Critical softening in Cam-Clay plasticity: 'adaptive' viscous regularization and numerical integration across stress-strain jump discontinuities

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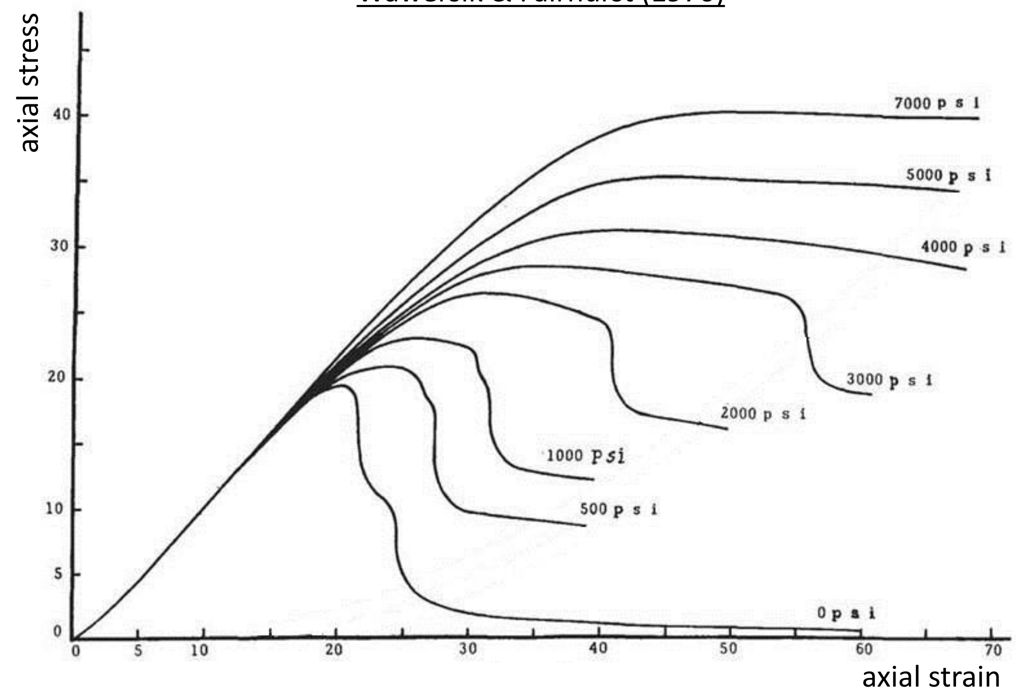
outline

1. Motivation
   - experimental observations
   - constitutive modelling
2. Cam-Clay plasticity
   - evolution equations
   - well-posedness
3. Viscoplastic regularization
   - evolution equations
   - slow/fast dynamics
4. Numerical integration
   - strategy
   - applications (VE/BVP)
5. Conclusions & Perspectives

standard compression tests on geomaterials
(rock, sand, fine-grained soils)

Wawersik & Fairhurst (1970)



hardening/softening

softening may lead to both:

(i) spatial discontinuities (strain localization)

(ii) time discontinuities (critical softening)

Ludovico-Marques et al. (2012)

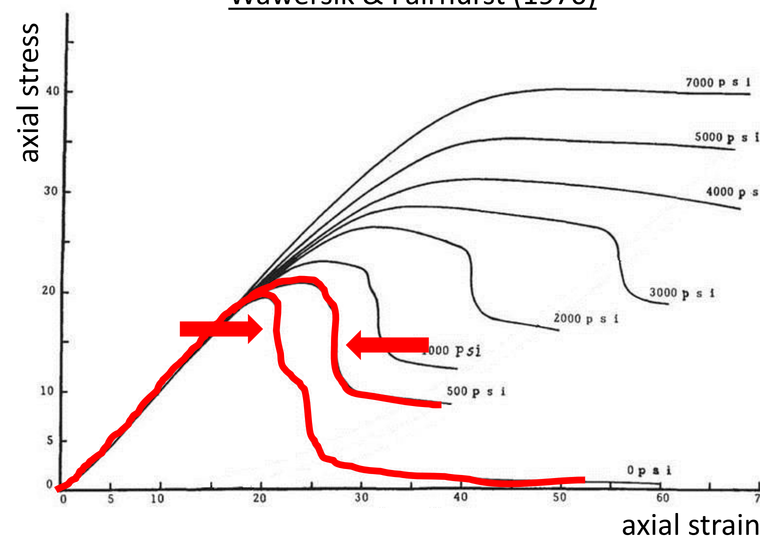
Colliat (1986)

critical softening (displacement controlled test):

- loss of test controllability
- perfectly brittle behaviour
- sharp drop (jump) of load-carrying ability

response of the material evolves at a different time scale (faster) with respect to the applied perturbation

Wawersik & Fairhurst (1970)



classical approach in continuum mechanics:

rate-independent elastoplasticity:

- evolution laws derived from the interpretation of simple laboratory tests
- assumption: stress/strain fields homogeneous within the sample

strain localization and critical softening:

- local instabilities in the constitutive equations
- ill-posedness of the evolution problem
- non-uniqueness in the incremental response

critical softening in a strain controlled process:

- vanishing of the determinant of the elastoplastic compliance matrix
- critical value of the hardening modulus

classical approach in continuum mechanics:

possibility to guarantee well-posedness of the evolution problem even beyond the onset of critical softening: ‘adaptive’ viscoplastic regularization (Dal Maso *et al*. 2009, 2010, 2011)

- Cam-clay plasticity

problems to be tackled…

- why the evolution problem becomes ill-posed
- how to handle critical softening (viscoplastic approximation)
- how to integrate the regularized equations

Cam-Clay plasticity

It exhibits both hardening and softening, depending on the loading conditions.

The variables and constraints of the model are:

displacement: $\mathbf{u} \in \mathbb{R}^{\mathrm{n}}$

total strain: $\boldsymbol{\varepsilon} := \frac{1}{2}(\nabla \mathbf{u} + \nabla^{\top} \mathbf{u})$

additive decomposition of deformation: $\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}^{\mathrm{e}} + \boldsymbol{\varepsilon}^{\mathrm{p}}$

stress: $\boldsymbol{\sigma} \in \mathbb{R}^{\mathrm{n}}_{\mathrm{sym}}$

internal variable: $z \in \mathbb{R}$

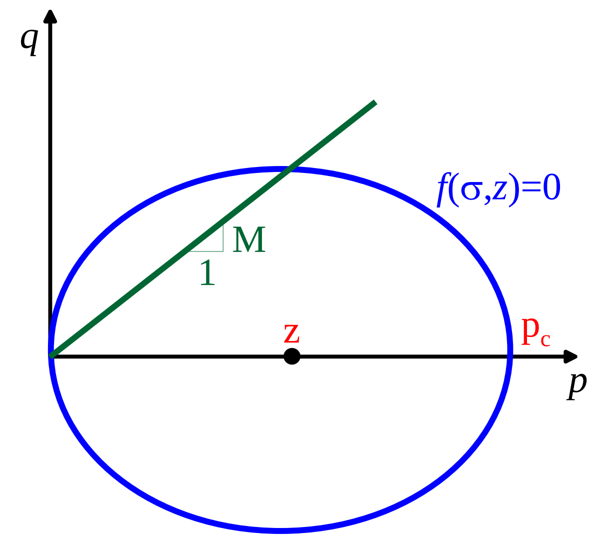
preconsolidation pressure: $p_c = 2z$

stress constraint: $\mathbb{E}_{\sigma} := \{(\boldsymbol{\sigma}, z) | f(\boldsymbol{\sigma}, z) \leq 0\}$

yield surface: $f : \mathbb{R}^{\mathrm{n}}_{\mathrm{sym}} \times \mathbb{R} \mapsto \mathbb{R}$

The yield surface is an ellipsoid in the stress space passing through the origin:

$$f(p,q,z) = \frac{q^2}{M^2} + p(p-2z)$$



where *p*, *q* are stress invariants:

$$p := \frac{1}{3}\,\mathrm{tr}(\boldsymbol{\sigma}) \qquad q := \sqrt{\frac{3}{2}}\|\mathbf{s}\|$$

$$\mathbf{s} := \boldsymbol{\sigma} - \frac{1}{3}\,\mathrm{tr}(\boldsymbol{\sigma})\mathbf{I}$$

If $\dot{z} > 0$ the yield surface expands leading to a hardening response.
If $\dot{z} < 0$ the yield surface shrinks leading to a softening response.

additive decomposition of deformations:

$$\dot{\boldsymbol{\varepsilon}} = \dot{\boldsymbol{\varepsilon}}^{\mathrm{e}} + \dot{\boldsymbol{\varepsilon}}^{\mathrm{p}}$$

constitutive equation:

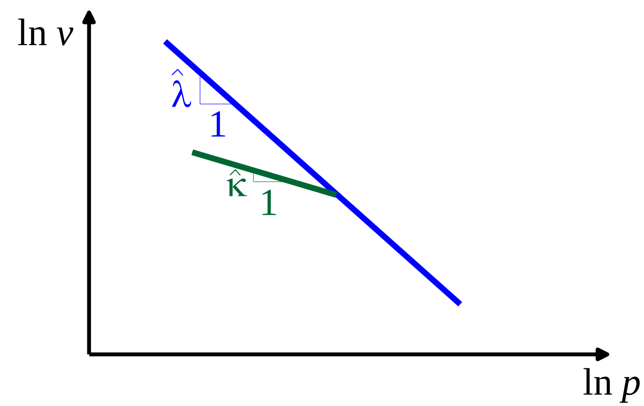
$$\dot{\boldsymbol{\sigma}} = \mathbb{C}\dot{\boldsymbol{\varepsilon}}^{\mathrm{e}}$$ where $$\mathbb{C} := K\mathbf{I}\otimes\mathbf{I} + 2G\Big(\mathbb{I} - \frac{1}{3}\mathbf{I}\otimes\mathbf{I}\Big)$$

flow rule:

$$\dot{\boldsymbol{\varepsilon}}^{\mathrm{p}} = \dot{\gamma}\frac{\partial f}{\partial \boldsymbol{\sigma}}$$ where $\dot{\gamma} \geq 0$ is the consistency parameter

hardening law:

$$\dot{z} = \rho_c z \,\mathrm{tr}\left(\dot{\boldsymbol{\varepsilon}}^{\mathrm{p}}\right)$$ where $$\rho_c = \frac{1}{\hat{\lambda} - \hat{k}}$$



Kuhn-Tucker conditions:

$$\dot{\gamma} \geq 0, f \leq 0, \dot{\gamma} f = 0$$

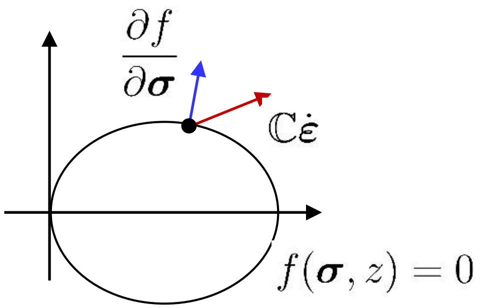
consistency condition:

$$\dot{\gamma}\dot{f} = 0$$

initial condition at yield: $f(\boldsymbol{\sigma}, z) = 0$

prescribe a total strain increment: $\dot{\varepsilon} > 0$

assume plastic loading: $\frac{\partial f}{\partial \boldsymbol{\sigma}} \cdot \mathbb{C}\dot{\varepsilon} > 0$



Kuhn-Tucker conditions: $\dot{\gamma} \geq 0, \quad f \leq 0, \quad \dot{\gamma} f = 0$

consistency condition: $\dot{\gamma} \dot{f} = 0$

$$\dot{f}(\boldsymbol{\sigma}, z) = \frac{\partial f}{\partial \boldsymbol{\sigma}} \cdot \dot{\boldsymbol{\sigma}} + \frac{\partial f}{\partial z} \cdot \dot{z} =$$

$$= \frac{\partial f}{\partial \boldsymbol{\sigma}} \cdot \mathbb{C}\dot{\varepsilon} - \dot{\gamma}\left[\frac{\partial f}{\partial \boldsymbol{\sigma}} \cdot \mathbb{C}\frac{\partial f}{\partial \boldsymbol{\sigma}} - \frac{\partial f}{\partial z} \cdot h\right] =$$

$$= \frac{\partial f}{\partial \boldsymbol{\sigma}} \cdot \mathbb{C}\dot{\varepsilon} - \dot{\gamma}[H - H_c]$$

hardening modulus

$$H = -\frac{\partial f}{\partial z} \cdot h$$

critical hardening modulus

$$H_c = -\frac{\partial f}{\partial \boldsymbol{\sigma}} \cdot \mathbb{C}\frac{\partial f}{\partial \boldsymbol{\sigma}}$$

Kuhn-Tucker conditions: $\dot{\gamma} \geq 0, \quad f \leq 0, \quad \dot{\gamma} f = 0$

consistency condition: $\dot{\gamma} \dot{f} = 0$

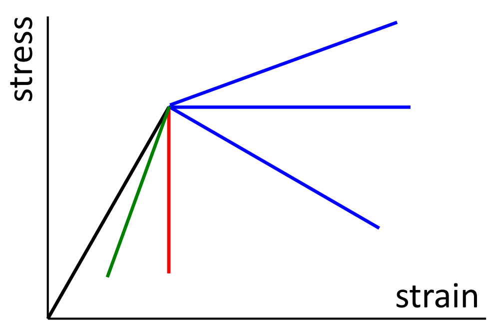
plastic modulus (K_P) or modulus of instability

$$0 = \dot{f} = \frac{\partial f}{\partial \boldsymbol{\sigma}} \cdot \mathbb{C}\dot{\boldsymbol{\varepsilon}} - \dot{\gamma}[H - H_c]$$

$$\dot{\gamma} = \frac{1}{H - H_c} \frac{\partial f}{\partial \boldsymbol{\sigma}} \cdot \mathbb{C}\dot{\boldsymbol{\varepsilon}}$$

The evolution problem is well posed only as long as $H - H_c > 0$ (Maier & Hueckel, 1979).

This condition is always assumed in the literature, in order to ensure the positiveness of the consistency parameter.



$H - H_c > 0$ (Hardening/Normal Softening)

$H - H_c = 0$ (Critical Softening)

$H - H_c < 0$ (Subcritical Softening)

viscoplastic regularization

We use a viscoplastic regularization in the Duvaut-Lions format.
Given a viscosity parameter $\tau>0$, the evolution equations are:

Additive decomposition of deformations:

$$\dot{\boldsymbol{\varepsilon}} = \dot{\boldsymbol{\varepsilon}}^{\mathrm{e}} + \dot{\boldsymbol{\varepsilon}}^{\mathrm{p}}$$

Constitutive equation:

$$\dot{\boldsymbol{\sigma}} = \mathbb{C}\dot{\boldsymbol{\varepsilon}}^{\mathrm{e}} \qquad \text{where} \qquad \mathbb{C} := K\mathbf{I}\otimes\mathbf{I} + 2G\left(\mathbb{I} - \frac{1}{3}\mathbf{I}\otimes\mathbf{I}\right)$$

Flow rule:

$$\dot{\boldsymbol{\varepsilon}}^{\mathrm{p}} = \frac{1}{\tau}\mathbb{A}\left[\boldsymbol{\sigma} - \pi_{\mathbb{A}}(\boldsymbol{\sigma})\right]$$

$\pi_{\mathbb{A}}(\boldsymbol{\sigma})$ projection onto $\mathbb{E}_{\sigma}$

Hardening law:

$$\dot{z} = \rho_c z \,\mathrm{tr}\left(\dot{\boldsymbol{\varepsilon}}^{\mathrm{p}}\right)$$

Unconstrained problem: the stress state is no longer constrained to lie on the yield surface during a plastic process.

viscoplastic regularization

($\tau \neq 0$) standard viscoplasticity

evolution problem always well posed
rate-dependency
continuous solution

($\tau \to 0$) limit solution

...

During a generic loading process, by solving the regularized evolution equations in the limit as $\tau \to 0$, the viscous dynamics presents three possible regimes:

Elastic regime

Loading process entirely inside the yield surface

Slow dynamics ($H - H_c > 0$ or $K_p > 0$)

corresponding rate-independent evolution problem well-posed
viscous limit solution is continuous (tends to solution of rate-independent problem)
both hardening and softening can occur.

Fast dynamics ($H - H_c \leq 0$ or $K_p \leq 0$)

corresponding rate-independent evolution problem ill-posed
viscous limit solution is discontinuous (jumps)
introduce a dilated time $s := t/\tau$ to rescale the equations
study the evolution of (σ, z) along the jump

rescaling the equations (in the limit as $\tau \to 0$):

$$\frac{\mathrm{d}}{\mathrm{d}s}(\cdot) = \tau \frac{\mathrm{d}}{\mathrm{d}t}(\cdot)$$

slow dynamics:

$$\frac{\mathrm{d}}{\mathrm{d}t}(\cdot) < \infty \quad \Rightarrow \quad \frac{\mathrm{d}}{\mathrm{d}s}(\cdot) = 0$$

fast dynamics (jumps):

$$\frac{\mathrm{d}\boldsymbol{\sigma}}{\mathrm{d}t} = \infty \quad \Rightarrow \quad \frac{\mathrm{d}}{\mathrm{d}s}(\cdot) = 0 \cdot \infty$$

rescaling the equations (in the limit as τ → 0):

$$\frac{\mathrm{d}}{\mathrm{d}s}(\cdot) = \tau \frac{\mathrm{d}}{\mathrm{d}t}(\cdot)$$

Flow rule:

$$\cancel{\frac{1}{\tau}} \frac{\mathrm{d}\boldsymbol{\varepsilon}^{\mathrm{p}}}{\mathrm{d}s} = \cancel{\frac{1}{\tau}} \mathbb{A}\big[\boldsymbol{\sigma} - \pi_{\mathbb{A}}(\boldsymbol{\sigma})\big] \qquad \frac{\mathrm{d}\boldsymbol{\varepsilon}^{\mathrm{p}}}{\mathrm{d}s} = \mathbb{A}\big[\boldsymbol{\sigma} - \pi_{\mathbb{A}}(\boldsymbol{\sigma})\big]$$

Hardening law:

$$\cancel{\frac{1}{\tau}} \frac{\mathrm{d}z}{\mathrm{d}s} = \cancel{\frac{1}{\tau}} \rho_c z \,\mathrm{tr}\left(\frac{\mathrm{d}\boldsymbol{\varepsilon}^{\mathrm{p}}}{\mathrm{d}s}\right) \qquad \frac{\mathrm{d}z}{\mathrm{d}s} = \rho_c z \,\mathrm{tr}\big[\mathbb{A}\big(\boldsymbol{\sigma} - \pi_{\mathbb{A}}(\boldsymbol{\sigma})\big)\big]$$

Constitutive equation:

$$\cancel{\frac{1}{\tau}} \frac{\mathrm{d}\boldsymbol{\sigma}}{\mathrm{d}s} = \cancel{\frac{1}{\tau}} \mathbb{C} \frac{\mathrm{d}\boldsymbol{\varepsilon}}{\mathrm{d}s} - \cancel{\frac{1}{\tau}} \mathbb{C} \frac{\mathrm{d}\boldsymbol{\varepsilon}^{\mathrm{p}}}{\mathrm{d}s}$$

$$\frac{\mathrm{d}\boldsymbol{\sigma}}{\mathrm{d}s} = \tau \cancel{\mathbb{C}\dot{\boldsymbol{\varepsilon}}} - \mathbb{C} \frac{\mathrm{d}\boldsymbol{\varepsilon}^{\mathrm{p}}}{\mathrm{d}s} \qquad \frac{\mathrm{d}\boldsymbol{\sigma}}{\mathrm{d}s} = \mathbb{C}\mathbb{A}\big[\pi_{\mathbb{A}}(\boldsymbol{\sigma}) - \boldsymbol{\sigma}\big]$$

rescaling the equations (in the limit as $\tau \to 0$):

slow dynamics

$$\frac{\mathrm{d}}{\mathrm{d}s}(\cdot) = \tau \frac{\mathrm{d}}{\mathrm{d}t}(\cdot)$$

Flow rule:

$$\cancel{\frac{1}{\tau}}\frac{\mathrm{d}\boldsymbol{\varepsilon}^{\mathrm{p}}}{\mathrm{d}s} = \cancel{\frac{1}{\tau}}\mathbb{A}\left[\boldsymbol{\sigma} - \pi_{\mathbb{A}}(\boldsymbol{\sigma})\right] \qquad \cancel{\frac{\mathrm{d}\boldsymbol{\varepsilon}^{\mathrm{p}}}{\mathrm{d}s}} = \mathbb{A}\left[\boldsymbol{\sigma} - \pi_{\mathbb{A}}(\boldsymbol{\sigma})\right]$$

Hardening law:

$$\cancel{\frac{1}{\tau}}\frac{\mathrm{d}z}{\mathrm{d}s} = \cancel{\frac{1}{\tau}}\rho_c z \,\mathrm{tr}\left(\frac{\mathrm{d}\boldsymbol{\varepsilon}^{\mathrm{p}}}{\mathrm{d}s}\right) \qquad \cancel{\frac{\mathrm{d}z}{\mathrm{d}s}} = \rho_c z \,\mathrm{tr}\left[\mathbb{A}\left(\boldsymbol{\sigma} - \pi_{\mathbb{A}}(\boldsymbol{\sigma})\right)\right]$$

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$$\frac{\mathrm{d}\boldsymbol{\sigma}}{\mathrm{d}s} = \tau\cancel{\mathbb{C}\dot{\boldsymbol{\varepsilon}}} - \mathbb{C}\frac{\mathrm{d}\boldsymbol{\varepsilon}^{\mathrm{p}}}{\mathrm{d}s} \qquad \cancel{\frac{\mathrm{d}\boldsymbol{\sigma}}{\mathrm{d}s}} = \mathbb{C}\mathbb{A}\left[\pi_{\mathbb{A}}(\boldsymbol{\sigma}) - \boldsymbol{\sigma}\right]$$

rescaling the equations (in the limit as $\tau \to 0$):

fast dynamics

$$\frac{\mathrm{d}}{\mathrm{d}s}(\cdot) = \tau \frac{\mathrm{d}}{\mathrm{d}t}(\cdot)$$

Flow rule:

$$\frac{1}{\tau}\frac{\mathrm{d}\boldsymbol{\varepsilon}^{\mathrm{p}}}{\mathrm{d}s} = \frac{1}{\tau}\mathbb{A}\left[\boldsymbol{\sigma} - \pi_{\mathbb{A}}(\boldsymbol{\sigma})\right]$$

$$0 \neq \frac{\mathrm{d}\boldsymbol{\varepsilon}^{\mathrm{p}}}{\mathrm{d}s} = \mathbb{A}\left[\boldsymbol{\sigma} - \pi_{\mathbb{A}}(\boldsymbol{\sigma})\right]$$

Hardening law:

$$\frac{1}{\tau}\frac{\mathrm{d}z}{\mathrm{d}s} = \frac{1}{\tau}\rho_c z \operatorname{tr}\left(\frac{\mathrm{d}\boldsymbol{\varepsilon}^{\mathrm{p}}}{\mathrm{d}s}\right)$$

$$0 \neq \frac{\mathrm{d}z}{\mathrm{d}s} = \rho_c z \operatorname{tr}\left[\mathbb{A}\left(\boldsymbol{\sigma} - \pi_{\mathbb{A}}(\boldsymbol{\sigma})\right)\right]$$

Constitutive equation:

$$\frac{1}{\tau}\frac{\mathrm{d}\boldsymbol{\sigma}}{\mathrm{d}s} = \frac{1}{\tau}\mathbb{C}\frac{\mathrm{d}\varepsilon}{\mathrm{d}s} - \frac{1}{\tau}\mathbb{C}\frac{\mathrm{d}\boldsymbol{\varepsilon}^{\mathrm{p}}}{\mathrm{d}s}$$

$$\frac{\mathrm{d}\boldsymbol{\sigma}}{\mathrm{d}s} = \tau\mathbb{C}\dot{\varepsilon} - \mathbb{C}\frac{\mathrm{d}\boldsymbol{\varepsilon}^{\mathrm{p}}}{\mathrm{d}s}$$

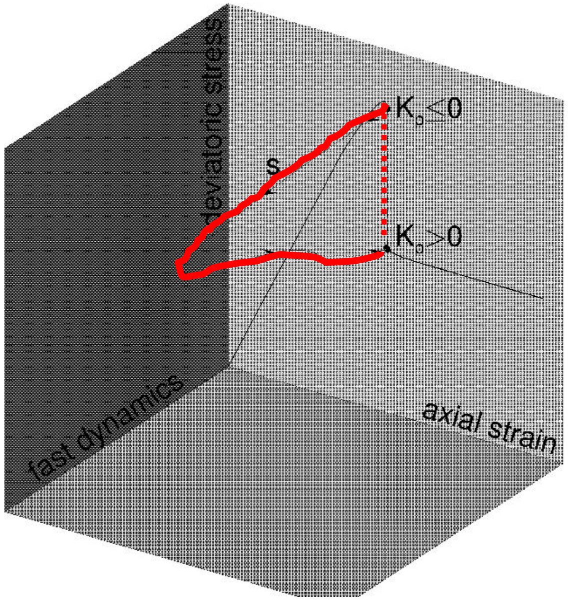
$$0 \neq \frac{\mathrm{d}\boldsymbol{\sigma}}{\mathrm{d}s} = \mathbb{C}\mathbb{A}\left[\pi_{\mathbb{A}}(\boldsymbol{\sigma}) - \boldsymbol{\sigma}\right]$$

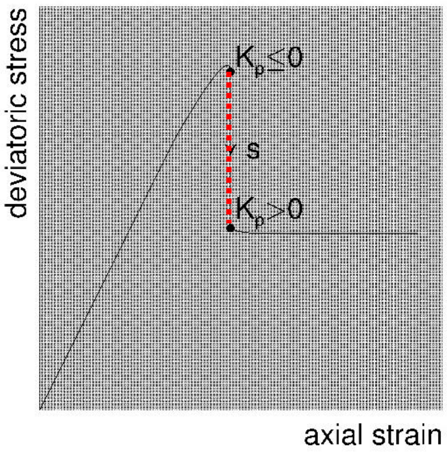
In the fast dynamics regime, the rescaled equations are:

$$\begin{cases} \dot{\boldsymbol{\sigma}}(s) = \mathbb{C}\mathbb{A}\big(\pi_{\mathbb{A}}(\boldsymbol{\sigma}(s)) - \boldsymbol{\sigma}(s)\big) \\ \dot{z}(s) = \rho_c z \,\mathrm{tr}\left[\mathbb{A}\big(\boldsymbol{\sigma}(s) - \pi_{\mathbb{A}}(\boldsymbol{\sigma}(s))\big)\right] \end{cases}$$

The asymptotic values of the solution at $s=\pm\infty$ give the asymptotic values of the viscosity solution before and after the jump, *i.e.*:

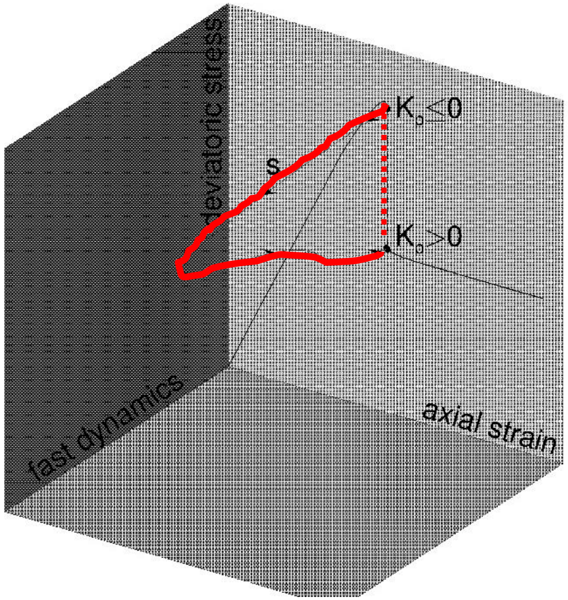
$$\lim_{s\to\pm\infty} \big(\boldsymbol{\sigma}(s), z(s)\big) = \left(\boldsymbol{\sigma}(t_1^{\pm}), z(t_1^{\pm})\right)$$

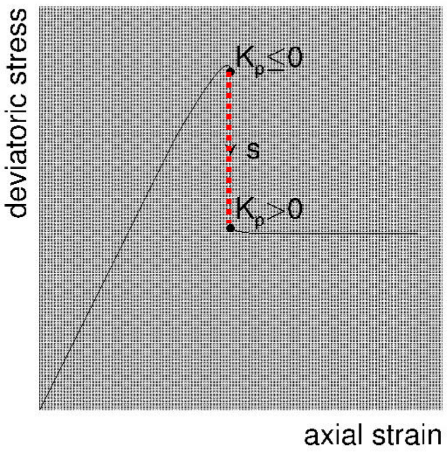




during the jump:
- internal variable strictly decreasing (softening).

at the end of the jump:
- the stress state lies on the yield surface: $f(\sigma,z)=0$
- the plastic modulus is positive: $K_p>0$
- the viscous solution evolves either in the elastic or in the slow dynamics regime





($\tau \neq 0$) standard viscoplasticity

evolution problem always well posed
rate-dependency
continuous solution

($\tau \to 0$) limit solution

evolution problem always well posed
discontinuous solution
- $K_p>0$ (slow dynamics) rate-independent problem
- $K_p \leq 0$ (fast dynamics) jumps

adaptive viscous regularization

We assume that the state of the material (σ_n,z_n) is given at time t_n.
Let $\Delta\boldsymbol{\varepsilon}_{n+1} = \boldsymbol{\varepsilon}_{n+1}$-$\boldsymbol{\varepsilon}_n$ be the incremental strain at time t_{n+1}, the problem to be addressed is to update the state variables (σ_{n+1},z_{n+1}) through the integration of the viscous equations, either in slow dynamics or in fast dynamics.

slow dynamics
standard return mapping algorithm
elastic predictor + plastic corrector
system well-conditioned for all $\tau \geq 0$

- $\tau > 0$ standard viscoplasticity
- $\tau = 0$ rate-independent limit

inception of jump discontinuities ($\tau = 0$):

- $\Delta\gamma_{n+1}$= consistency parameter
- if $\Delta\gamma_{n+1} < 0$ then solution rejected (critical softening)
- integrate the equation of fast dynamics

We assume that the state of the material (σ_n,z_n) is given at time t_n.
Let $\Delta\boldsymbol{\varepsilon}_{n+1} = \boldsymbol{\varepsilon}_{n+1} - \boldsymbol{\varepsilon}_n$ be the incremental strain at time t_{n+1}, the problem to be addressed is to update the state variables (σ_{n+1},z_{n+1}) through the integration of the viscous equations, either in slow dynamics or in fast dynamics.

fast dynamics
internal variable strictly decreasing (softening)
stress state outside the yield locus

idea:
use z as independent variable
shrink the elastic domain until the stress state come back to the yield surface

end of jump discontinuities:

- the stress state lies on the yield surface $f(\boldsymbol{\sigma}_{s+1}, z_{s+1}) = 0$
- the plastic modulus is positive $K_\mathrm{p}(\boldsymbol{\sigma}_{s+1}, z_{s+1}) > 0$
- integrate the equation of slow dynamics (τ =0)

numerical examples

1. single-element tests (undrained triaxial tests)
   1.1 accuracy and ‘adaptive’ ws ‘standard’
   1.2 preconsolidation pressure

2. BVP (plane strain compression tests/FEAP)
   2.1 standard viscoplasticity
   2.2 adaptive regularization

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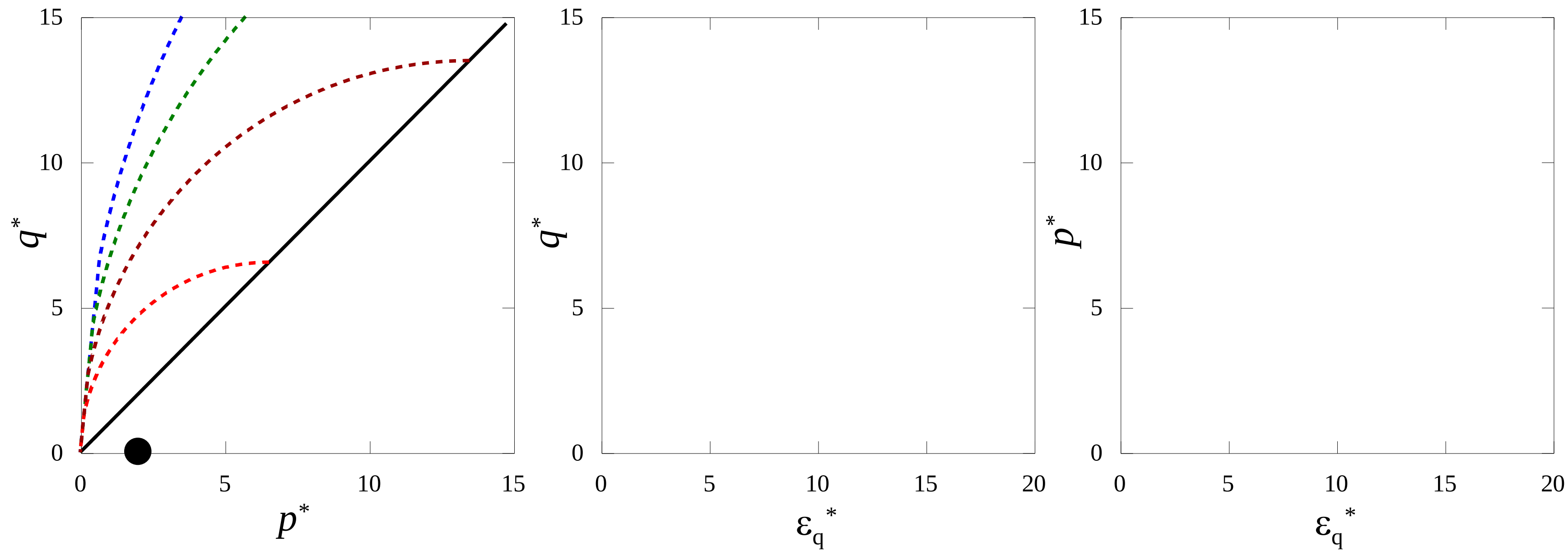
Dal Maso & DeSimone (2009)

simplified MMC model (elastic stiffness/hardening law)

'adaptive regularization', **τ=0**



$$\boldsymbol{\sigma}^* := \frac{1}{p_a}\boldsymbol{\sigma} \qquad z^* := \frac{z}{p_a} \qquad \boldsymbol{\varepsilon}^* := \frac{2G}{p_a}\boldsymbol{\varepsilon}$$

initial condition:

$p^*_0 = 2$

internal variable:

$z^*_0 = 6.54$

$z^*_0 = 13.48$

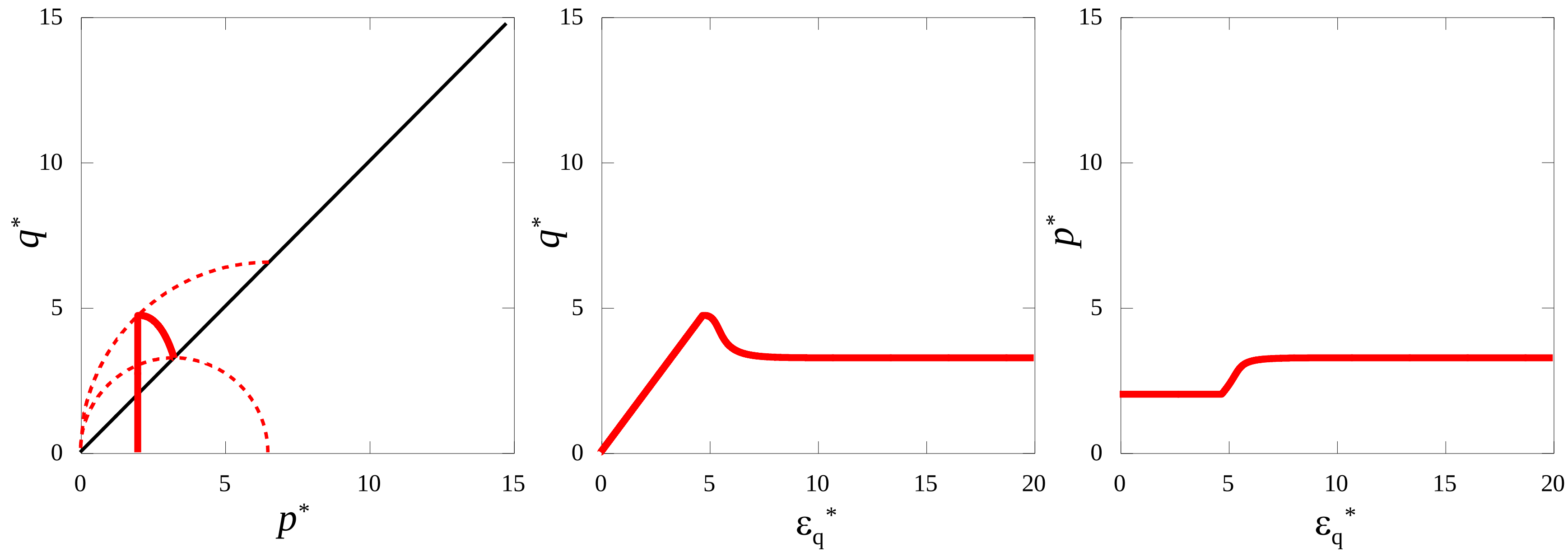
$z^*_0 = 22.50$

$z^*_0 = 33.71$

Dal Maso & DeSimone (2009)

simplified MMC model (elastic stiffness/hardening law)

'adaptive regularization', τ=0



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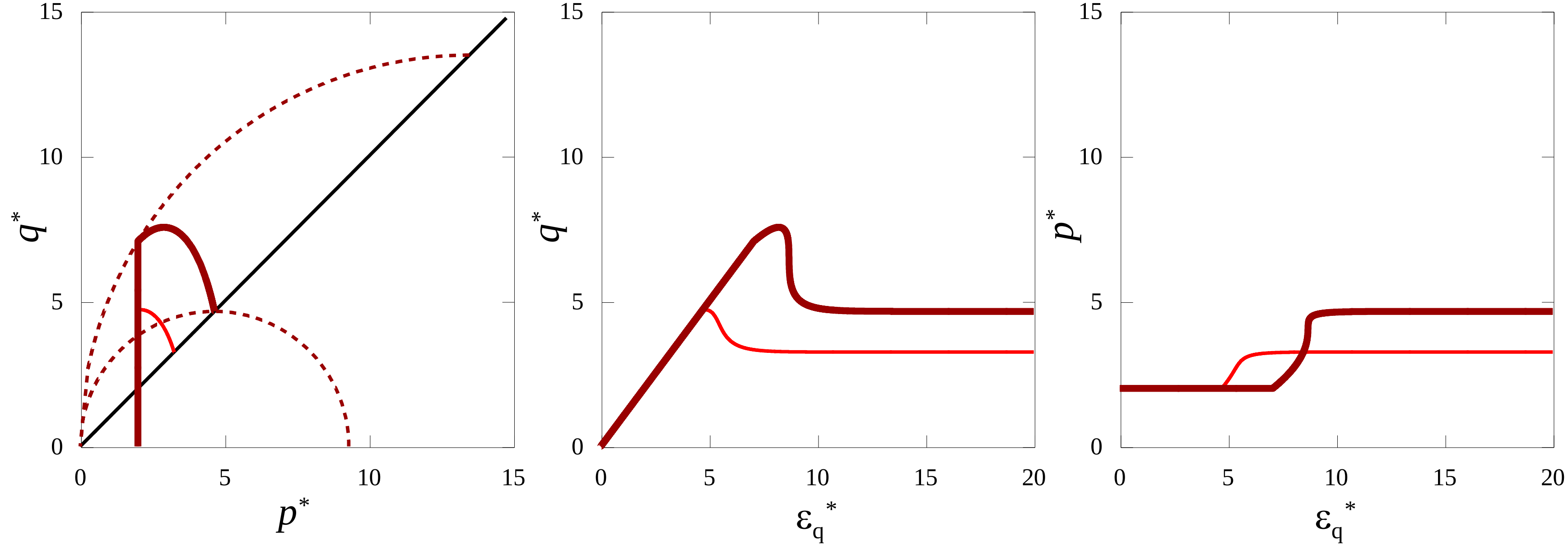

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numerical examples

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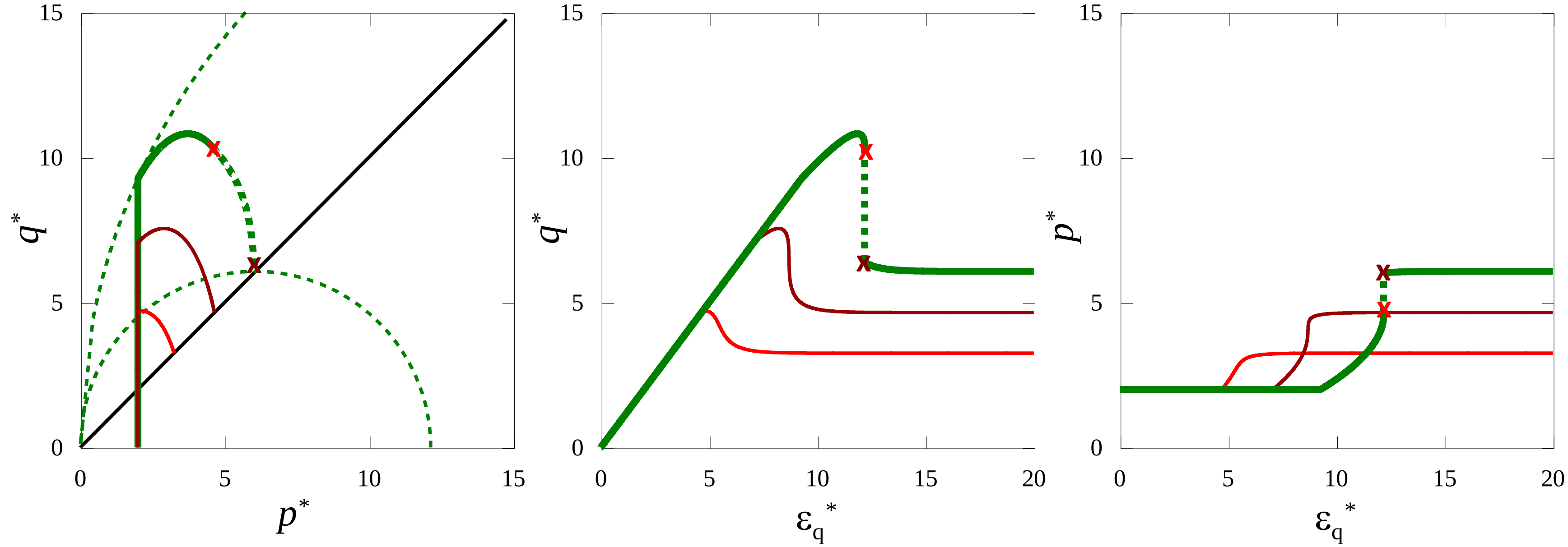

x inception of jump

$\Delta\gamma_{n+1} < 0$

x end of jump:

$f(\sigma_{s+1}, z_{s+1}) = 0$

$K_P(\sigma_{s+1}, z_{s+1}) > 0$

internal variable:

$z^*_0 = 6.54$

$z^*_0 = 13.48$

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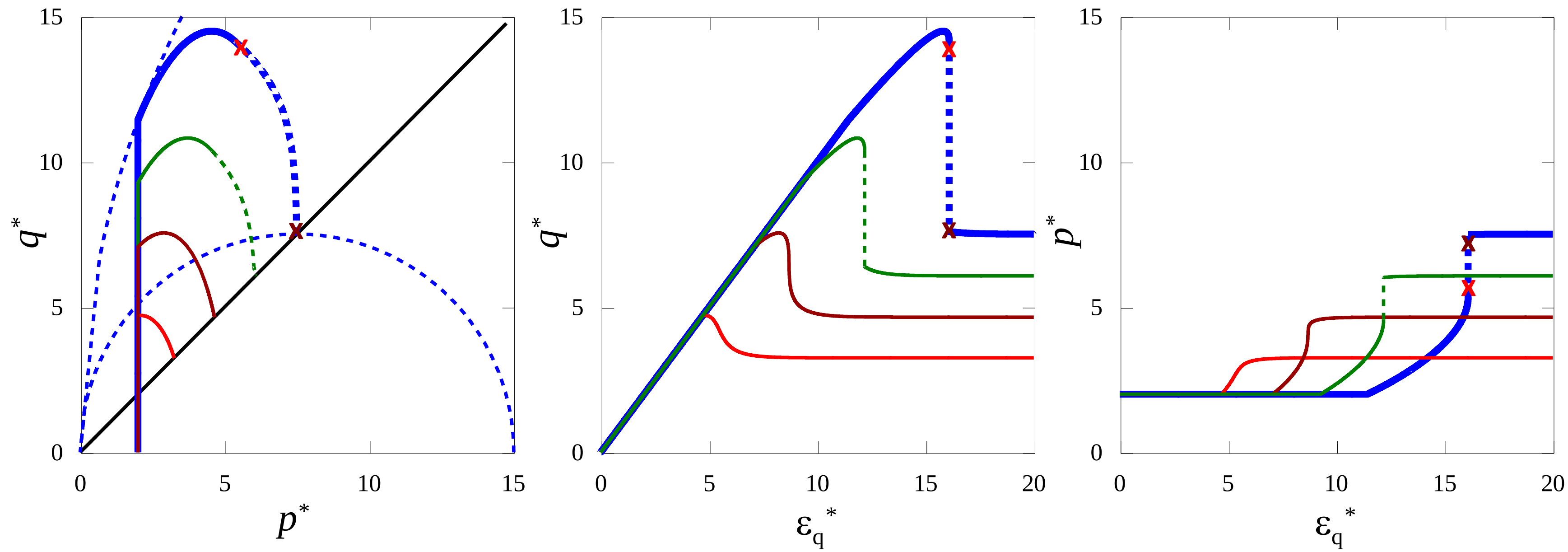
Dal Maso & DeSimone (2009)

simplified MMC model (elastic stiffness/hardening law)

'adaptive regularization', **τ=0**



x inception of jump

$\Delta\gamma_{n+1} < 0$

x end of jump:

$f(\sigma_{s+1}, z_{s+1}) = 0$

$K_P(\sigma_{s+1}, z_{s+1}) > 0$

internal variable:

$z^*_0 = 6.54$

$z^*_0 = 13.48$

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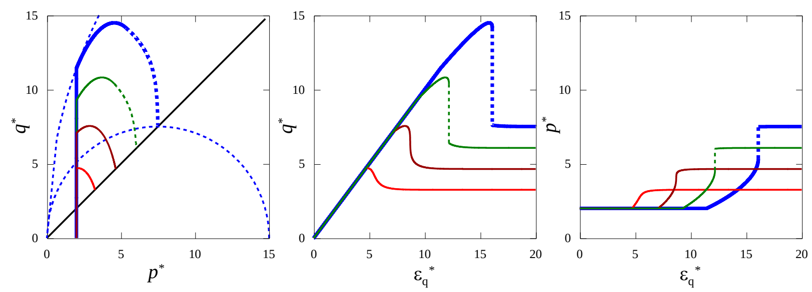
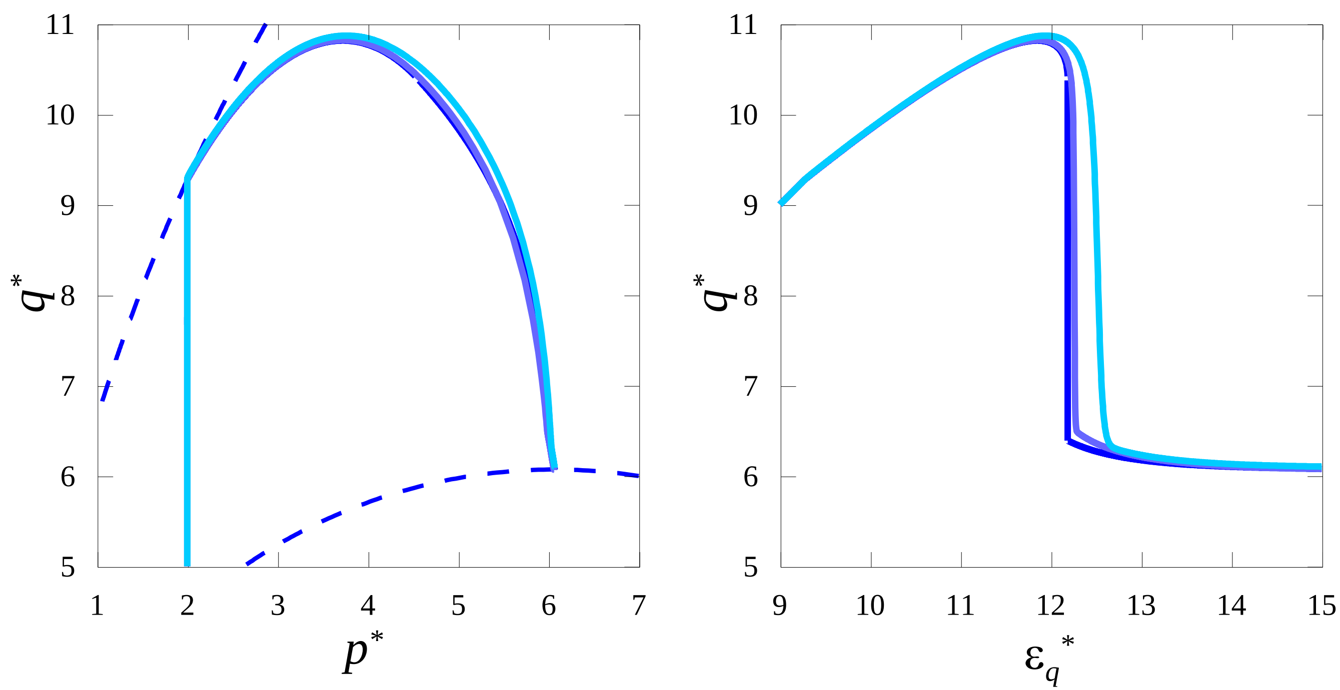
Dal Maso & DeSimone (2009)
simplified MMC model
'adaptive regularization'/standard viscoplasticity





regularization parameter:

τ=0
τ=1e-02
τ=1e-01

internal variable:

z^*_0 = 6.54
z^*_0 = 13.48
z^*_0 = 22.50
z^*_0 = 33.71

numerical examples

1. single-element tests (undrained triaxial tests)
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2. BVP (plane strain compression tests/FEAP)
   2.1 standard viscoplasticity
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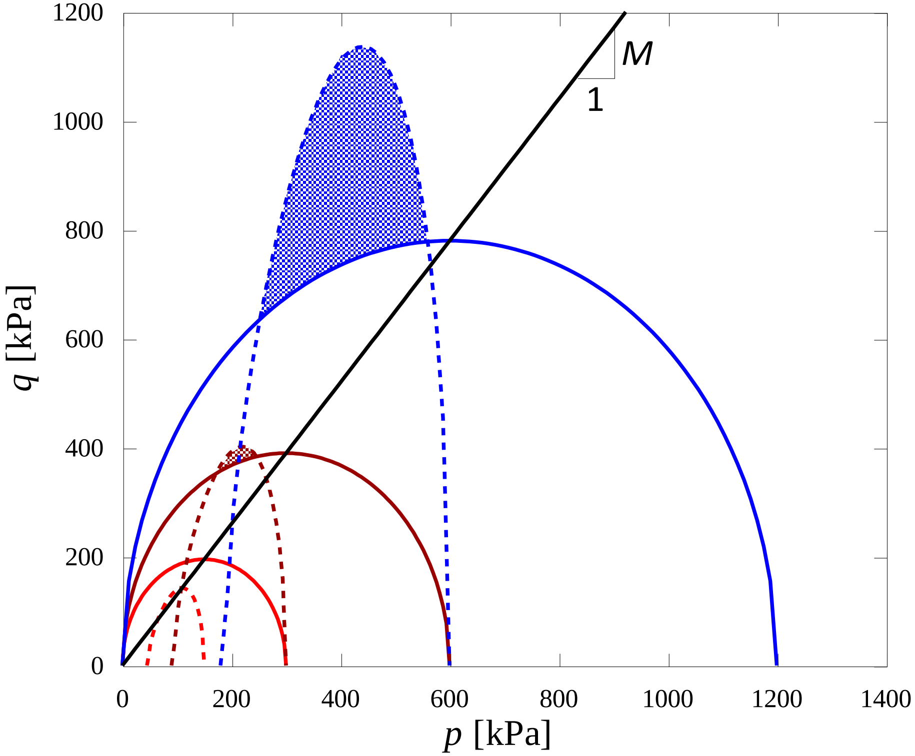


material parameters:

$$M = 1.3$$
$$G = 1 \text{ MPa}$$
$$\hat{\kappa} = 0.013$$
$$\hat{\lambda} = 0.032$$

preconsolidation pressure:

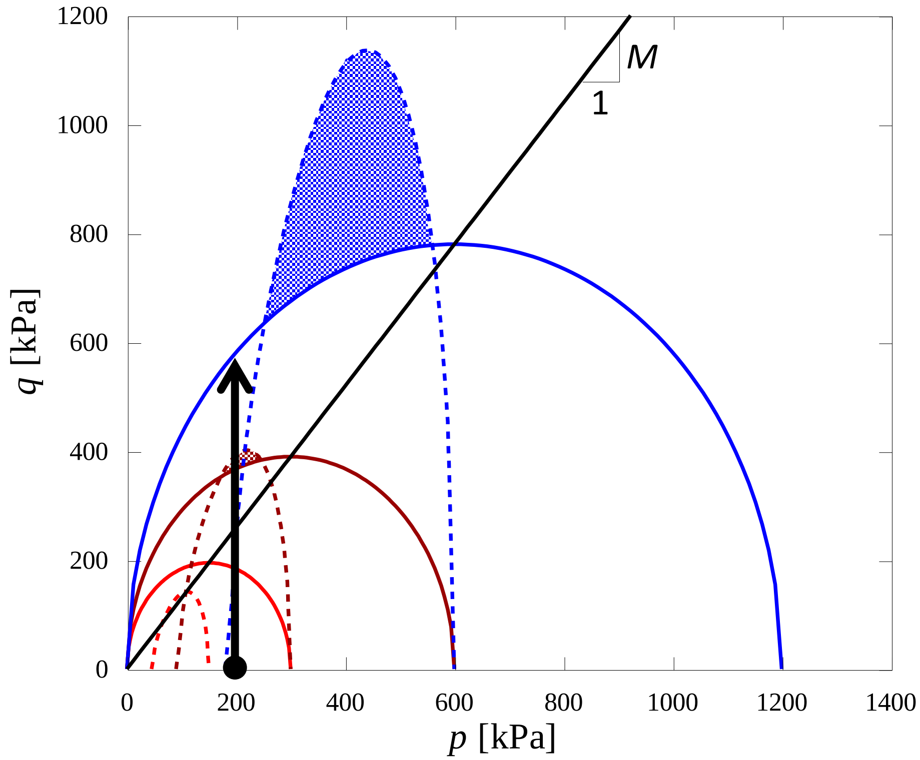
p_{c0} = 300kPa
p_{c0} = 600kPa
p_{c0} = 1200kPa

$f(q, p, z) = 0$

$K_p(q, p, z) = 0$

initial condition:

p_0 = 200kPa



material parameters:

$$M = 1.3$$
$$G = 1 \text{ MPa}$$
$$\hat{\kappa} = 0.013$$
$$\hat{\lambda} = 0.032$$

preconsolidation pressure:

p_{c0} = 300kPa
p_{c0} = 600kPa
p_{c0} = 1200kPa

——— $f(q, p, z) = 0$

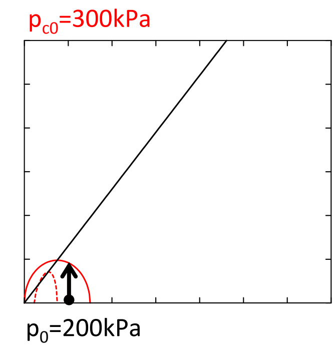
- - - - $K_p\,(q, p, z) = 0$

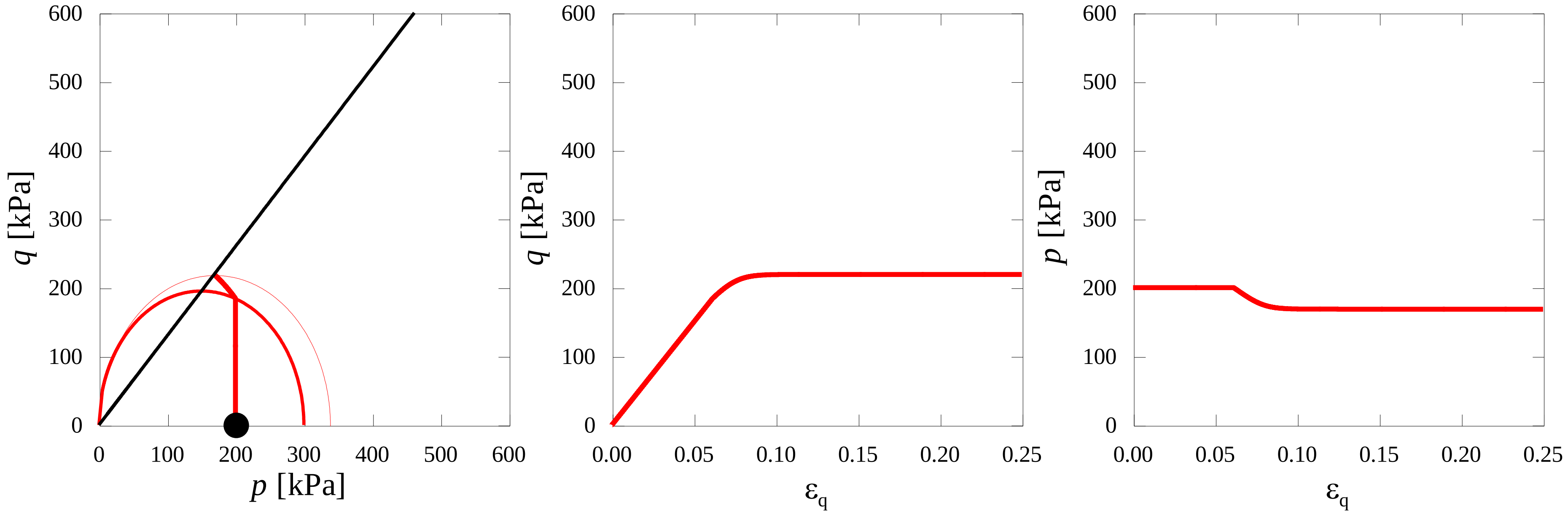
initial condition:

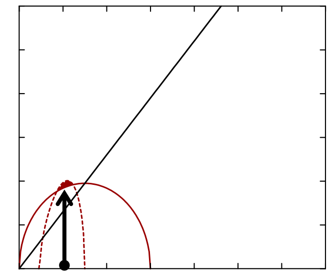
p_0 = 200kPa

preconsolidation pressure:

p_{c0} = 300kPa
p_{c0} = 600kPa
p_{c0} = 1200kPa





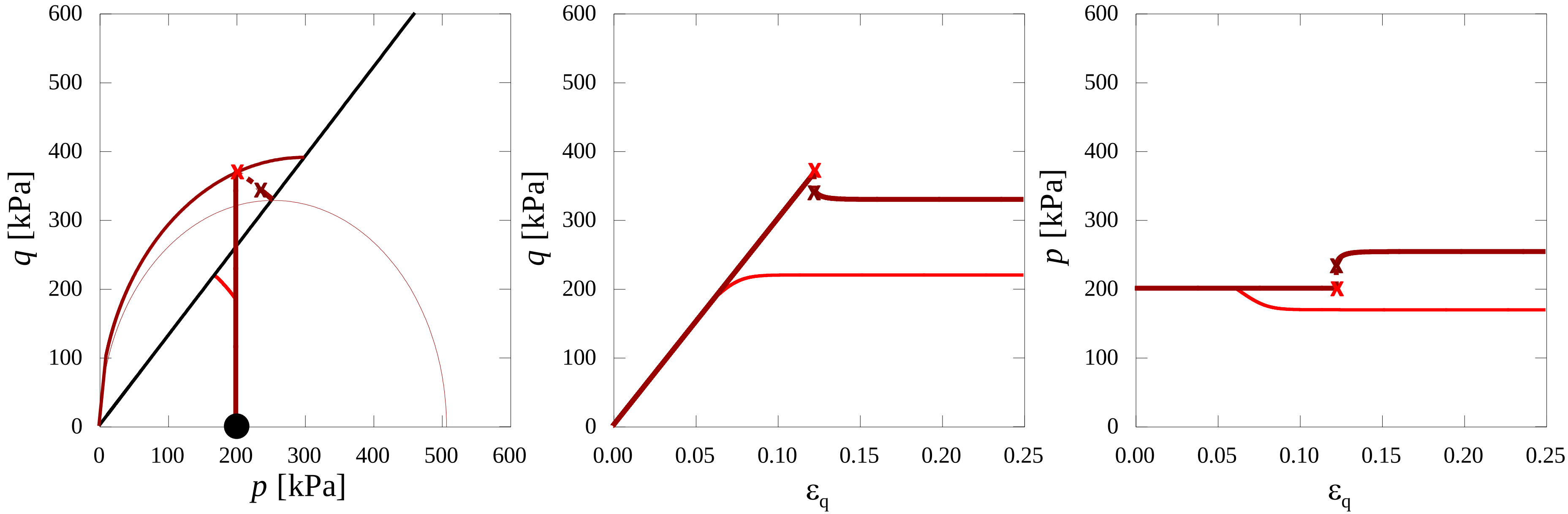


initial condition:

p_0 = 200kPa

preconsolidation pressure:

p_{c0} = 300kPa
p_{c0} = 600kPa
p_{c0} = 1200kPa

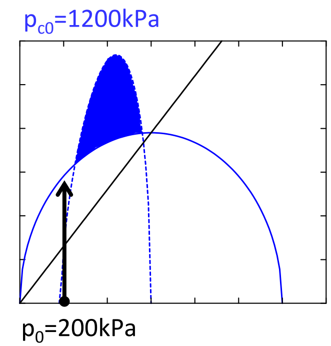


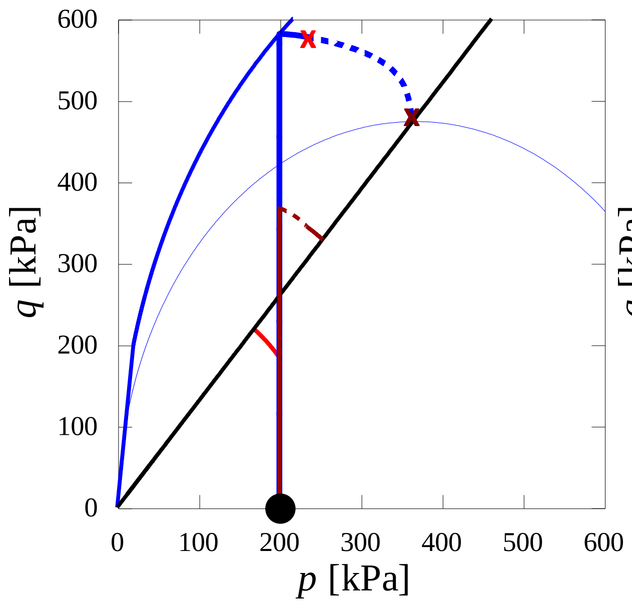
initial condition:

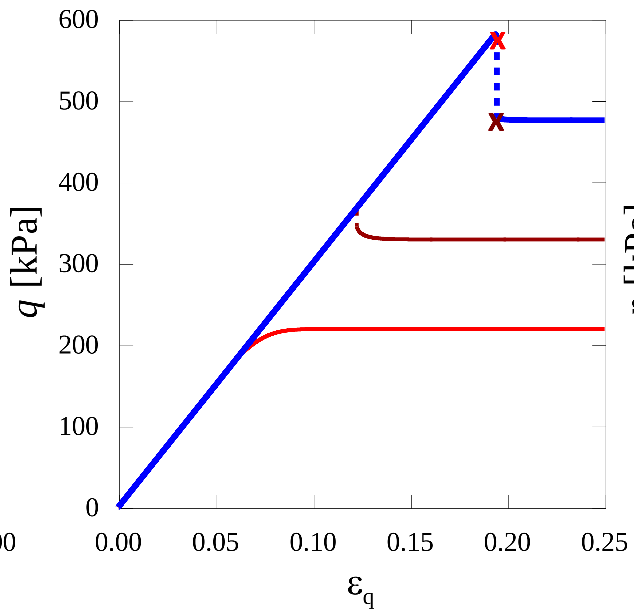
p_0 = 200kPa

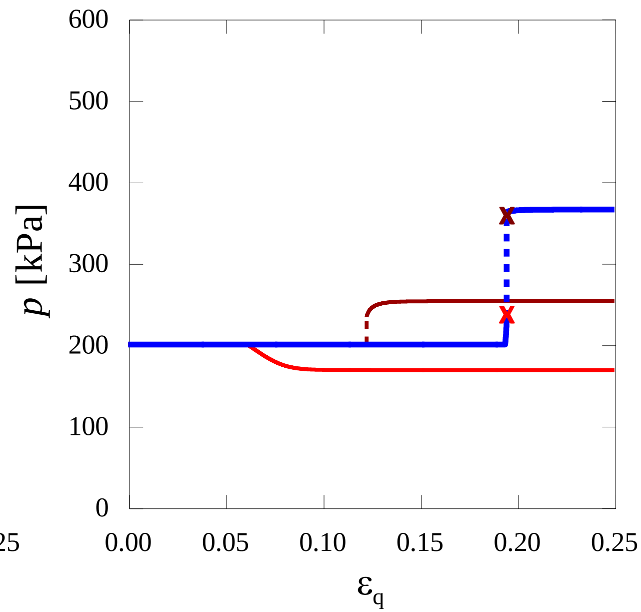
preconsolidation pressure:

p_{c0} = 300kPa
p_{c0} = 600kPa
p_{c0} = 1200kPa









numerical examples

1. single-element tests (undrained triaxial tests)
   1.1 accuracy and 'adaptive' ws 'standard'
   1.2 preconsolidation pressure

2. BVP (plane strain compression tests/FEAP)
   2.1 standard viscoplasticity
   2.2 adaptive regularization

numerical examples

standard plane strain compression tests

FE code FEAP

BVP #1

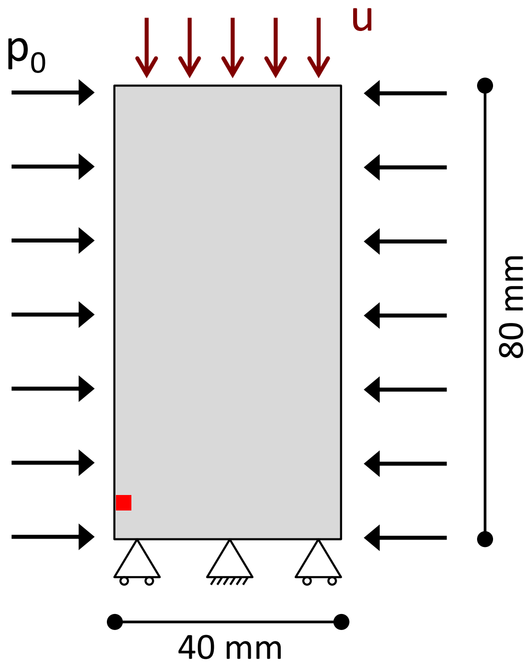
local: standard viscoplasticity (**$\tau \neq 0$**)

global: quasi-static (N-R algorithm)

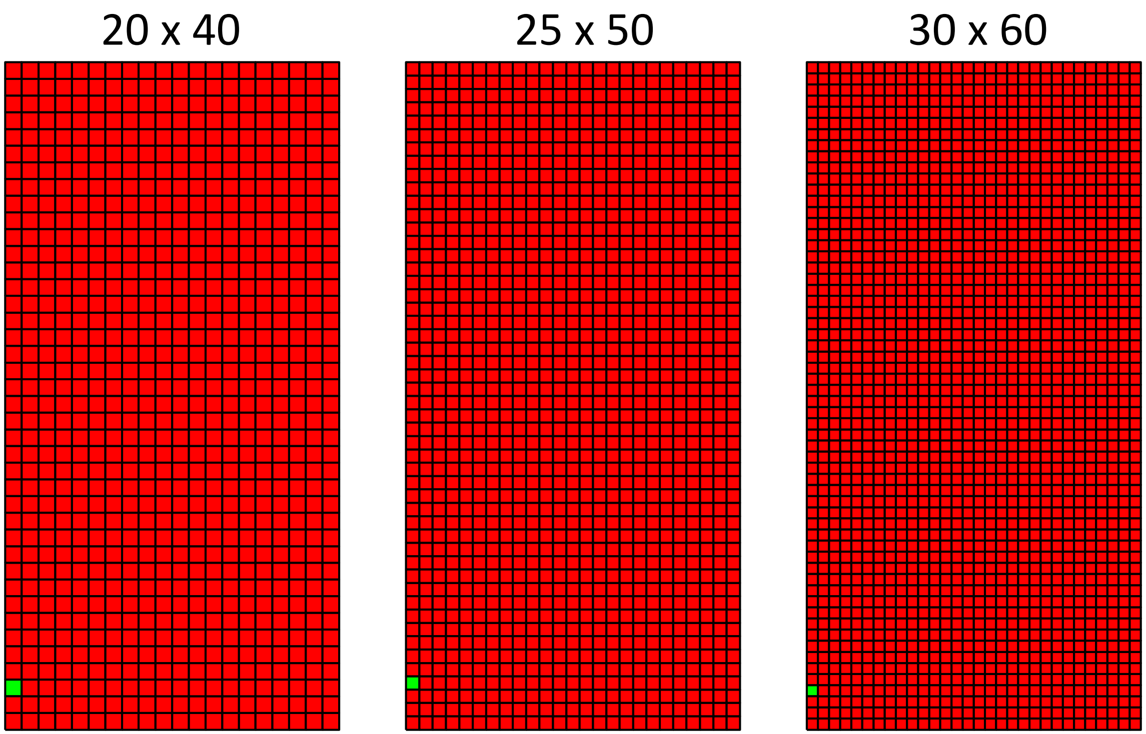
BVP #2

local: ‘adaptive’ regularization (**$\tau = 0$**)

global: dynamic (Newmark explicit)



local: standard viscoplasticity ($\tau \neq 0$)
global: quasi-static (N-R algorithm)



preconsolidation pressure:

p_{c0} = 1200kPa

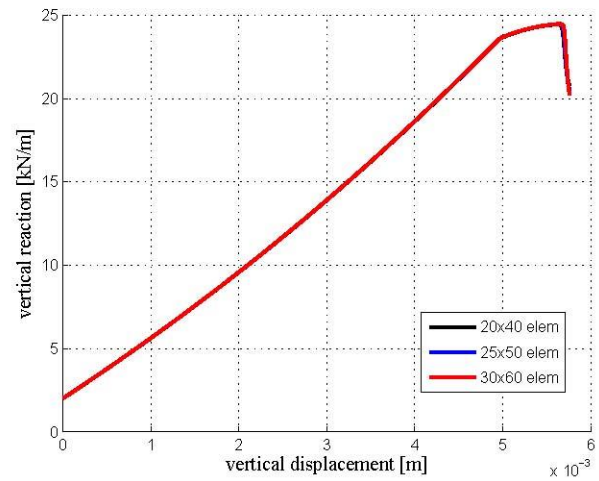
initial condition:

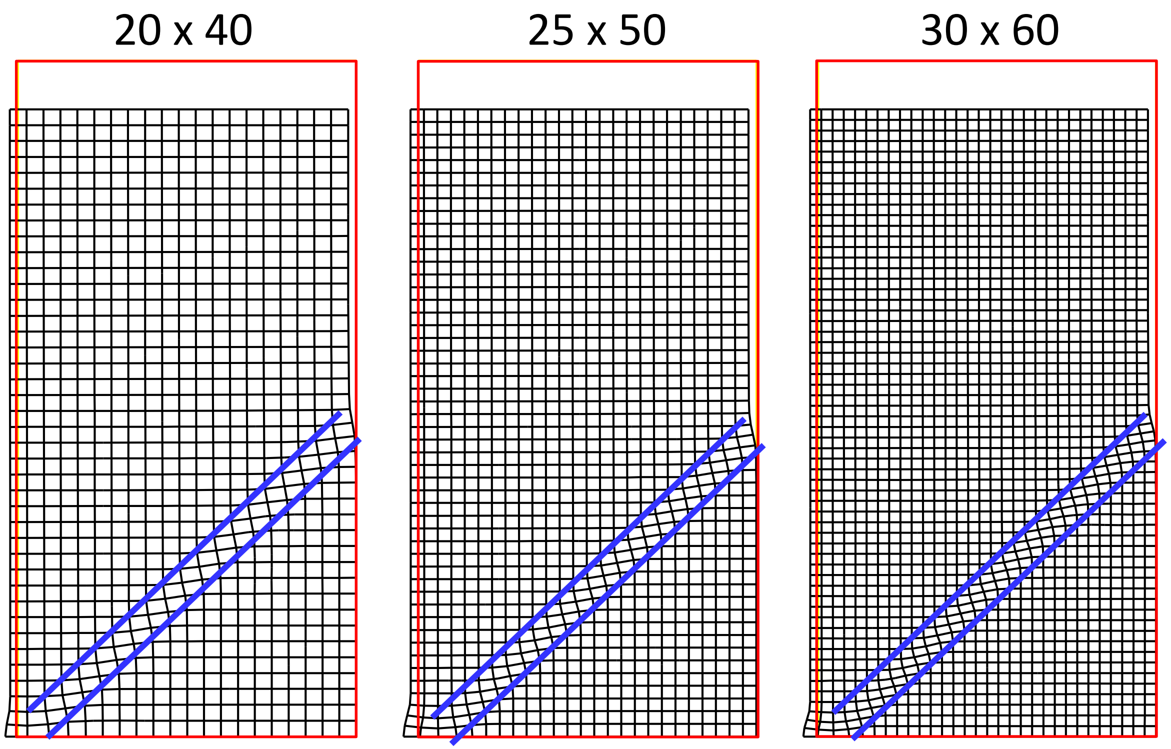
p_0 = 50kPa

material parameters:

$M = 1.3$
$G = 5$ MPa $G = 2$ MPa
$\hat{\lambda} = 0.100$ $\hat{\lambda} = 0.090$
$\hat{\kappa} = 0.050$
$\tau = 0.001s$

local: standard viscoplasticity ($\tau \neq 0$)
global: quasi-static (N-R algorithm)





preconsolidation pressure:

p_{c0} = 1200kPa

initial condition:

p_0 = 50kPa

material parameters:

$M = 1.3$

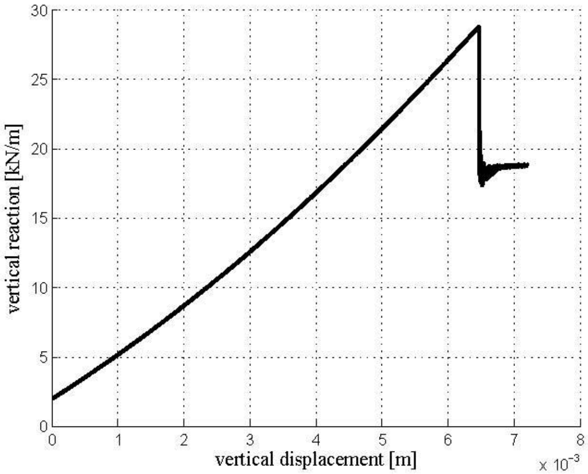
$G = 5$ MPa $G = 2$ MPa

$\hat{\lambda} = 0.100$ $\hat{\lambda} = 0.090$

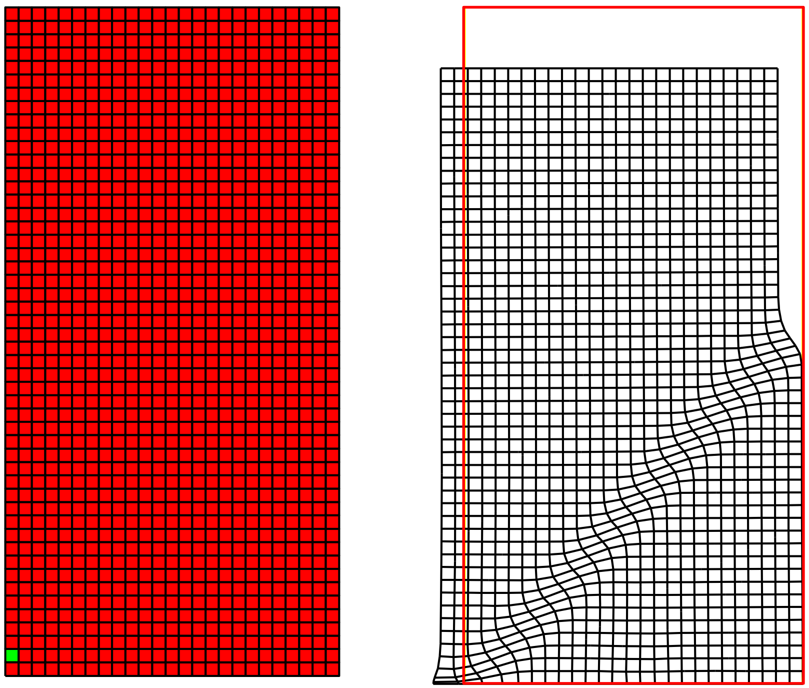
$\hat{\kappa} = 0.050$

$\tau = 0.001s$

local: 'adaptive' regularization (τ=0)
global: dynamic (Newmark explicit: β=0, γ=0.5)



25 x 50



<u>preconsolidation pressure:</u>

p_{c0} = 1200kPa

<u>initial condition:</u>

p_0 = 50kPa

<u>material parameters:</u>

$M = 1.3$
$G = 4$ MPa $G = 2.5$ MPa
$\hat{\lambda} = 0.100$ $\hat{\lambda} = 0.070$
$\hat{\kappa} = 0.045$
$\tau = 0s$

conclusions & perspectives

CONCLUSIONS

viscoul regularization

Cam-clay plasticity

Limit solution ($\tau \rightarrow 0$) – adaptive regularization

∃! solution beyond critical softening

numerical implementation

exploiting properties of viscous limit solution

jump discontinuities

mesh sensitivity (BVP)

PERSPECTIVES

extend the proposed approach to:

plasticity models more suitable for softening

critical softening driven by chemo-mechanical coupling effects

combine the proposed approach with:

non-local approach (localization phenomena)

references

[1] Conti, Tamagnini, DeSimone (2013). “Critical softening in Cam-Clay plasticity: adaptive viscous regularization, dilated time and numerical integration across stress-strain jump discontinuities”. *Comput. Methods Appl. Mech Engrg*. 258, 118-133.

[2] Dal Maso, DeSimone, Solombrino (2011). “Quasistatic evolution for cam-clay plasticity: a weak formulation via viscoplastic regularization and time rescaling”. *Calc. Var.* 40, 125-181.

[3] Dal Maso, Solombrino (2010). “Quasistatic evolution for cam-clay plasticity: the spatially homogeneous case”. *Netw. Heter. Media* 5, 97-132.

[4] Dal Maso, DeSimone (2009). “Quasistatic evolution for cam-clay plasticity: examples of spatially homogeneous solutions”. *Math. Models Methods Appl. Sci.* 19, 1643-1711.