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Modern Trends in Geomechanics
Assisi, May 16 -18 2016

Continuum modeling of delayed breakage: exploring the role of grain-scale fracture kinetics

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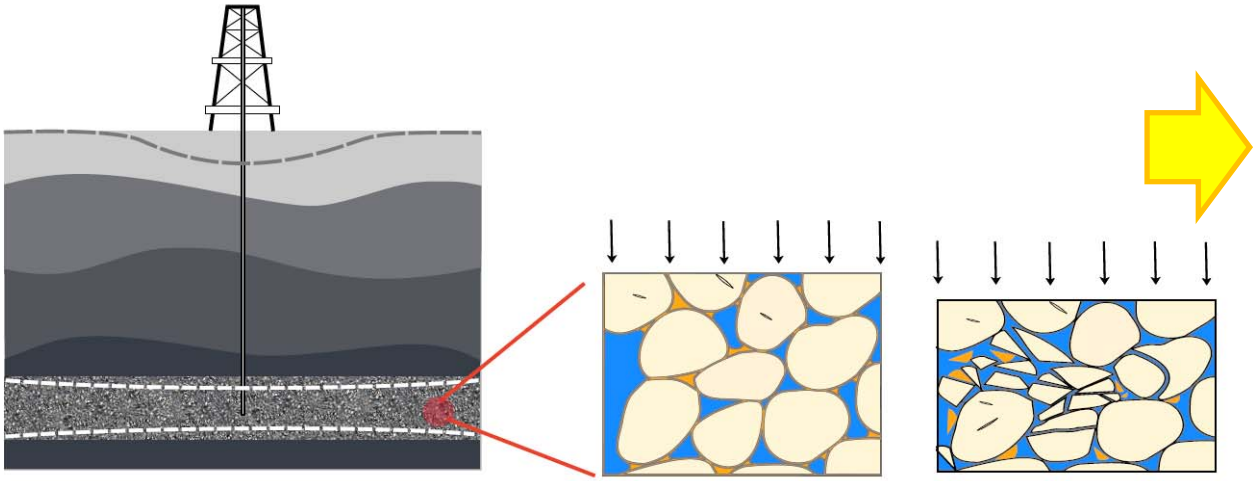
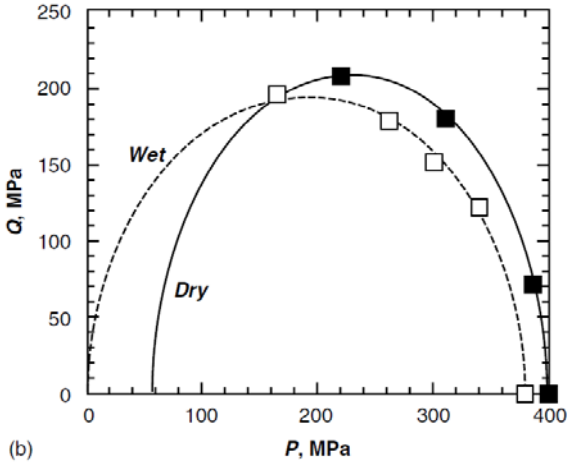
NORTHWESTERN UNIVERSITY

Crushing-Induced Compaction and Settlement

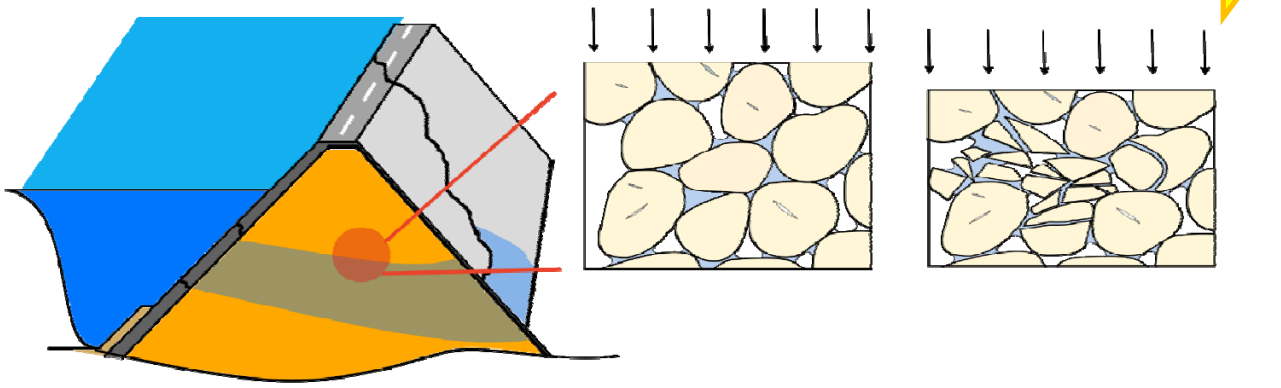
Depletion-Induced Subsidence

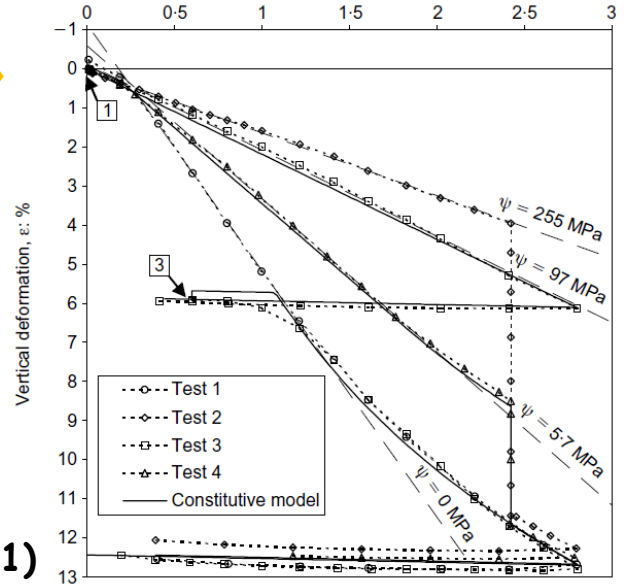




Wong et al. (2003)

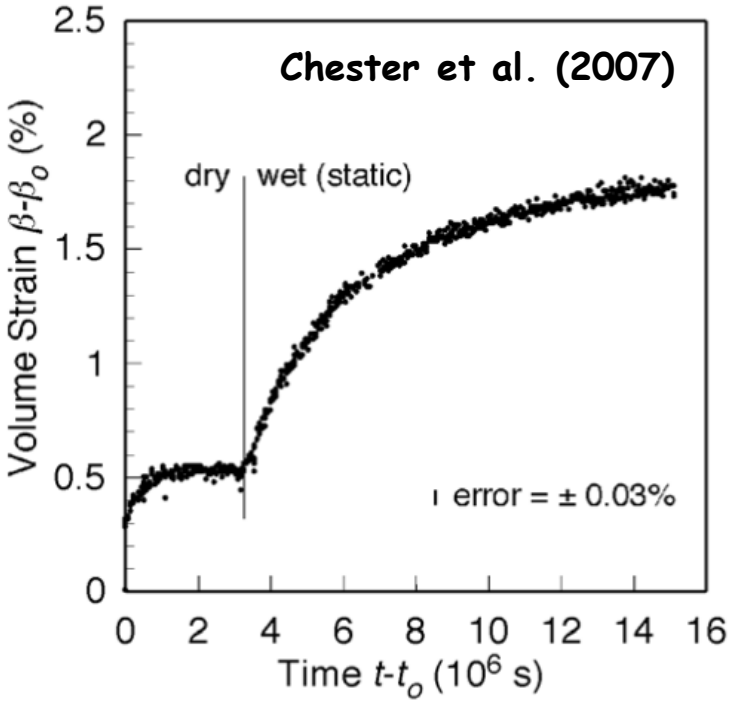
Settlements in Rockfill Dams

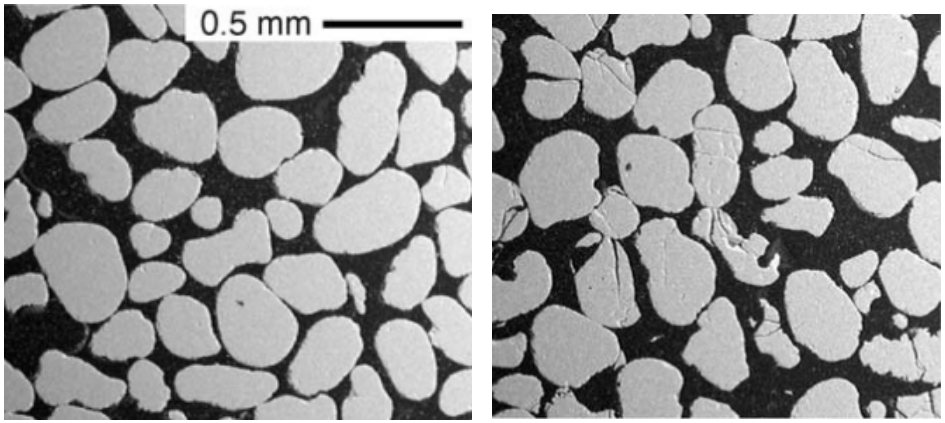


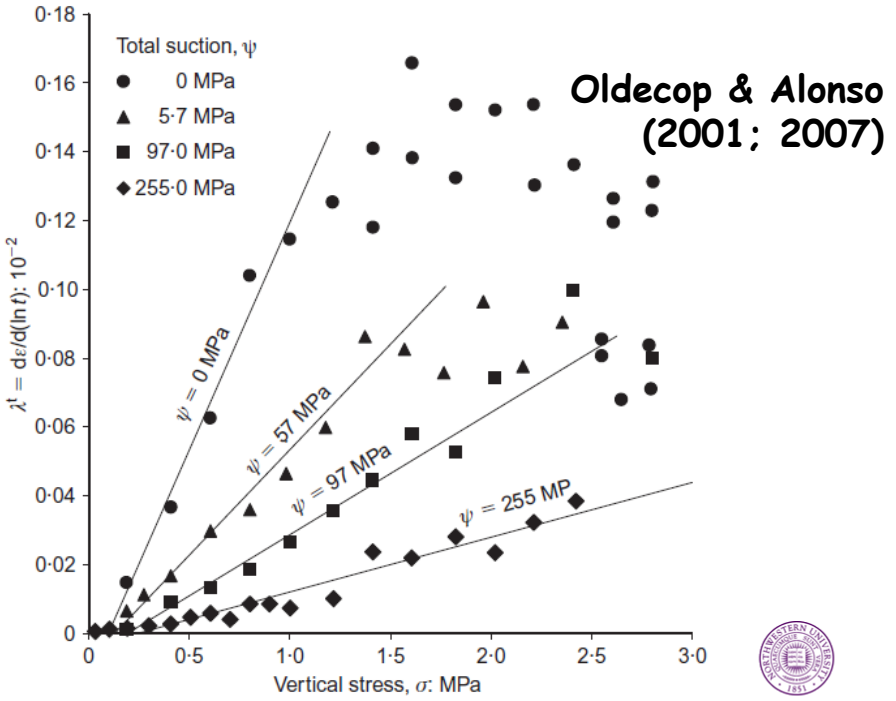


Oldecop & Alonso (2001)

Delayed Crushing and Moisture Effects

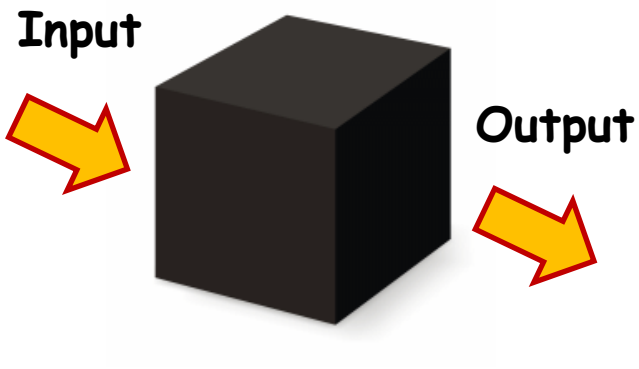




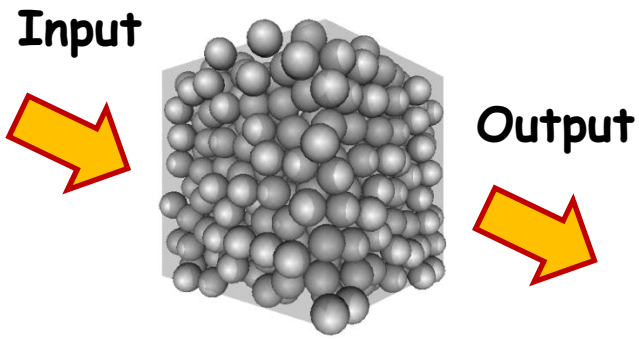


Delayed Crushing and Moisture Effects

Black-Box Modeling



Microstructure-Inspired Modeling

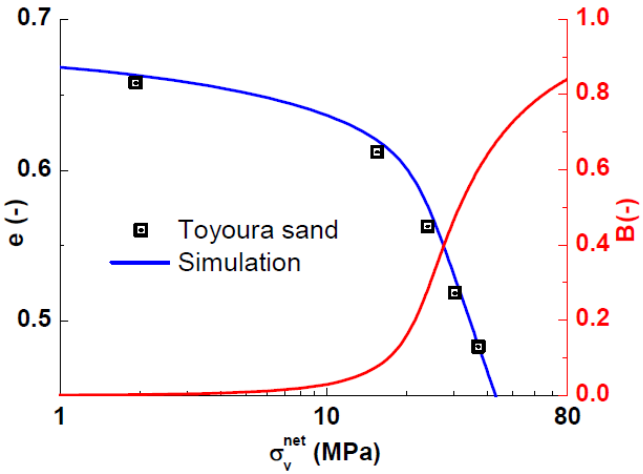


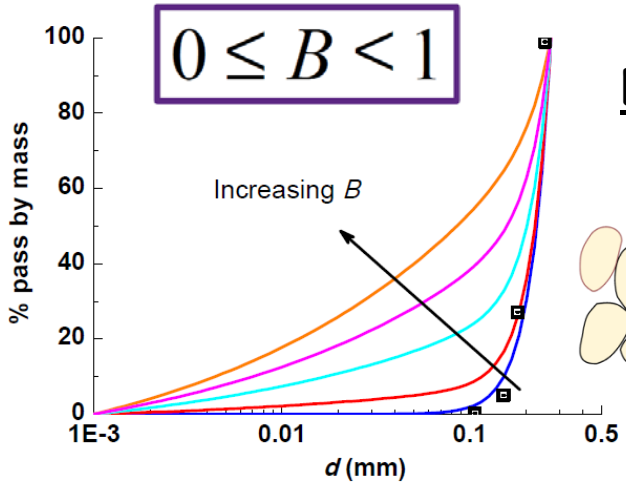
SIZE (GRAINS)

TIME (CREEP)

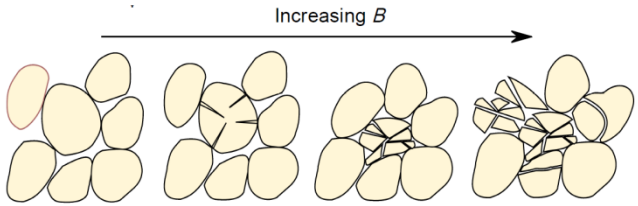
FLUID ADSORPTION

Grain Breakage in Partially Saturated Granular Systems





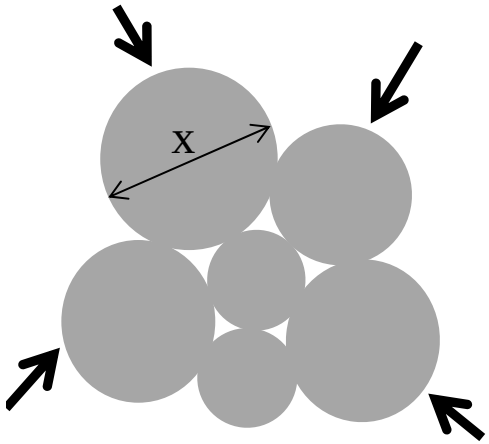
BREAKAGE MECHANICS



Einav (2007)

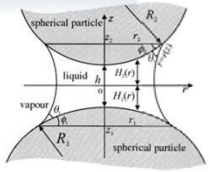
STRAIN-ENERGY STORAGE (CONTACT FORCES)

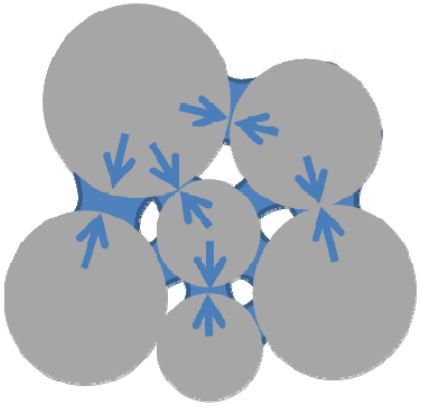
$\sim x^2$



CAPILLARY EFFECTS (SURFACE TENSIONS)

$\sim x^{-1}$

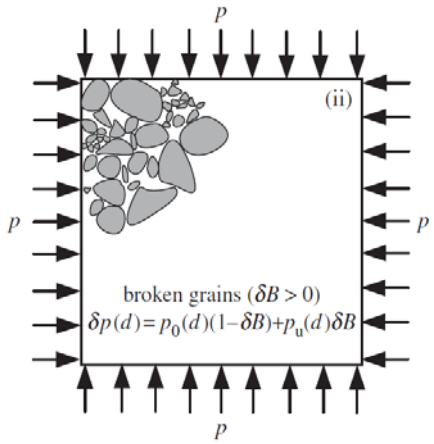




Buscarnera & Einav (2012)

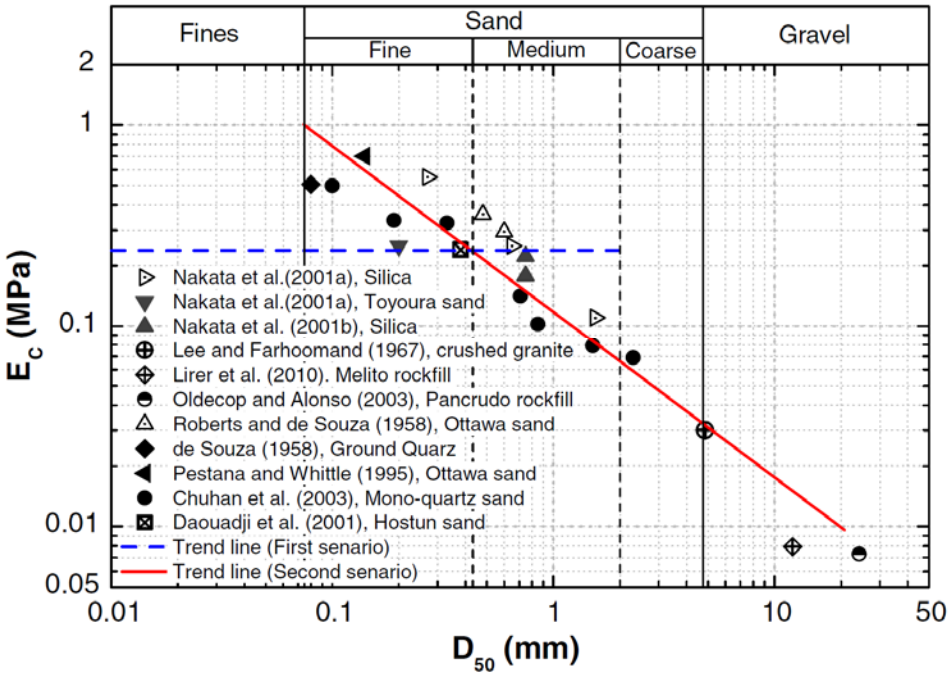
Missing Elements: Grain Size Effects and Fluid Adsorption

Size Effects



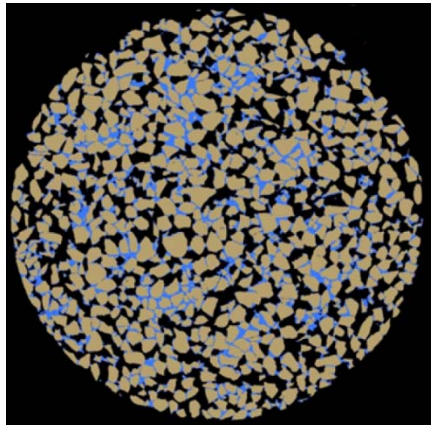
$$p_{cr} = \sqrt{\frac{2KE_c}{\vartheta}}$$

Einav (2007)

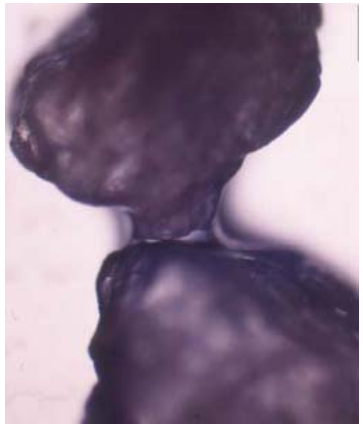


Zhang & Buscarnera (2014)

Moisture Effects



Riedel et al (2012)



Gili and Alonso (2002)

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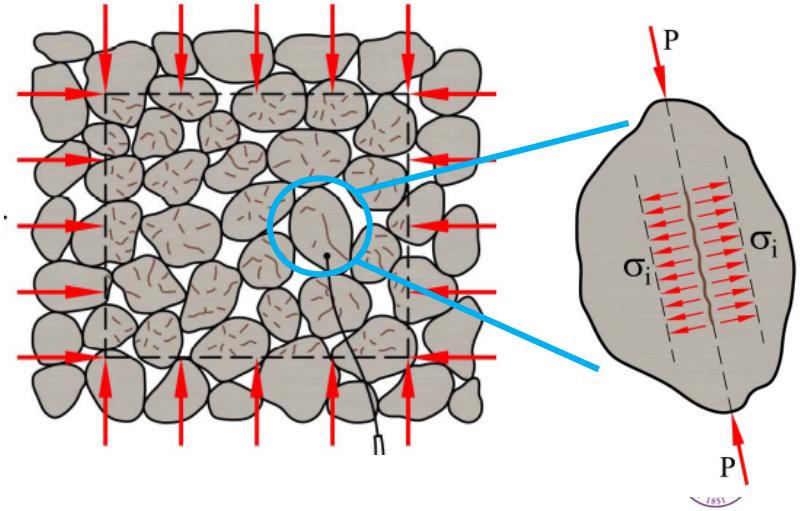
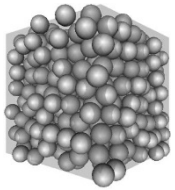


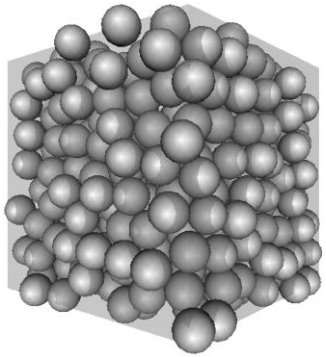
Grain Size and Crushing: From Micro to Macro

Size Effects

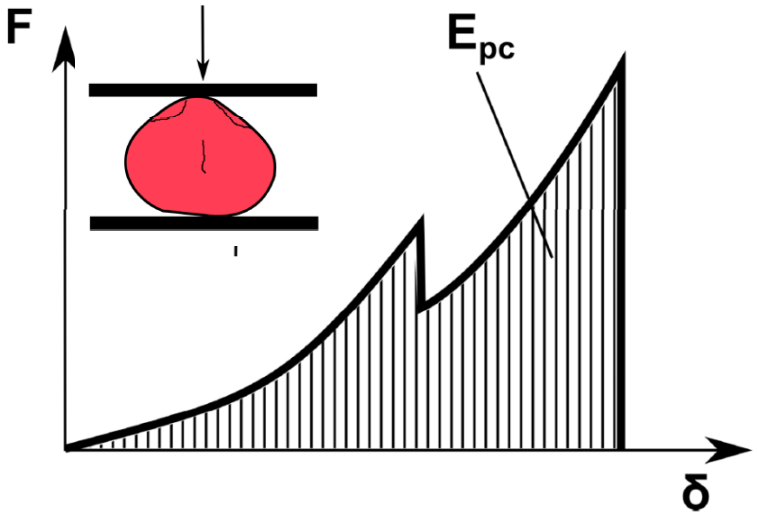
REV of small grains

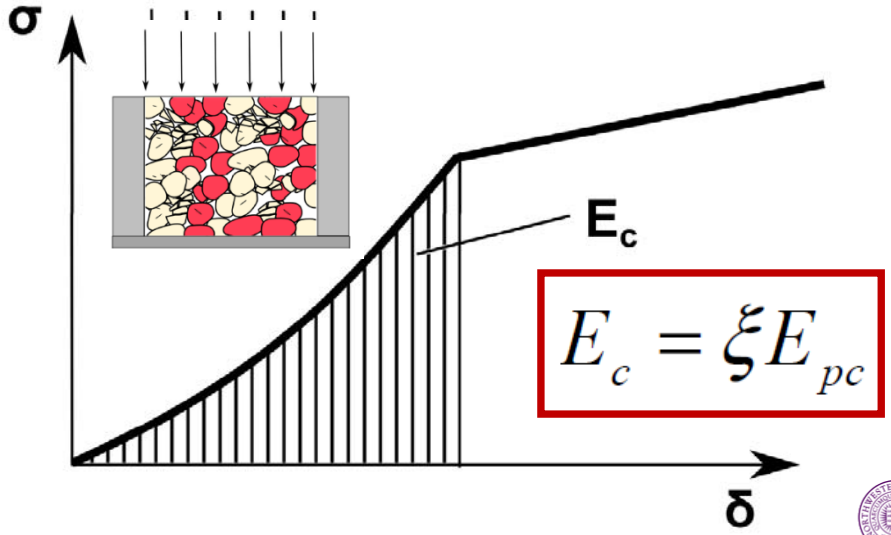


REV of large grains



Critical Energy at Crushing



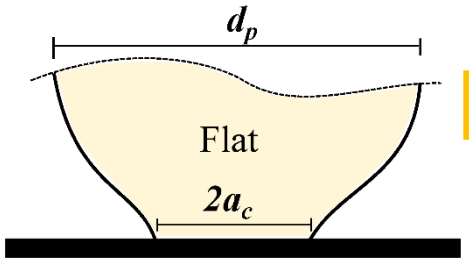


$$E_c = \xi E_{pc}$$

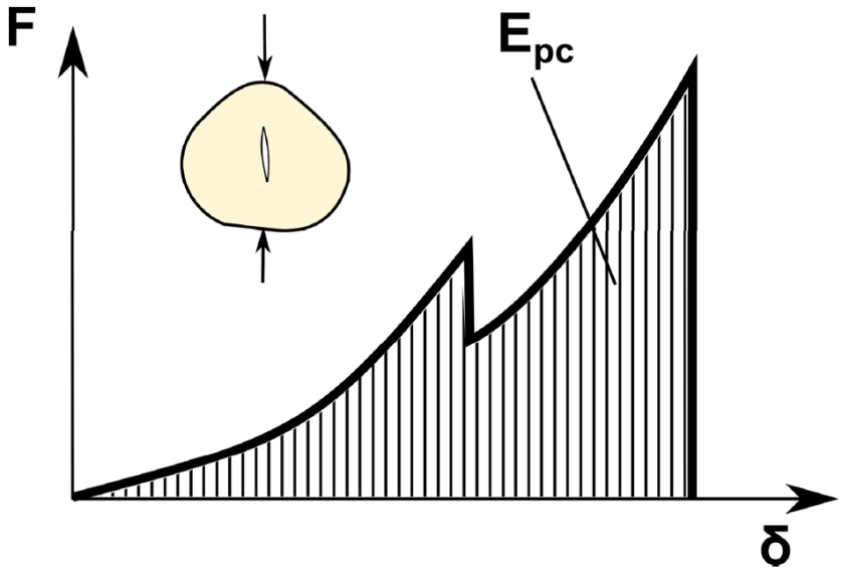
Grain Size and Crushing: From Micro to Macro

Contact Models

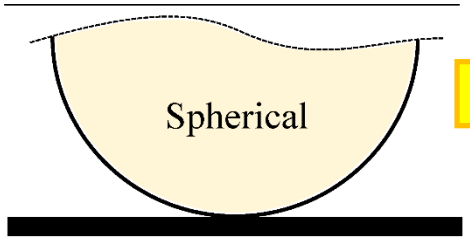
Linear Contact



$$F = \frac{E\pi a_c^2}{R}\delta$$

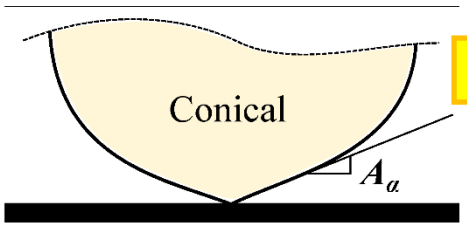


Hertzian Contact



$$F = \frac{4}{3}R^{1/2}\left(\frac{1-\nu^2}{E}\right)^{-1}\delta^{3/2}$$

Goddard Contact



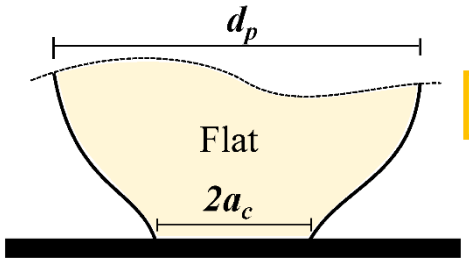
$$F = \frac{E}{1-\nu^2}\left(\frac{2}{\pi A_s}\right)^{1/2}\delta^2$$

Zhang, Buscarnera & Einav (2015)

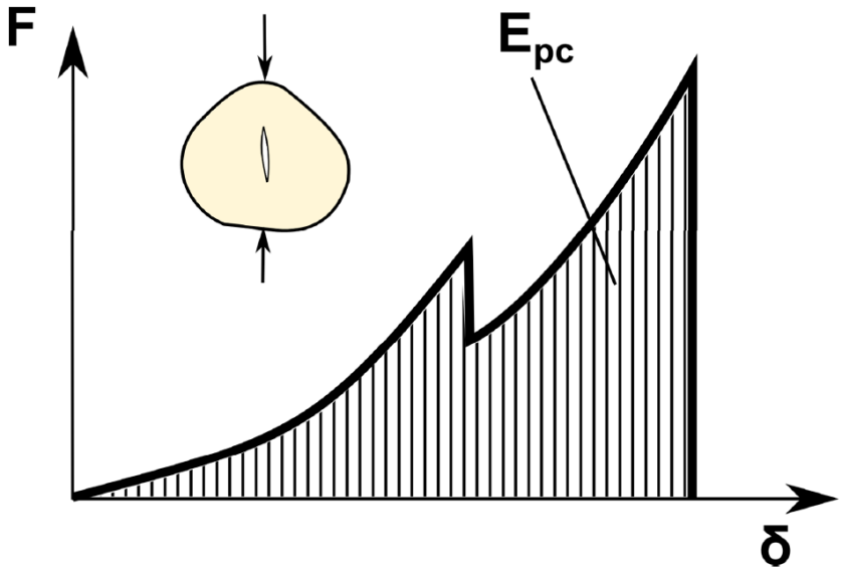
Grain Size and Crushing: From Micro to Macro

Contact Models

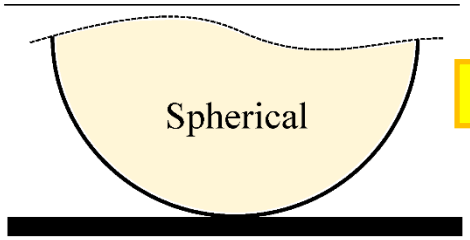
Linear Contact



$$E_{pc} = \frac{12}{E\pi^2}\left(\frac{a_c}{R}\right)^{-2}\sigma_{pc}^2$$

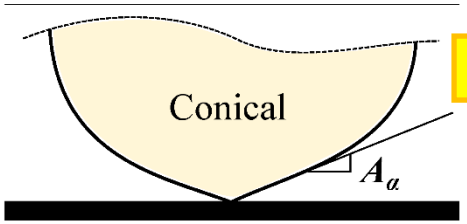


Hertzian Contact



$$E_{pc} = 4{\cdot}989\frac{1}{\pi}\left(\frac{1-\nu^2}{E}\right)^{2/3}\sigma_{pc}^{5/3}$$

Goddard Contact



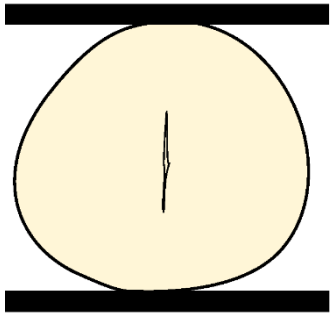
$$E_{pc} = 3{\cdot}360A_s^{1/4}\pi^{-(3/4)}\left(\frac{1-\nu^2}{E}\right)^{1/2}\sigma_{pc}^{3/2}$$

Zhang, Buscarnera & Einav (2015)

Grain Size and Crushing: From Micro to Macro

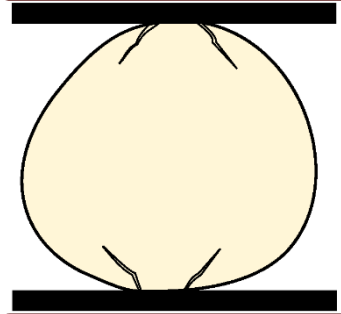
Failure Models

Centre Crack



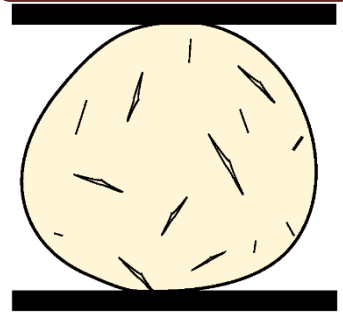
$$E_{pc} = 0{\cdot}897\pi^{2/3}a'^{5/3}\left(\frac{1-\nu^2}{E}\right)^{-(1/6)} G_{IC}^{5/6} d_p^{-(5/6)}$$

Contact Crack



$$E_{pc} = 6{\cdot}071\pi^{-1}b'^{5}\left(\frac{\pi}{1-2\nu}\right)^{5}\left(\frac{1-\nu^2}{E}\right)^{3/2} G_{IC}^{5/2} d_p^{-(5/2)}$$

Weibull



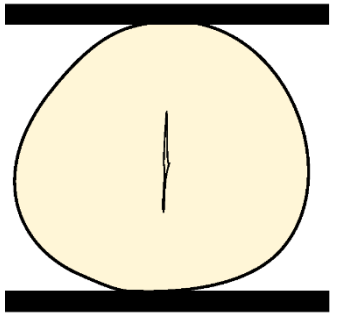
$$E_{pc} = 4{\cdot}989\pi^{-1}\left(\frac{1-\nu^2}{E}\right)^{2/3} \sigma_{pc,0}^{5/3} d_0^{5/w} d_p^{-5/w}$$

Zhang, Buscarnera & Einav (2015)

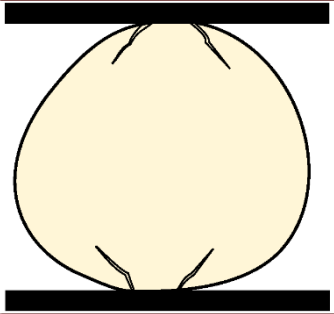
Grain Size and Crushing: From Micro to Macro

Failure Models

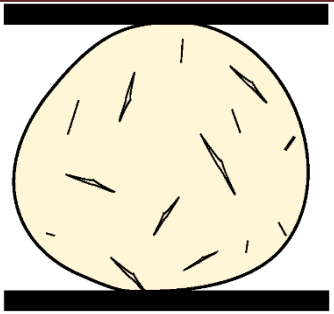
Centre Crack

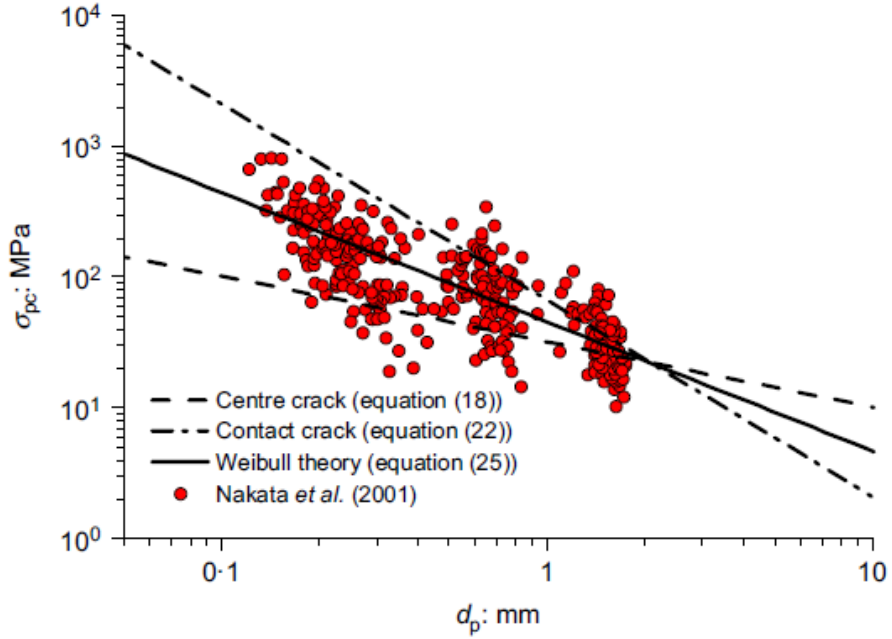


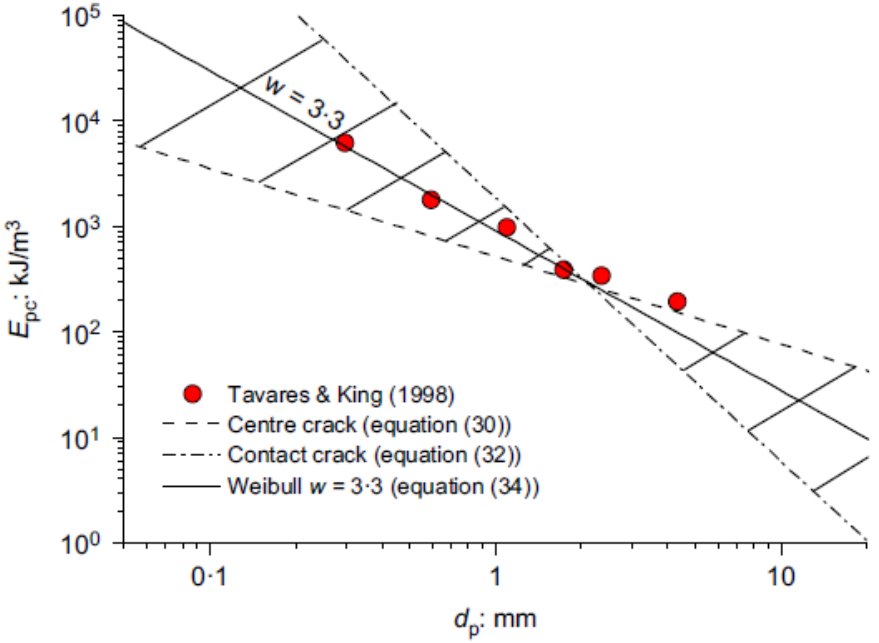
Contact Crack



Weibull



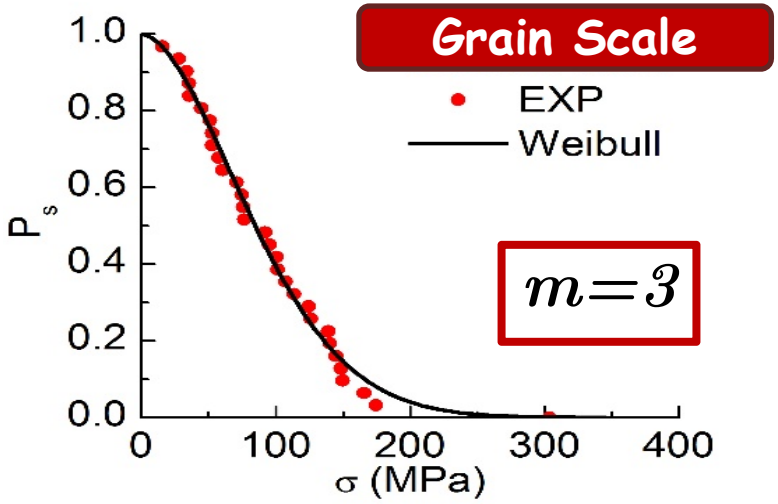


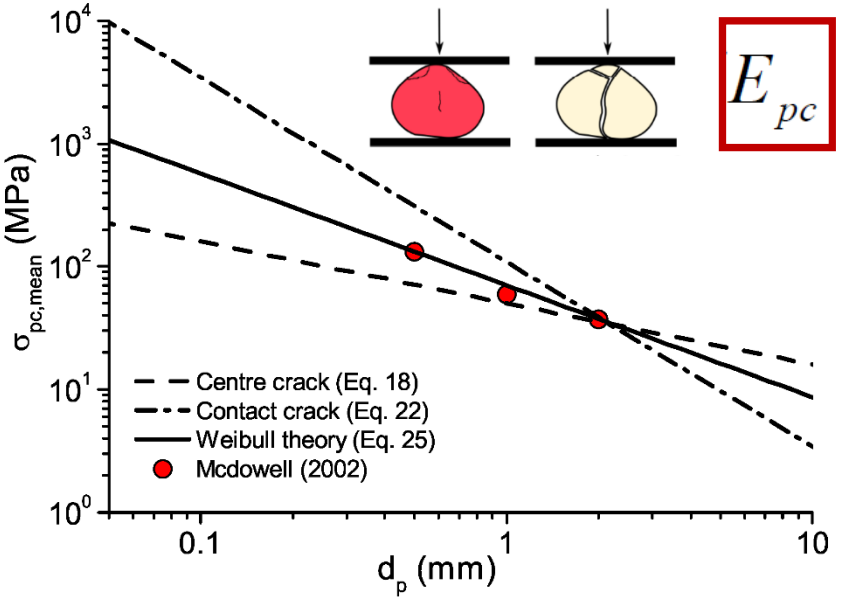


Zhang, Buscarnera & Einav (2015)

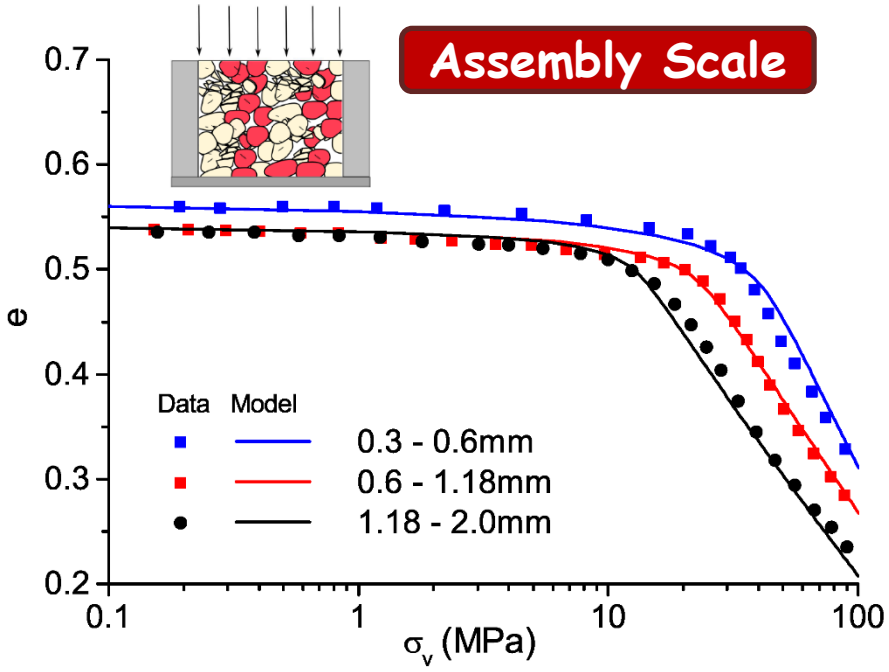
NORTHWESTERN UNIVERSITY

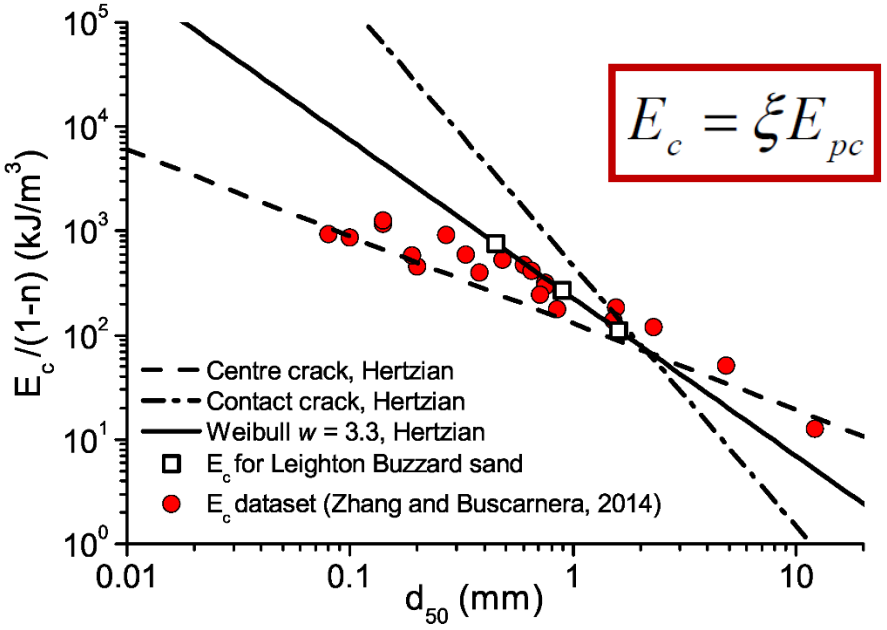
Grain Size and Crushing: From Micro to Macro





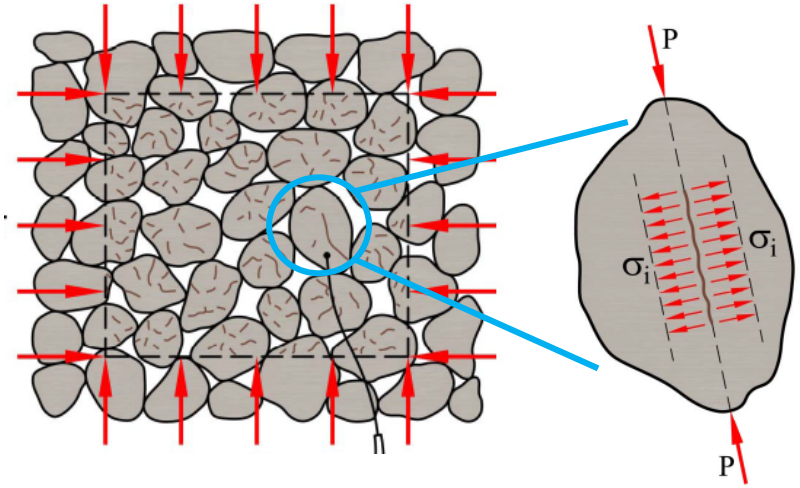
Zhang, Buscarnera & Einav (2015)

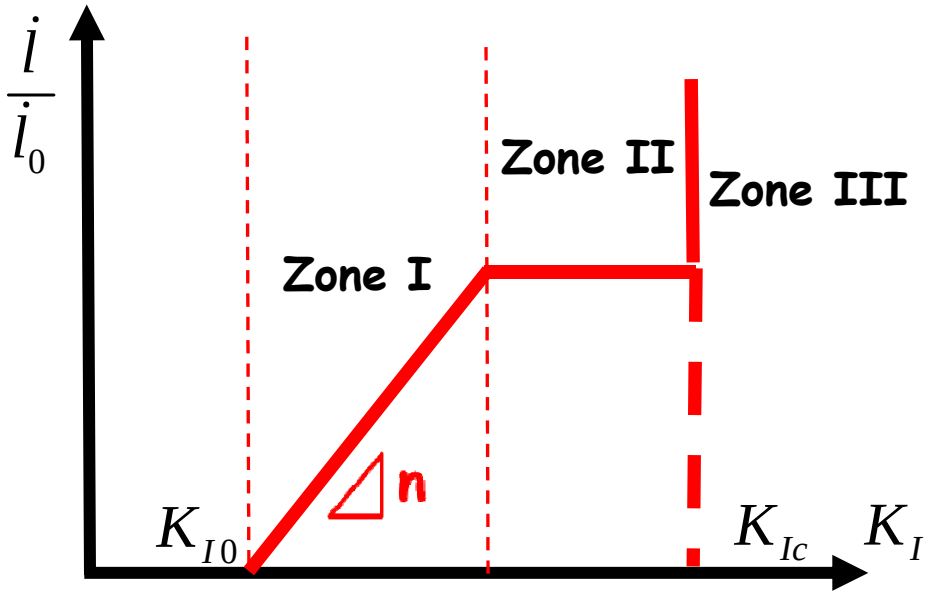




Time Dependent Grain Fracture

Subcritical Crack Growth



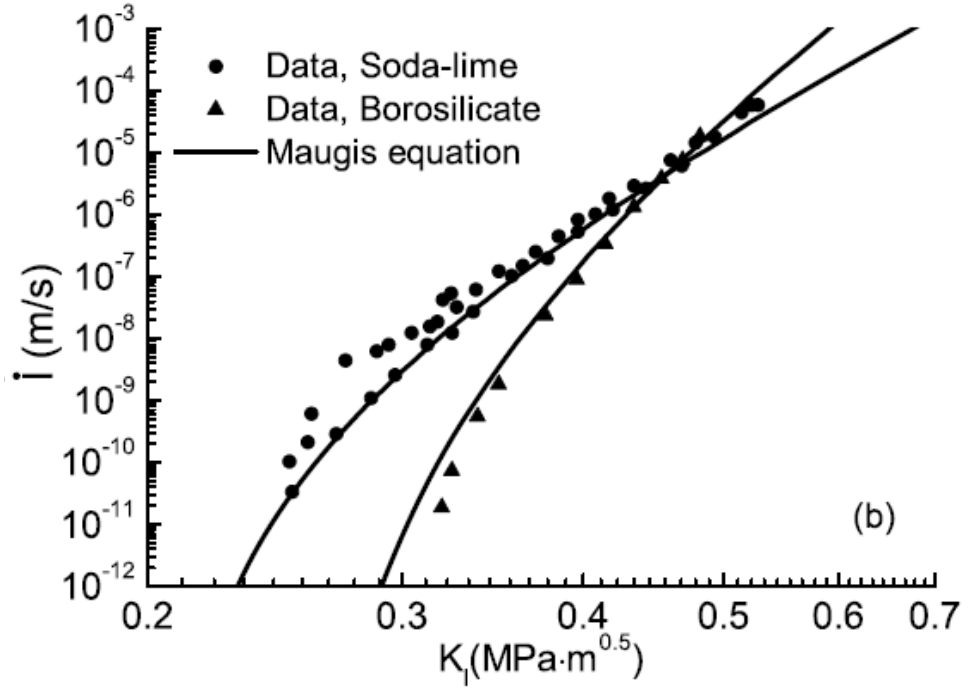


Charles Law

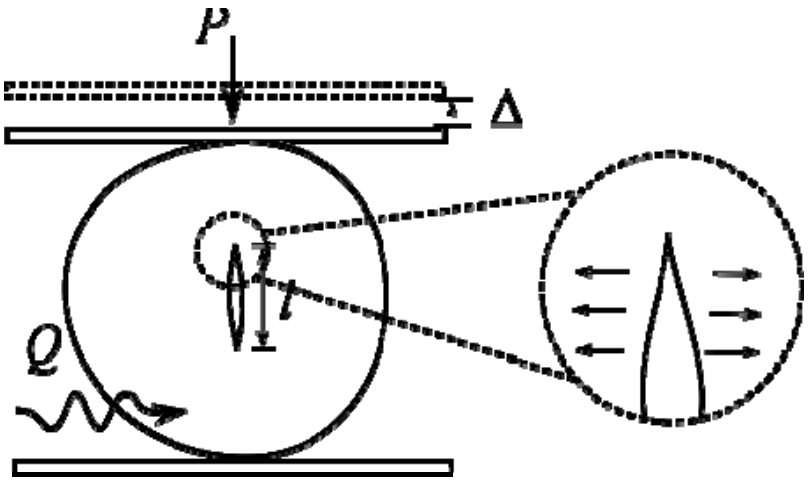
$$i = i_0 \left(\frac{K_I}{K_{IC}} \right)^n$$

Maugis Law

$$i = i_0 \left\langle \frac{G_I}{2\gamma} - 1 \right\rangle^{n/2}$$



Time Dependent Grain Fracture



Zhang & Buscarnera (2016)

Thermodynamics of Crack Growth

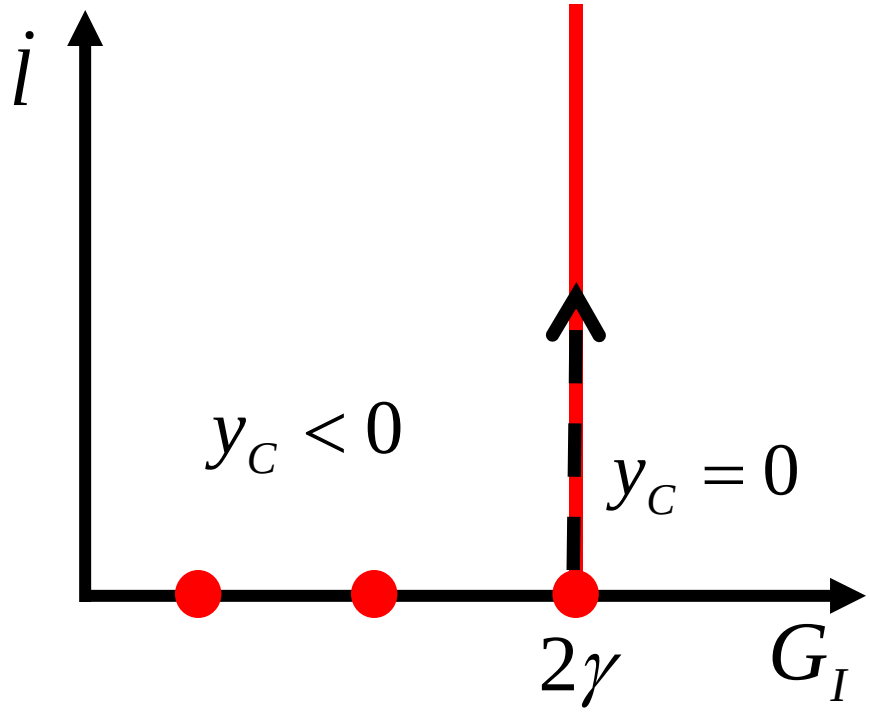
$$\left(P-\frac{\partial\Psi(\Delta,l)}{\partial\Delta}\right)\dot{\Delta}+\left(-\frac{\partial\Psi(\Delta,l)}{\partial l}\right)\dot{l}=\tilde{\Phi}\geq 0$$

Dissipation for Surface Area Creation

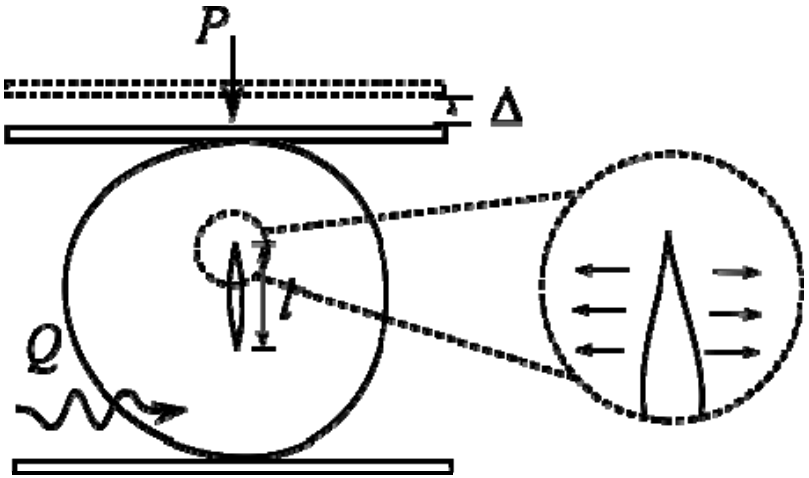
$$\tilde{\Phi}=2\gamma\dot{l}\geq 0$$

Energy Threshold for Crack Growth

$$y_C=\frac{G_I}{2\gamma}-1\leq 0$$



Time Dependent Grain Fracture



Zhang & Buscarnera (2016)

Thermodynamics of Crack Growth

$$\left(P-\frac{\partial\Psi(\Delta,l)}{\partial\Delta}\right)\dot{\Delta}+\left(G_I-2\gamma\right)\dot{l}=T\Lambda_c\geq 0$$

Rice (1978)

Kinetics Restriction

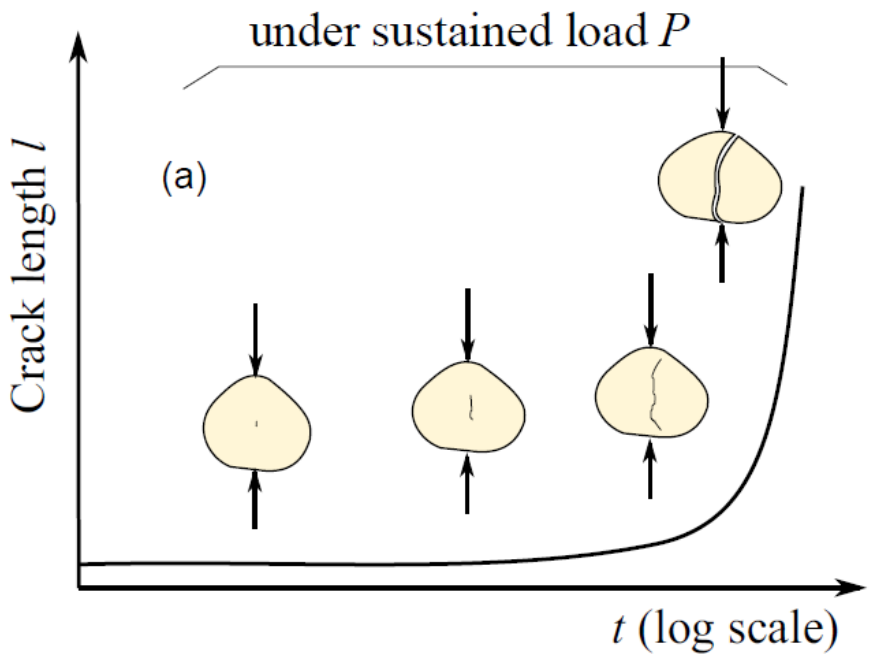
$$\left(G_I-2\gamma\right)\dot{l}\geq 0$$

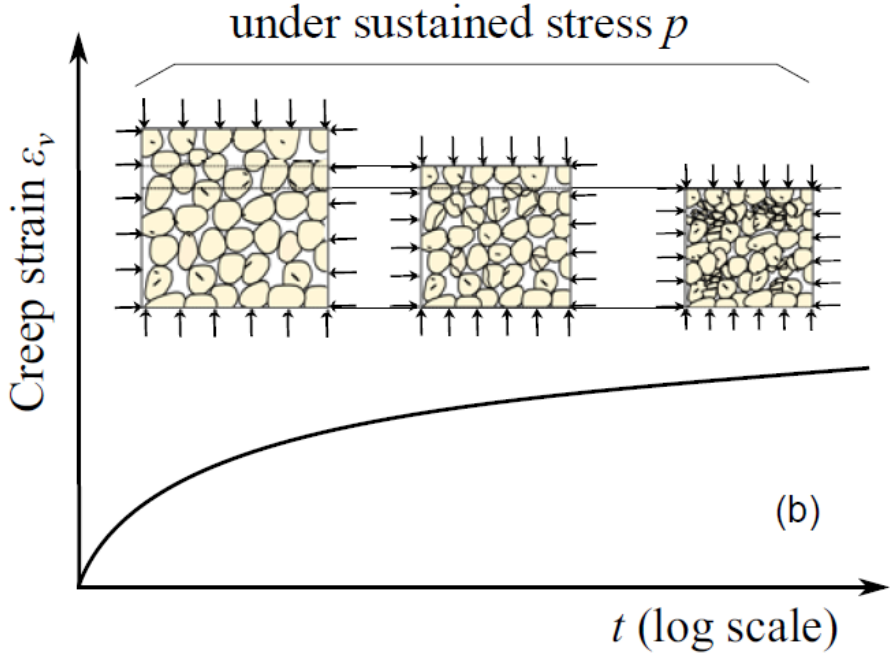
Force and Flow Potentials

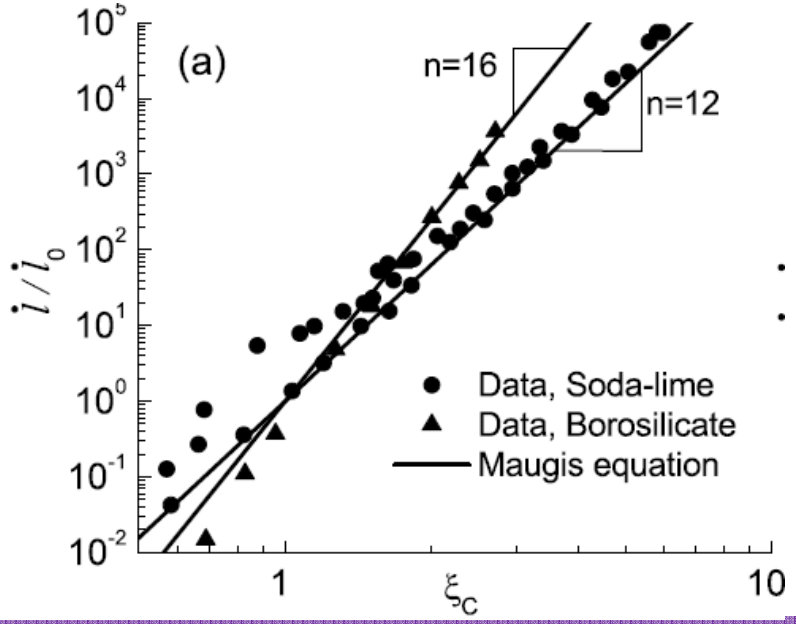
$$\boldsymbol{\chi}=\frac{\partial z}{\partial\dot{\boldsymbol{\alpha}}} \qquad \dot{\boldsymbol{\alpha}}=\frac{\partial w}{\partial\boldsymbol{\chi}} \qquad w(\mathbf{A},\boldsymbol{\chi})=\boldsymbol{\chi}:\dot{\boldsymbol{\alpha}}-z=\Phi-z$$

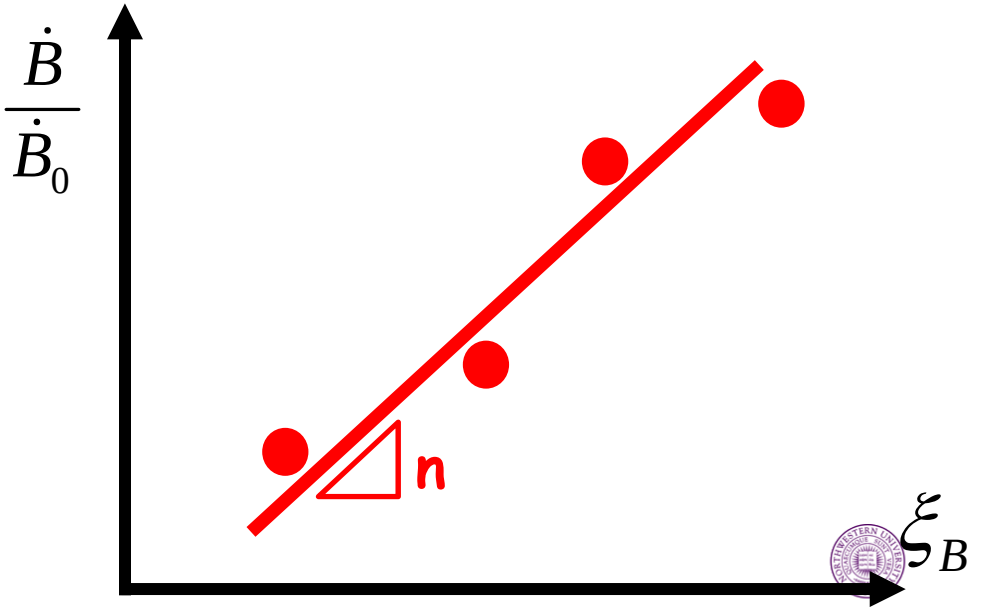
Houlsby & Puzrin (2007)

Time Dependent Grain Fracture



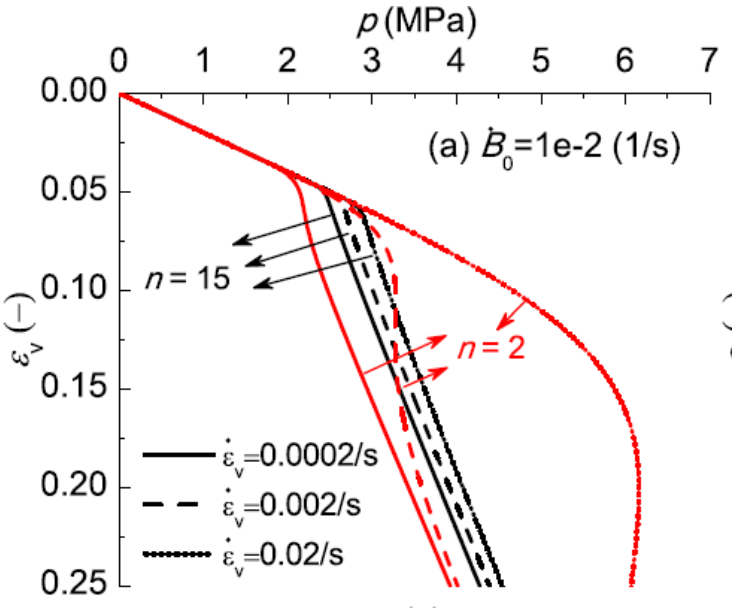


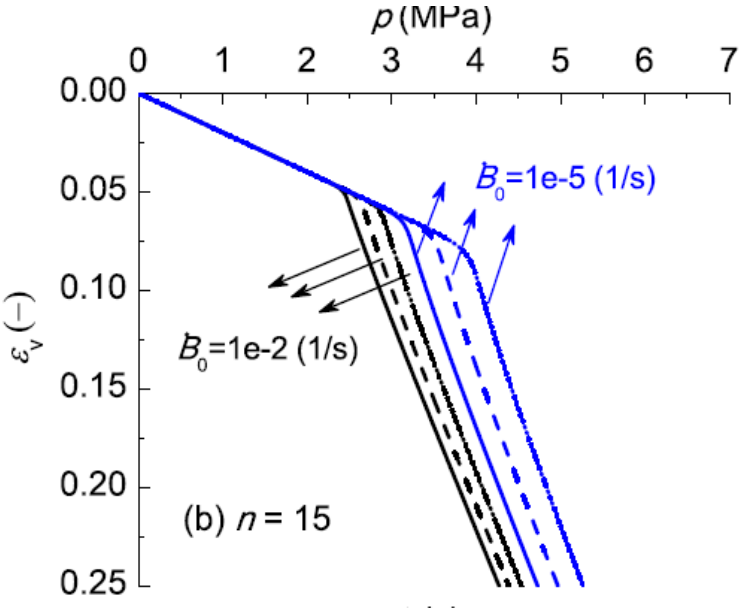




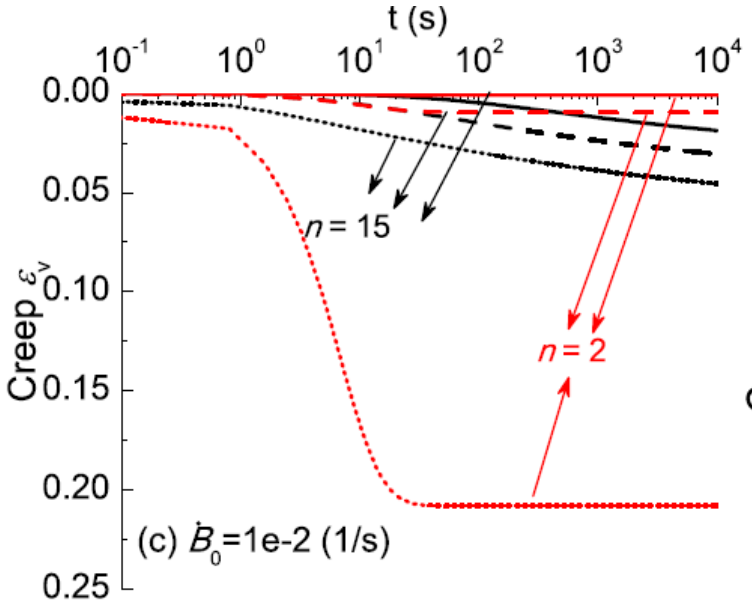
Continuum Description of Crushing-Induced Creep

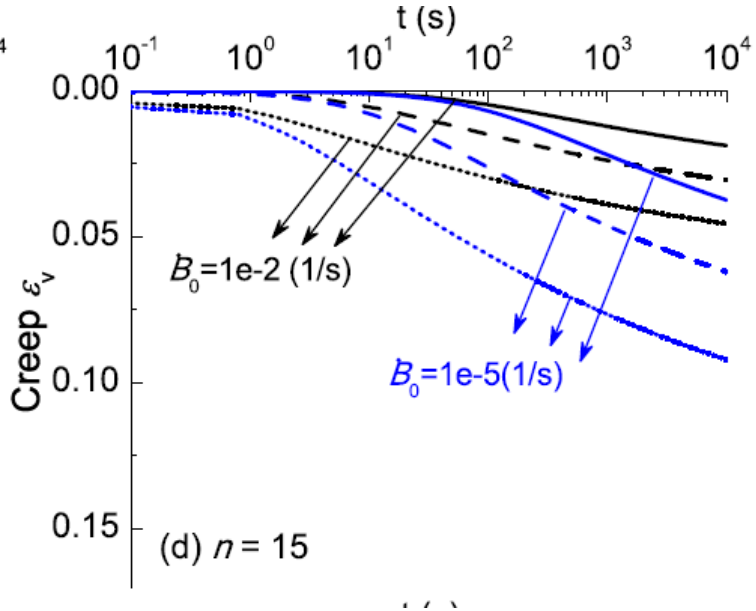
Strain-Rate Dependence





Creep Strains





Conclusions

- The incorporation of measurable micro-scale features and basic physical arguments can augment HM continuum models for granular soils/rocks.

- Energy scalings inspired by an analogy between grain fracture and sample comminution can inspire relations between yielding pressure and fracture properties ($G_{IC}=2\gamma$) and particle size.

- Thermodynamic analogies between subcritical crack growth in grains and comminution suggests a non-homogeneous dissipation form useful to model creep, relaxation and rate-dependent breakage.

- Concepts from the physics of fluid adsorption into solids can be inserted into continuum models to explain the dependence of the crushing strength of particulate continua on the reactivity of the environment.