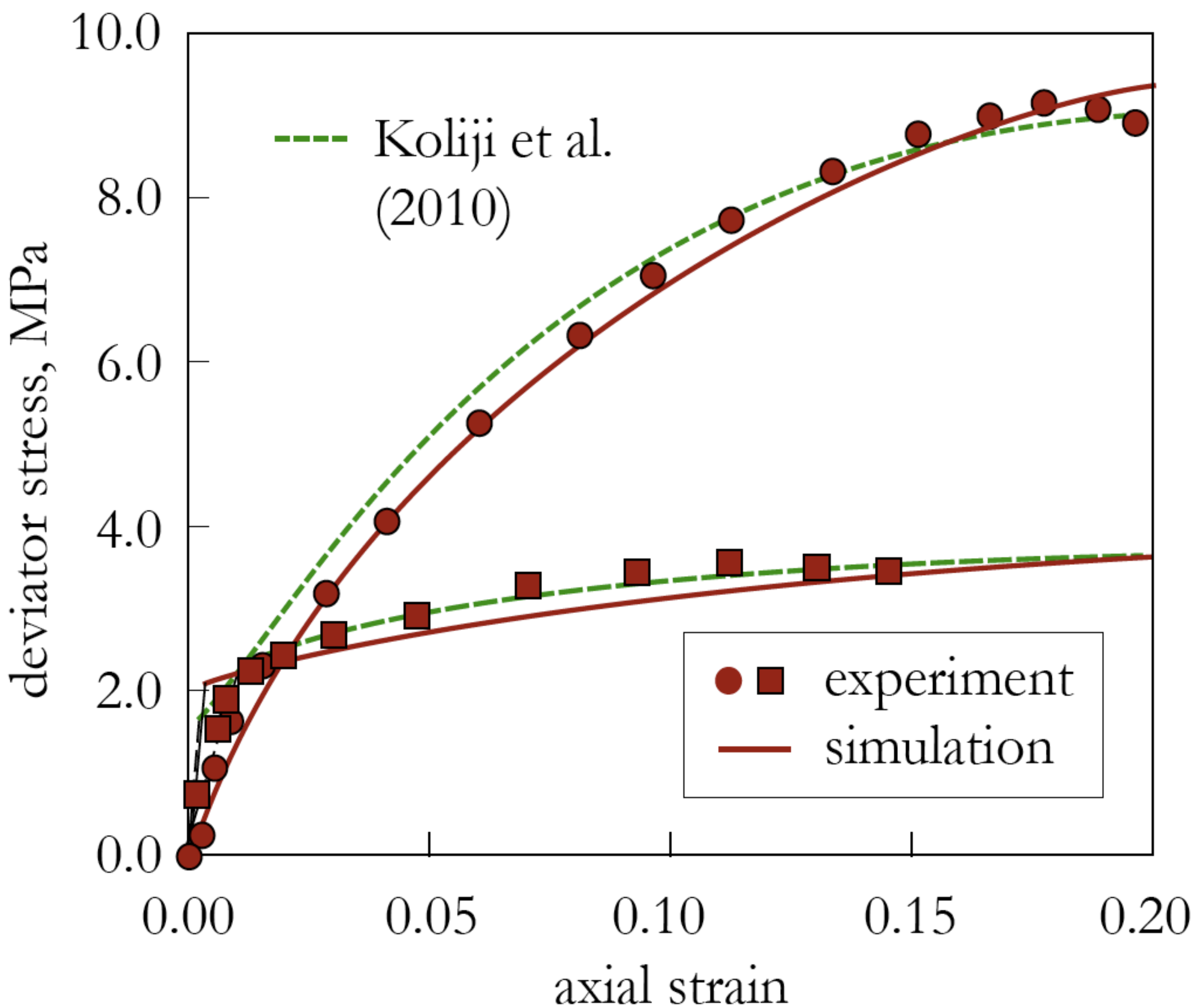

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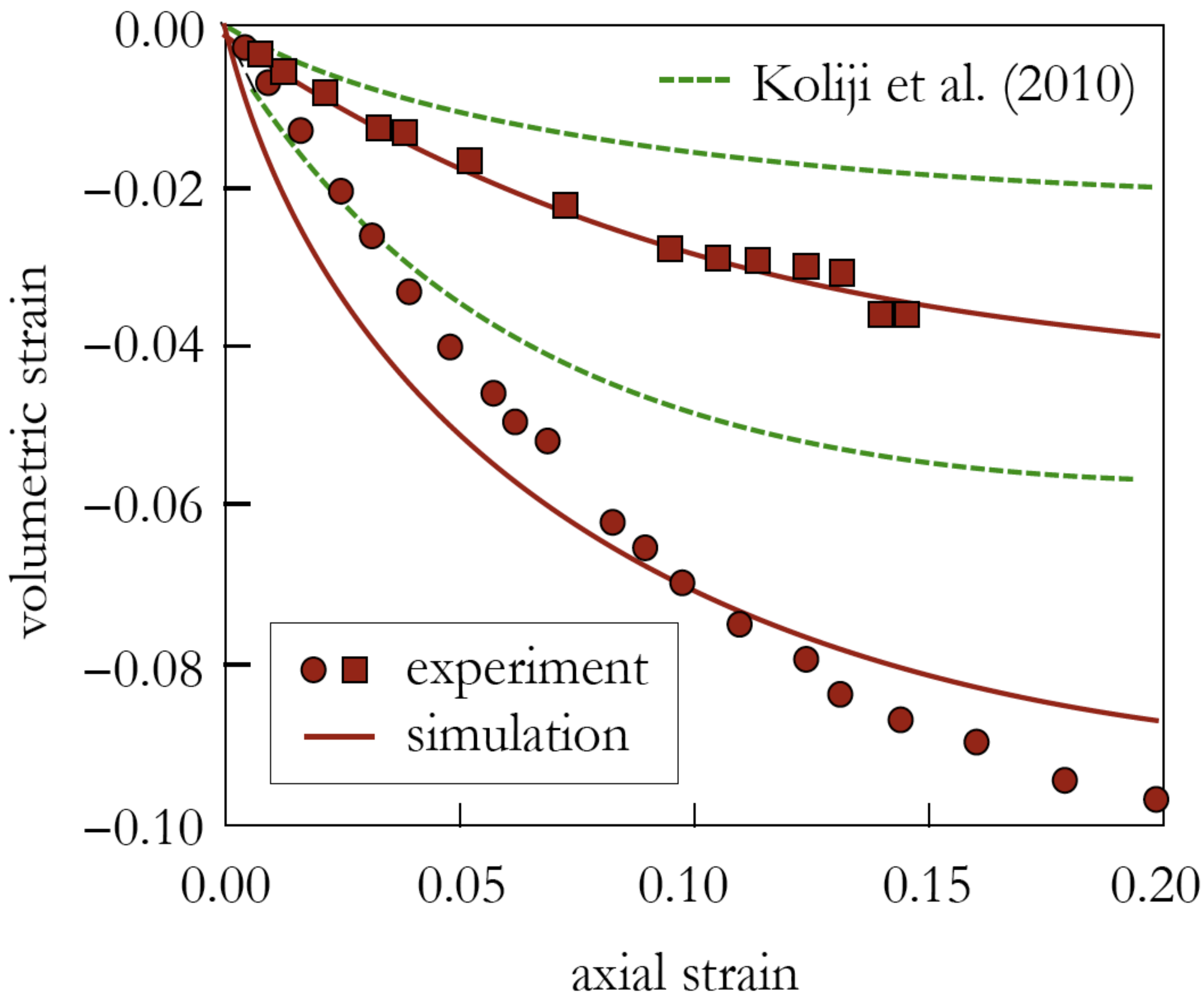
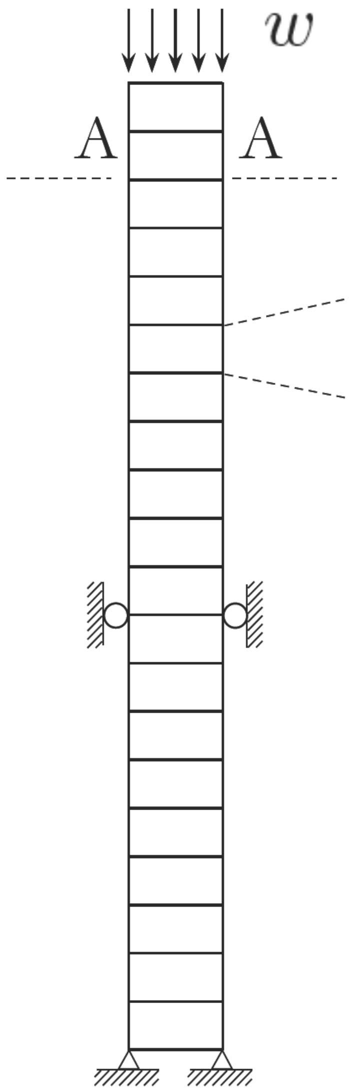
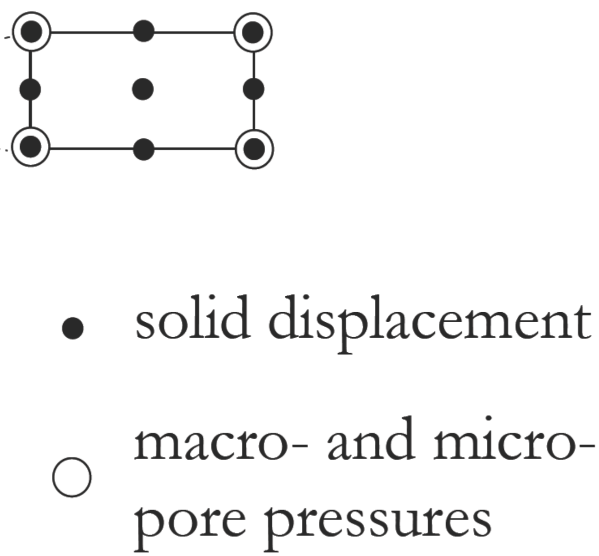
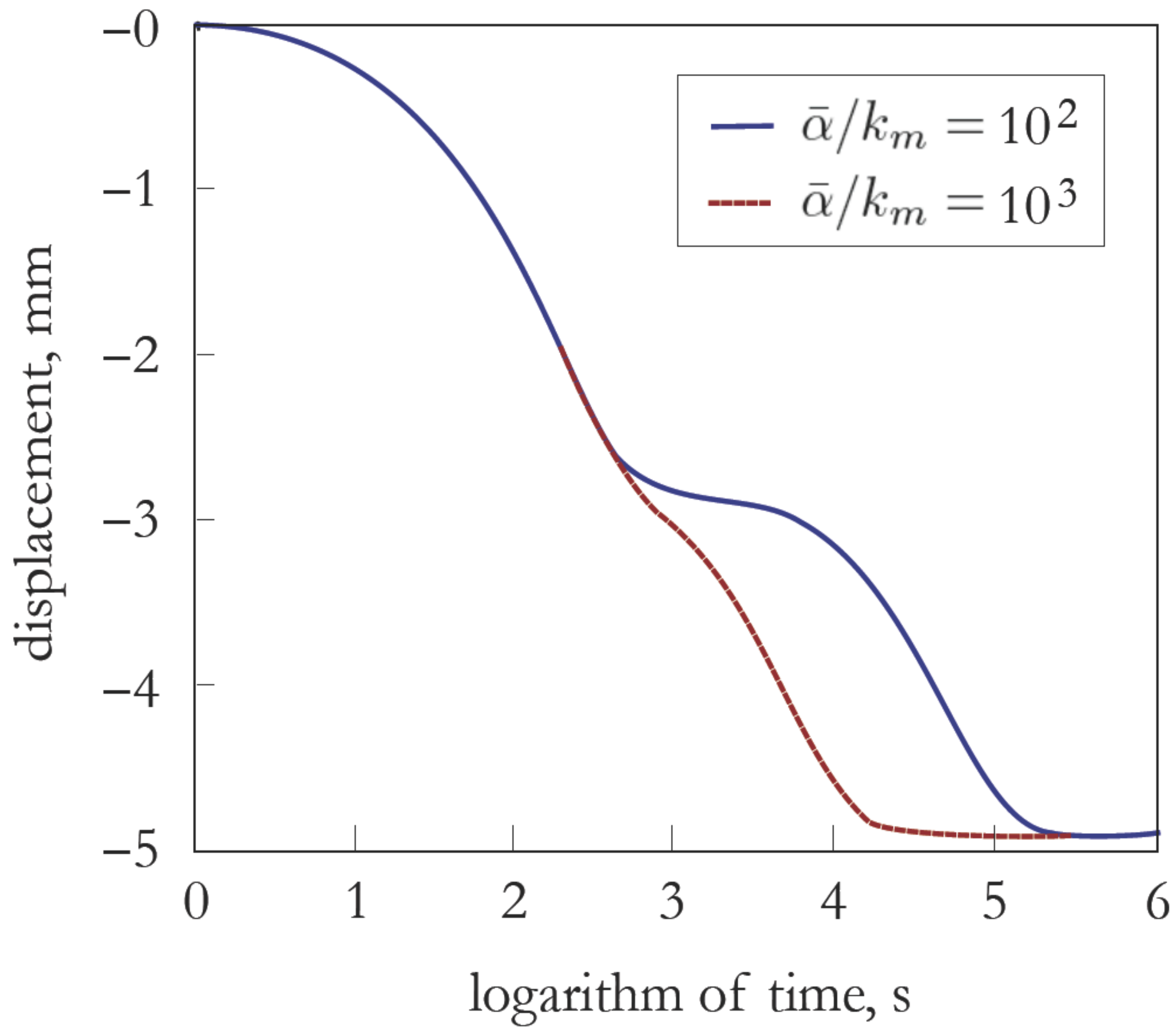


1D consolidation

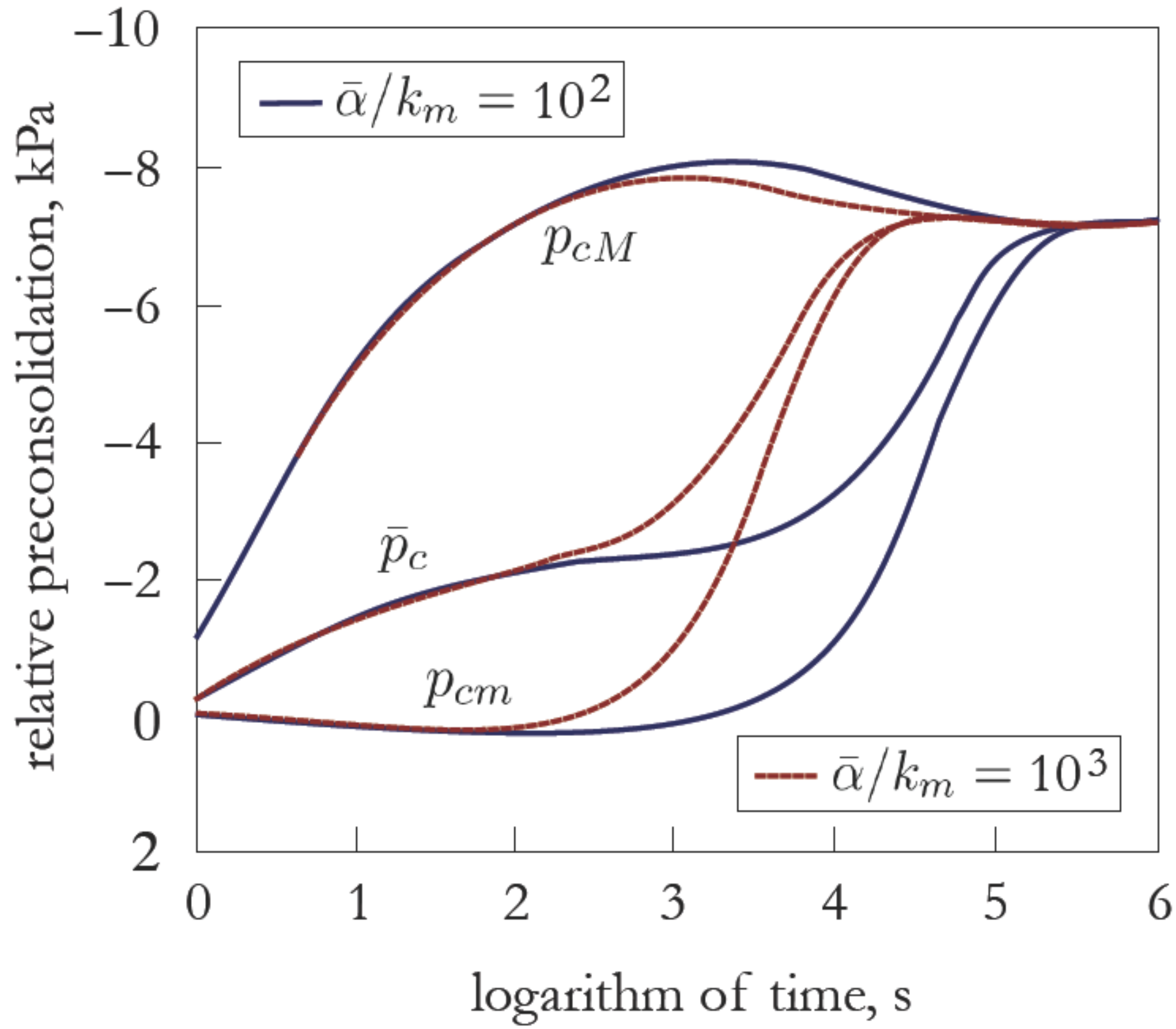




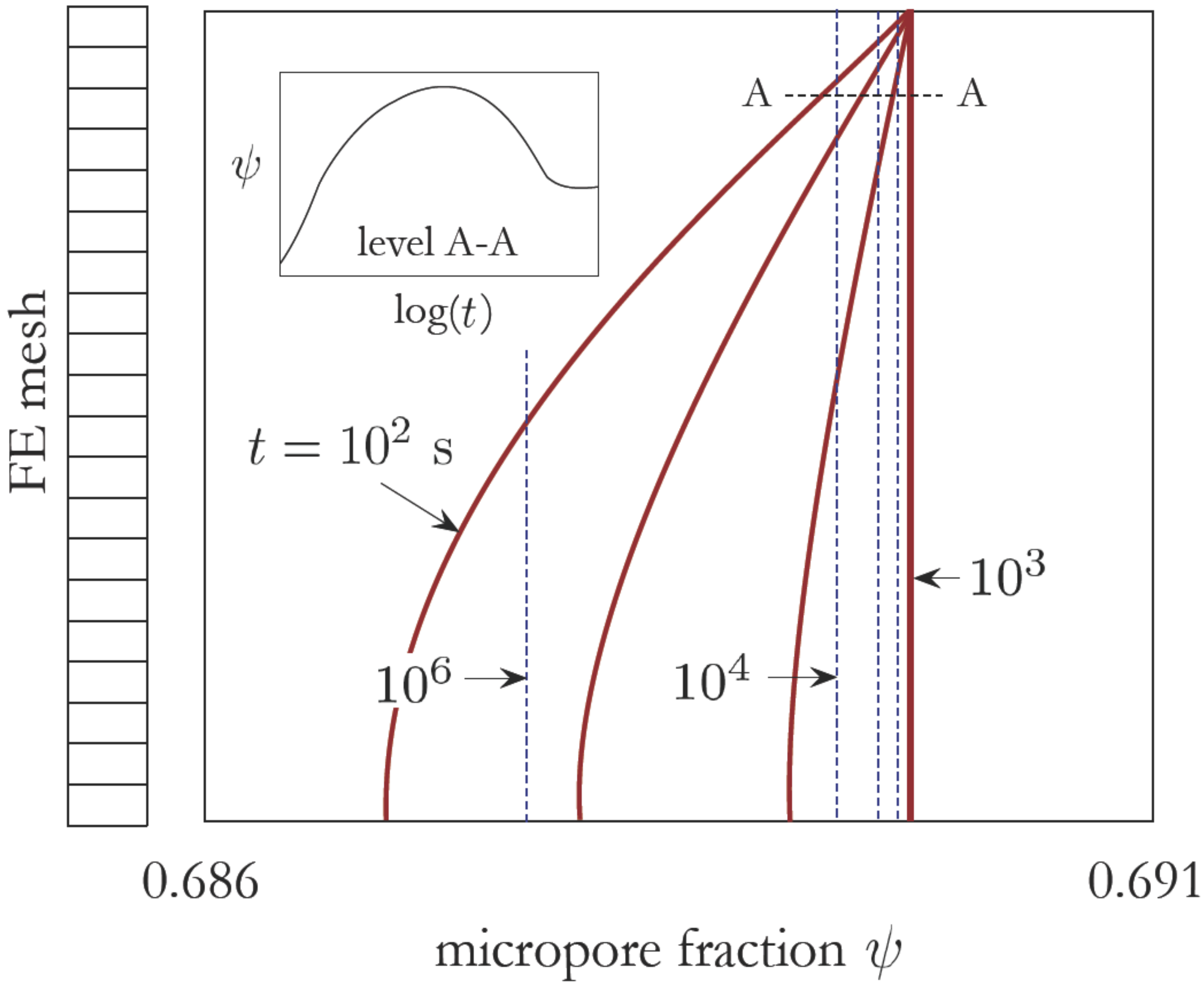
Bioley silt simulation – displacement at A-A



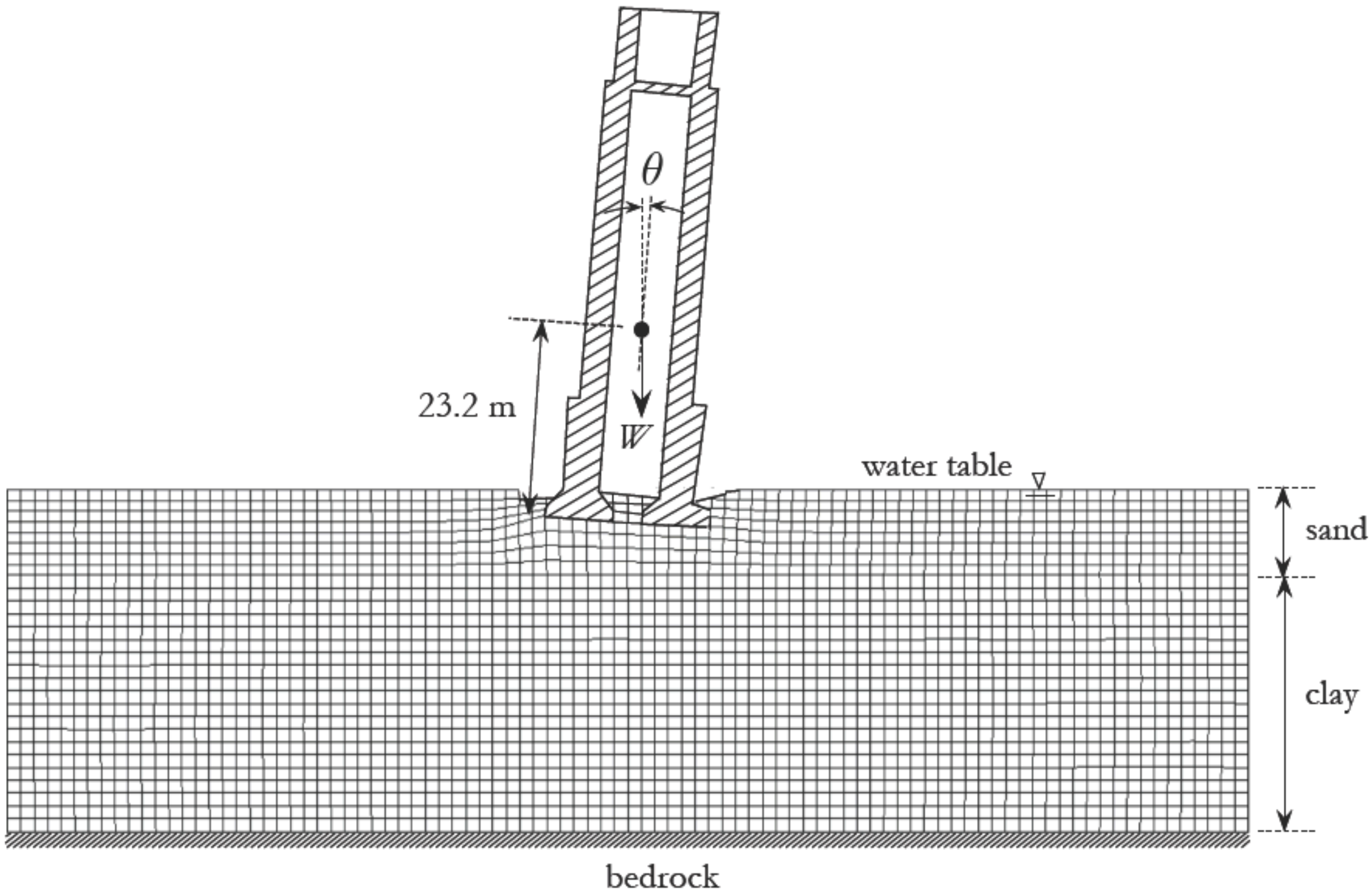
Evolution of preconsolidation pressures at A-A



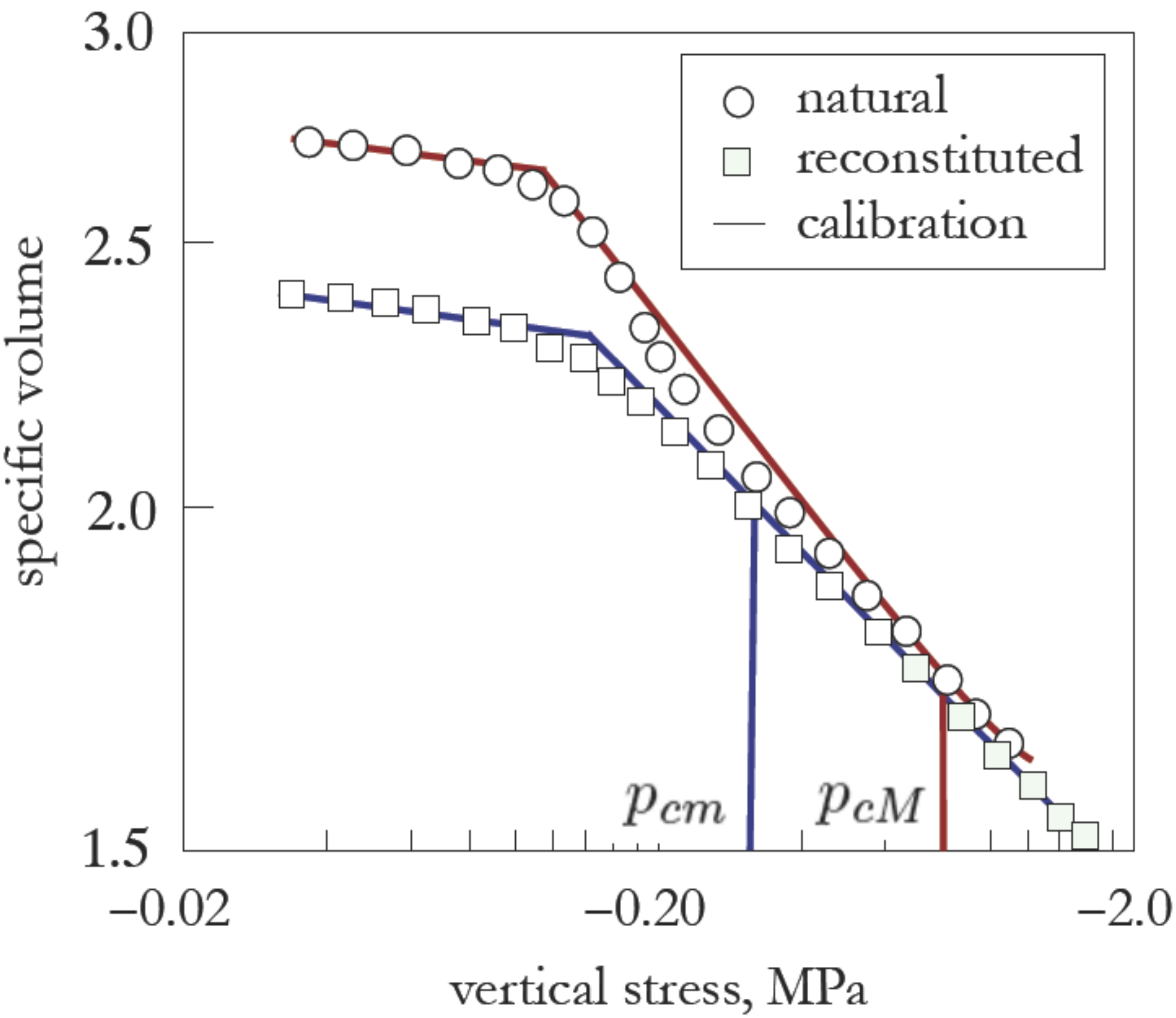
Evolution of micropore fraction



Leaning tower



Pancone clay



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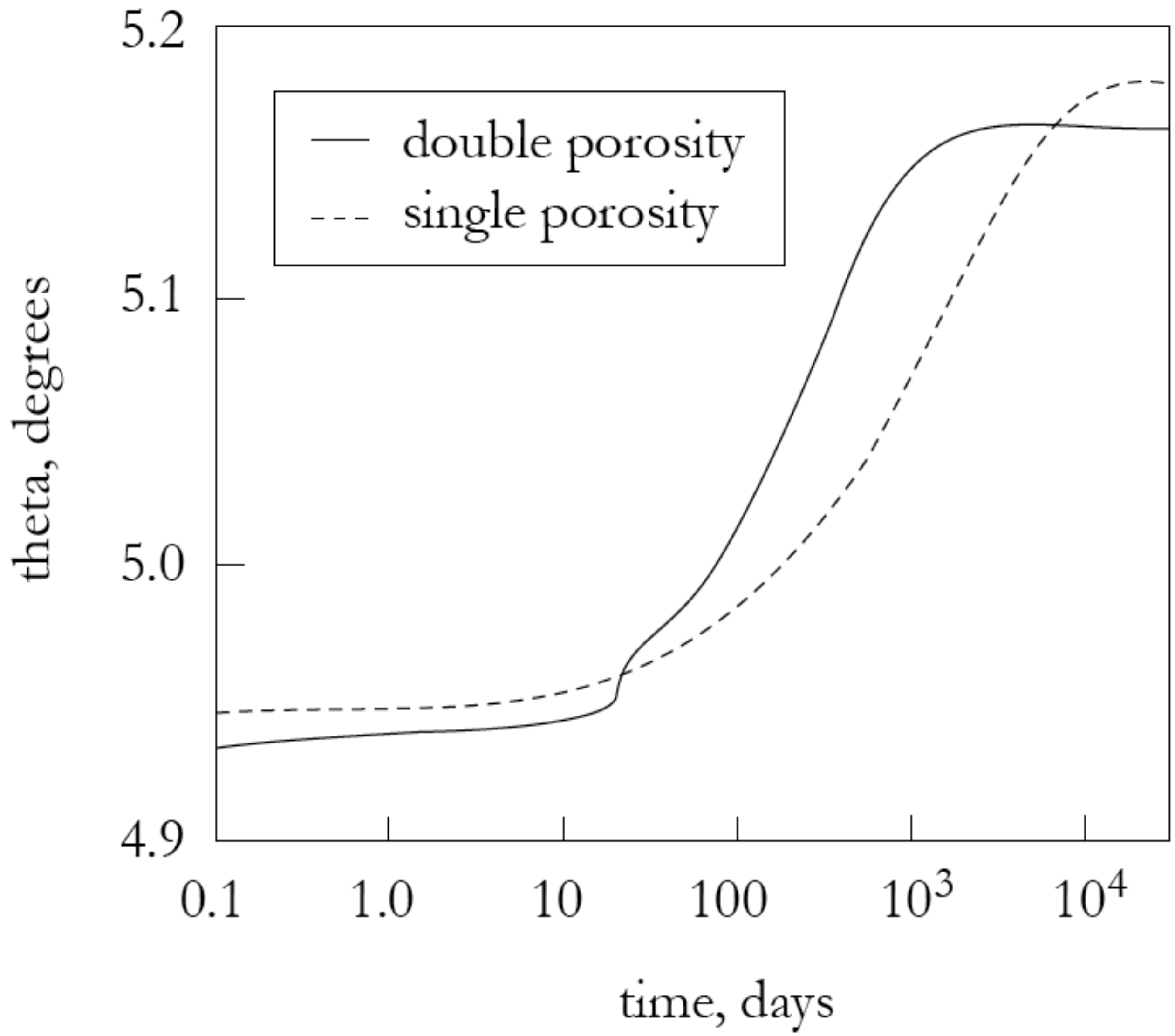
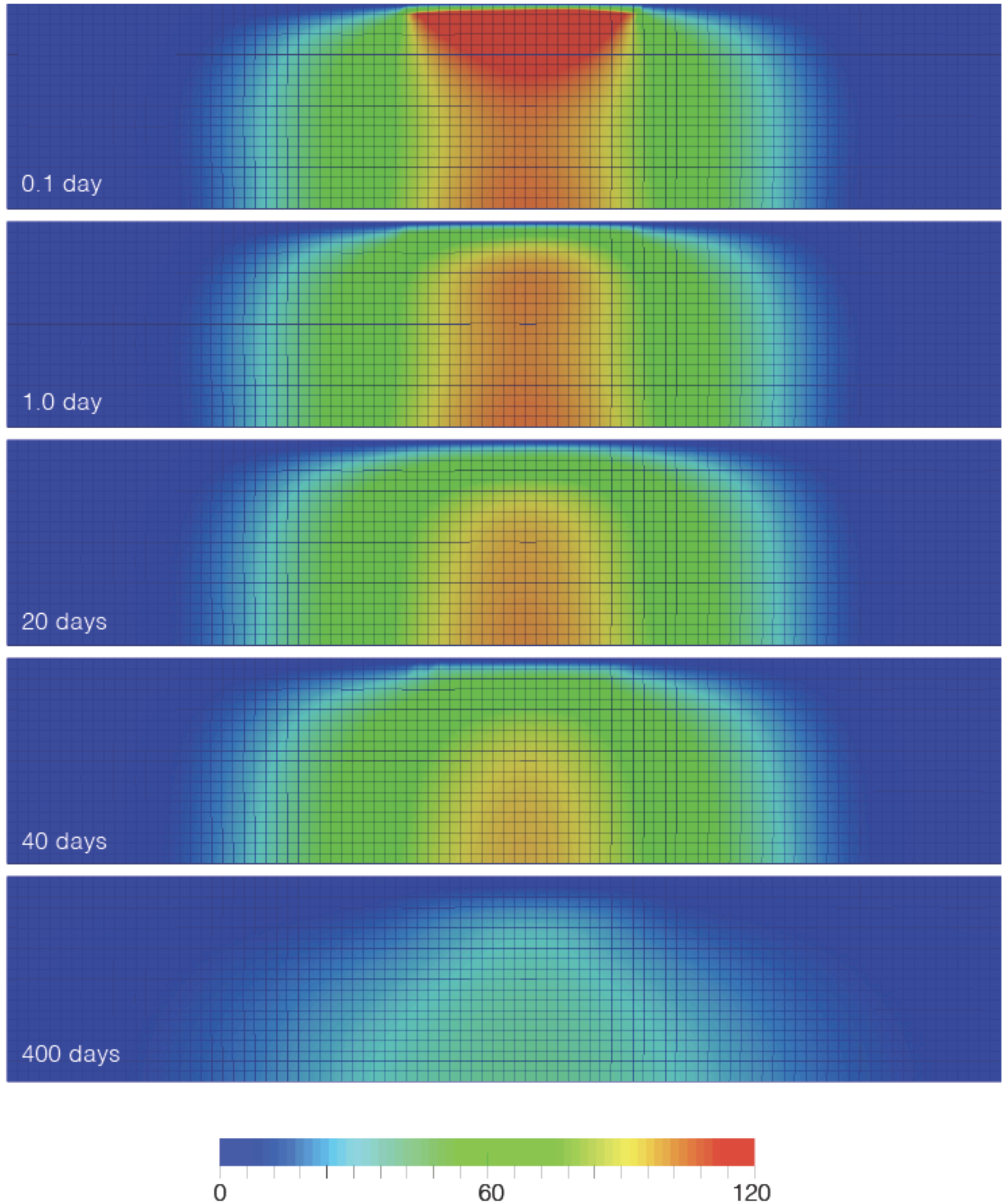


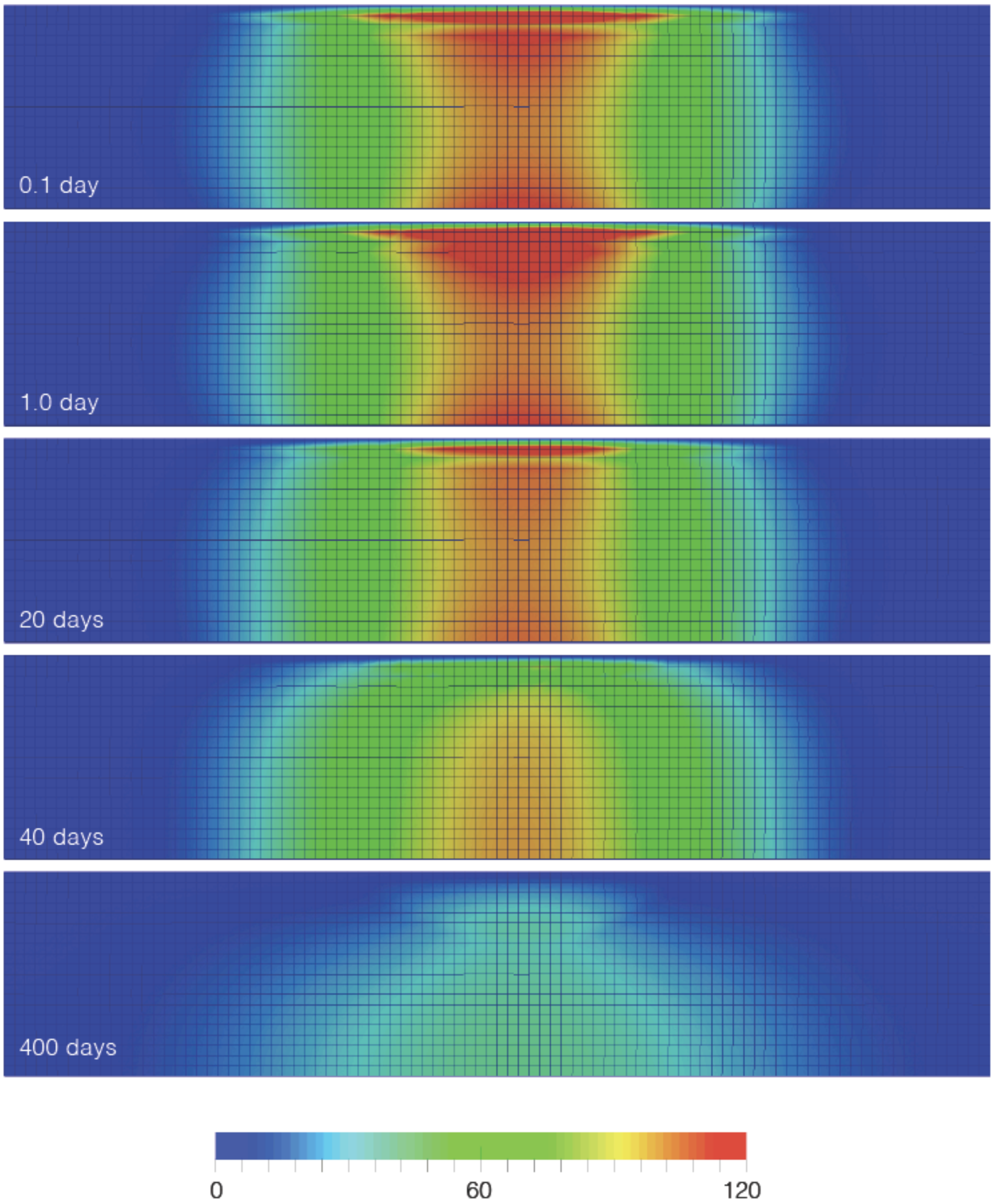
Leaning tower

Evolution of macropore pressures in Pancone clay



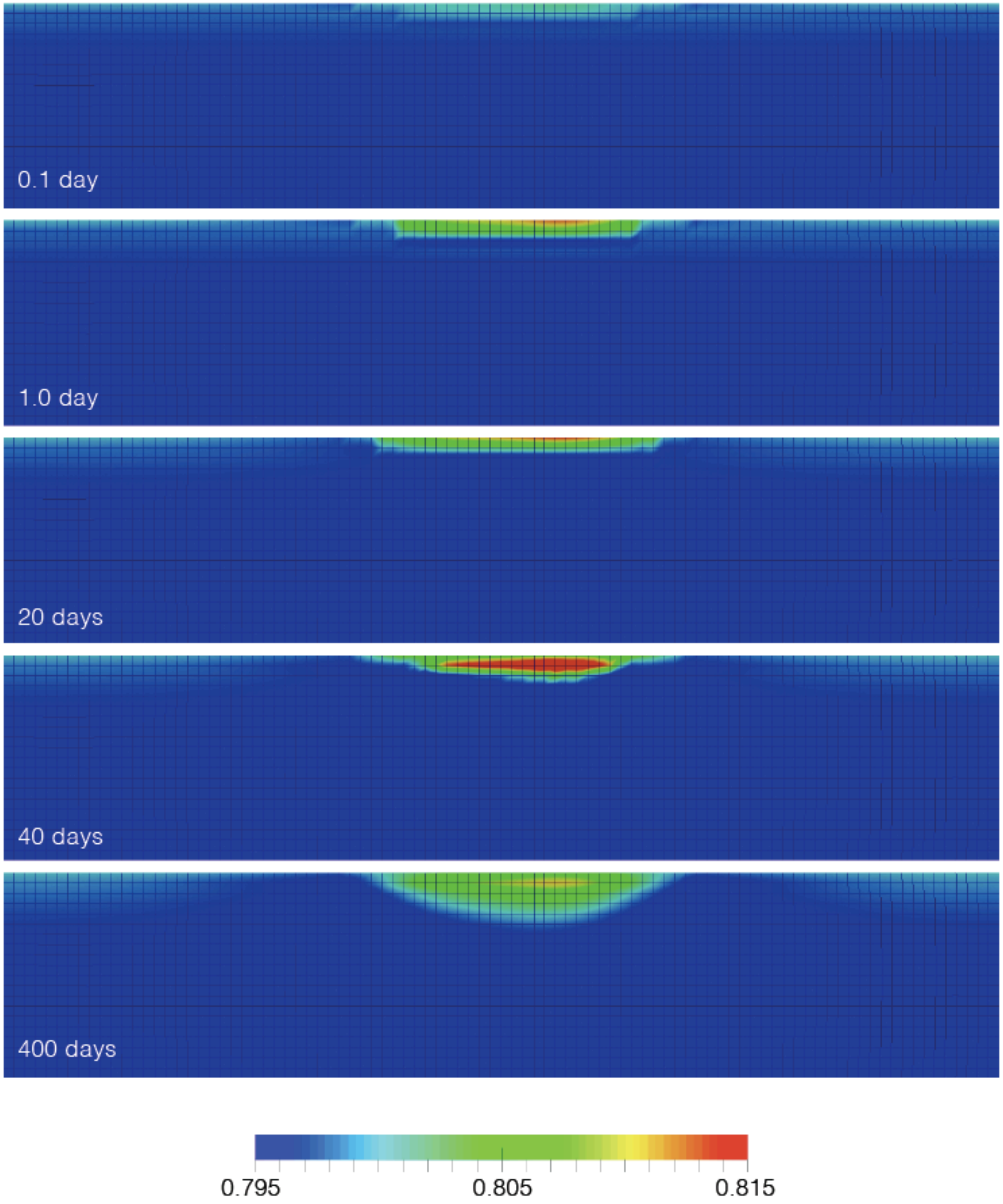
Leaning tower

Evolution of micropore pressures in Pancone clay



Leaning tower

Evolution of micropore fraction in Pancone clay



Summary

- The (new) double porosity formulation is motivated by continuum principles of thermodynamics
- The (new) finite element formulation can track the evolution of internal structure (micropore fraction)
- Numerical predictions agree well with experimental responses of aggregated soils

Acknowledgments

Sponsors

- US Department of Energy (DOE)
- US National Science Foundation (NSF)

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Thank You!

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