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A microstructural model for hydraulic conductivity evolution due to brittle damage

G. Della Vecchia [(1)], A. Pandolfi [(1)], M.L. De Bellis [(2)], & M. Ortiz [(3)]

(1) Politecnico di Milano

(2) PoliMI, currently Università del Salento

(3) California Institute of Technology

Introduction

Fractures and discontinuities: among the most important features of geological structures

Natural rock formations: fractures and discontinuities facilitate storage and movements of fluids

Introduction

Fracture processes exploited in engineering technology

- Prediction of reservoir integrity hazardeous waste storage

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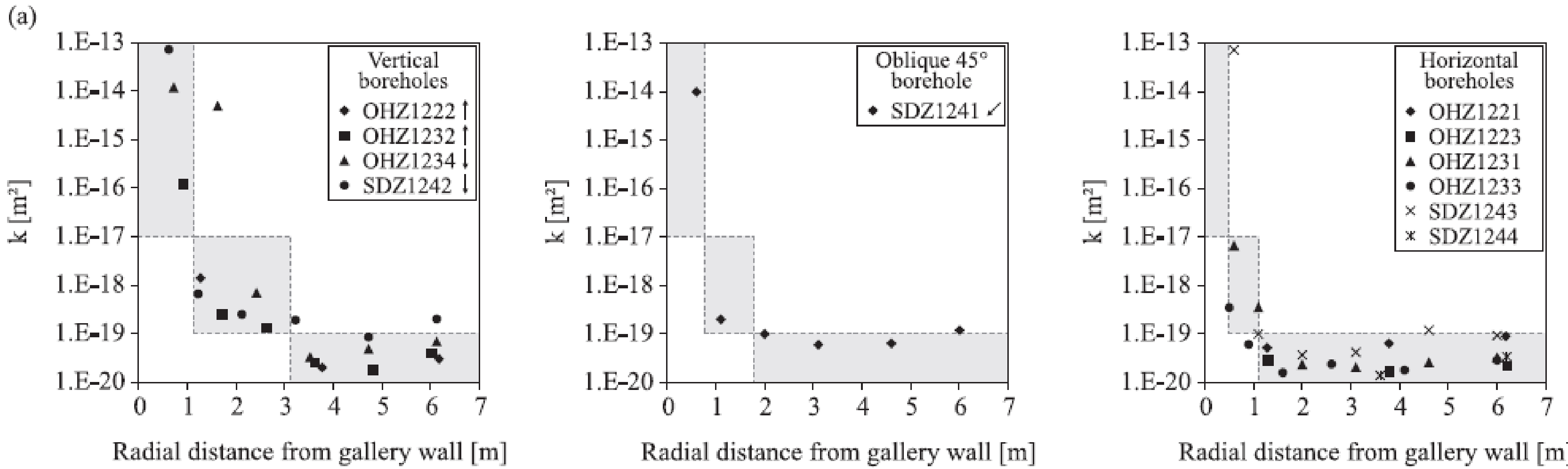
Fracture processes exploited in engineering technology

- Prediction of reservoir integrity hazardeous waste storage

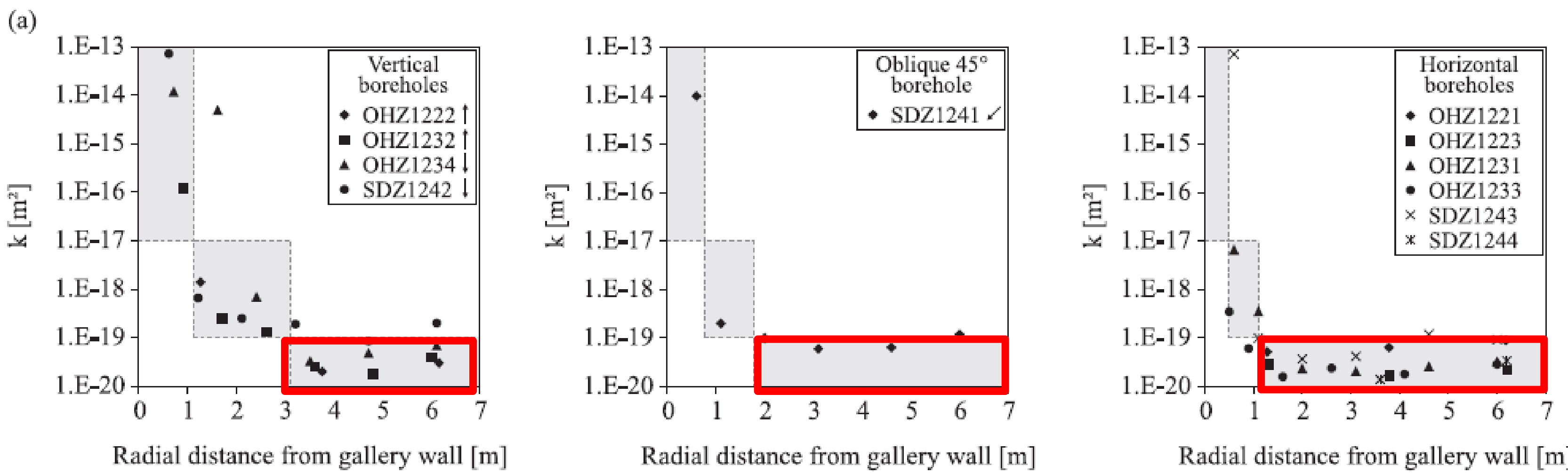


Pardoen et al., 2016

URL experimental tunnel (Andra) in Collovo-Oxfordian claystone: permeability increase by measurements performed in boreholes drilled in different orientations

Fracture processes exploited in engineering technology

- Prediction of reservoir integrity hazardeous waste storage



Pardoen et al., 2016

Undisturbed zone, $k<10^{-19}$ m^2

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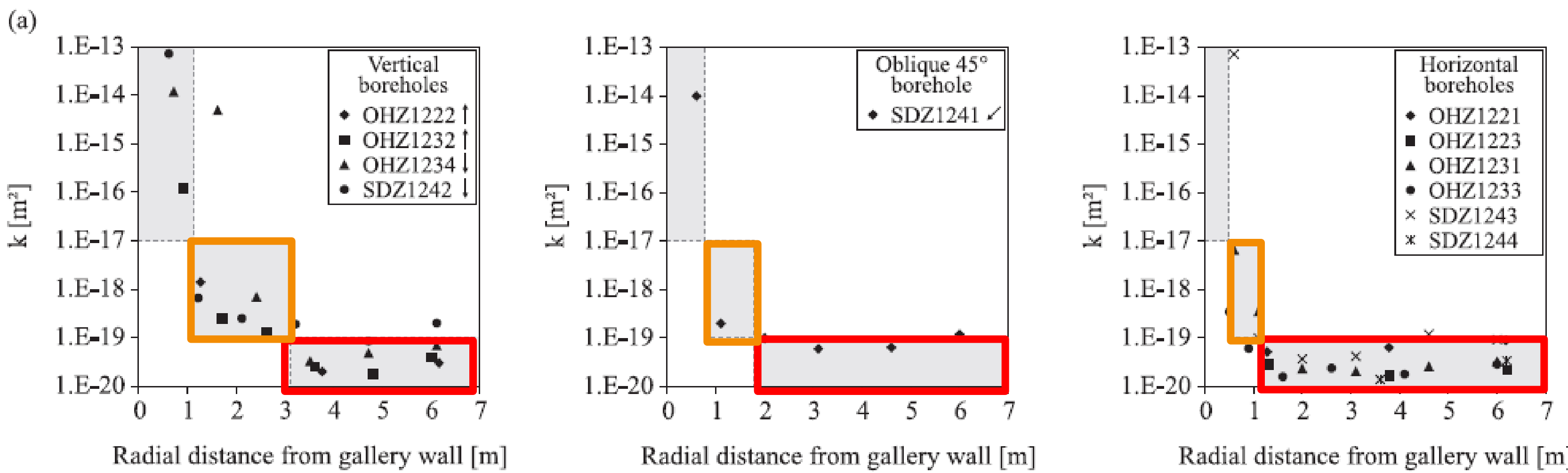

Pardoen et al., 2016

Undisturbed zone, $k<10^{-19}$ m^2

Slightly disturbed zone, k from 10^{-19} m^2 to 10^{-17} m^2

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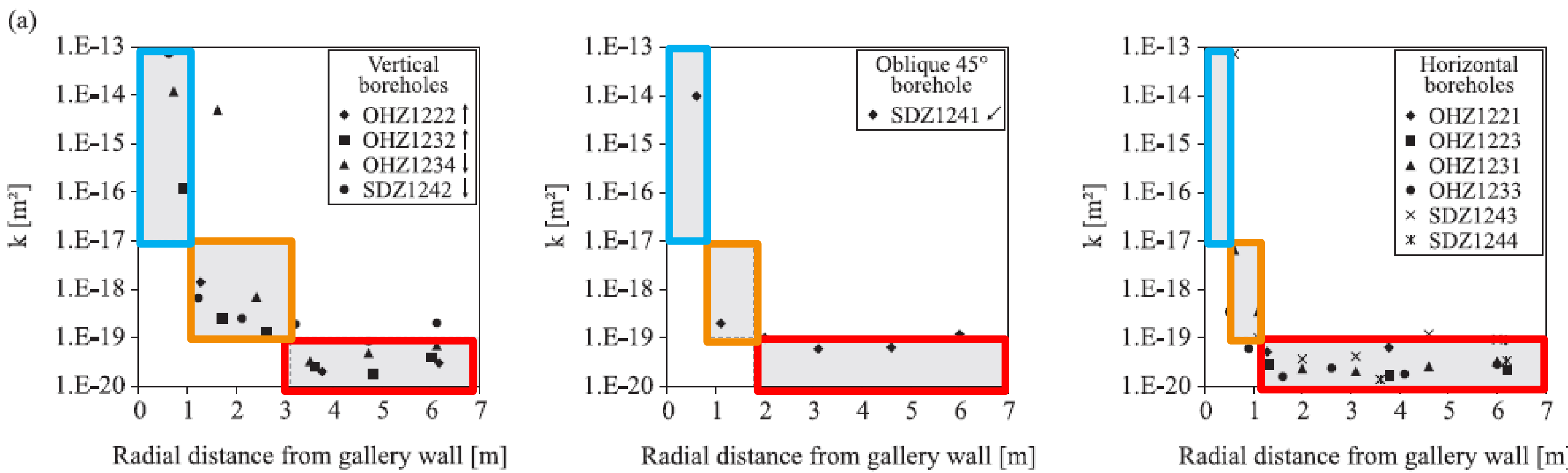

Pardoen et al., 2016

Undisturbed zone, k $<10^{-19}$ m^2

Slightly disturbed zone, k from 10^{-19} m^2 to 10^{-17} m^2

Highly disturbed zone, k $>10^{-17}$ m^2

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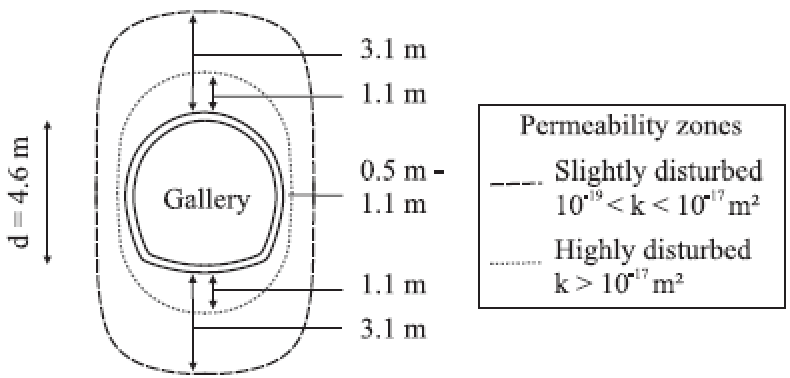
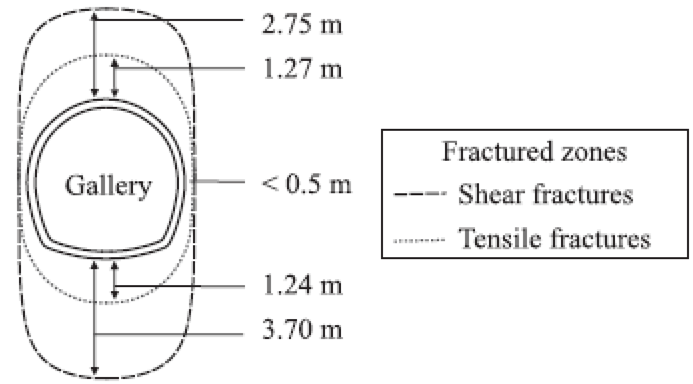
Fracture processes exploited in engineering technology

- Prediction of reservoir integrity hazardeous waste storage





Pardoen et al., 2016

URL experimental Tunnel (Andra) in Collovo-Oxfordian claystone: relation between permeability and shear/tensile fracture zones

Introduction

Fracture processes exploited in engineering technology

- Prevention of water/gas outburst into underground mines
- Prediction of reservoir integrity for CO2 sequestration
- Prediction of water flow into galleries.....

→ The excavation of underground structures in rock masses induces cracking → significant changes in flow and permeability → Modification of pore pressure → modification in the mechanical response

Outline

Which kind of model?

Governing equations

Linearized porous brittle damage material model with distributed frictional-cohesive faults

- Fault kinematics;
- Energetic contributions;
- Link with porosity and permeability;

Some examples:

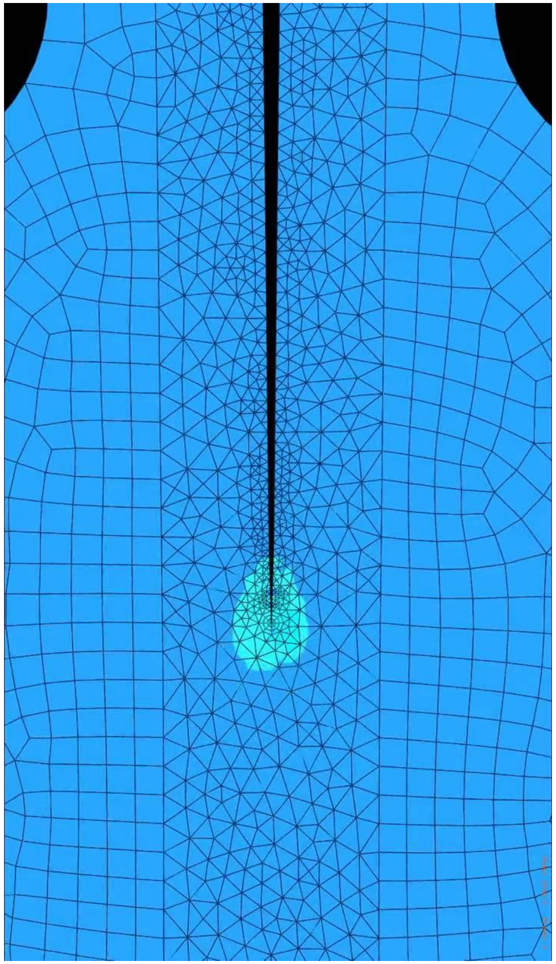
- Sensitivity analysis
- Experimental triaxial test simulations
- Excavation of a borehole

Which kind of model?

Yuan & Harrison (2006) "A review of the state of the art in modelling progressive mechanical breakdown and associated fluid flow in intact heterogeneous rocks" IJRMMS 43

Discrete models based on fracture mechanics

- Open crack and sliding crack models used to simulate the progressive microfracturing of rock upon loading
- Need of a well-defined crack or defect, with known orientation, spacing, length and frictional properties (normally undetectable properties)
- Difficulties in modelling the interaction between cracks (i.e. permeability increase due to the increased connectivity between cracks) (e.g. Pouya, 2015)



Which kind of model?

Yuan & Harrison (2006) "A review of the state of the art in modelling progressive mechanical breakdown and associated fluid flow in intact heterogeneous rocks" IJRMMS 43

Continuum damage mechanics models

- Phenomenological approach considering the averaged effect of microstructural changes to reproduce the hydro-mechanical response during the progressive degeneration of rocks
- Rock mass containing a large number of discontinuities as a homogeneous anisotropic porous medium
- Possibility to derive a permeability tensor from a damage crack tensor (Oda 1985, Shao et al 2005, Arson & Pereira 2013, Lavasseur et al 2013)

Which kind of model?

Our proposal

- Development of a continuum model of distributed fracturing of rock masses based on an explicit micromechanical construction of connected patterns of cracks (from Pandolfi et al 2006) and of the related permeability variation

- Explicit fracture patterns

- Not arbitrary fracture patterns→ inception, orientation and spacing of fractures derives from energetic consideration

- Analytical calculation of porosity and permeability

Governing equations

Solid

Linear momentum balance

$$\nabla \cdot \boldsymbol{\sigma} + \boldsymbol{b} = 0$$

Fluid

Fluid mass balance

$$\frac{\partial \rho_f n S_r}{\partial x} + \nabla \cdot \rho_f \boldsymbol{q} = 0$$

Assumptions

Linearized kinematics

$$\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}^m + \boldsymbol{\varepsilon}^f$$

Incompressible pore fluid

$$\rho_f = cost.$$

Fluid saturated medium

$$S_r = 1$$

Constitutive laws

Brittle damage constitutive law

$$\boldsymbol{\sigma}' = \boldsymbol{\sigma}'(\boldsymbol{\varepsilon}^{\mathrm{m}})$$

$$\boldsymbol{\sigma} = \boldsymbol{\sigma}' + p\boldsymbol{I}$$

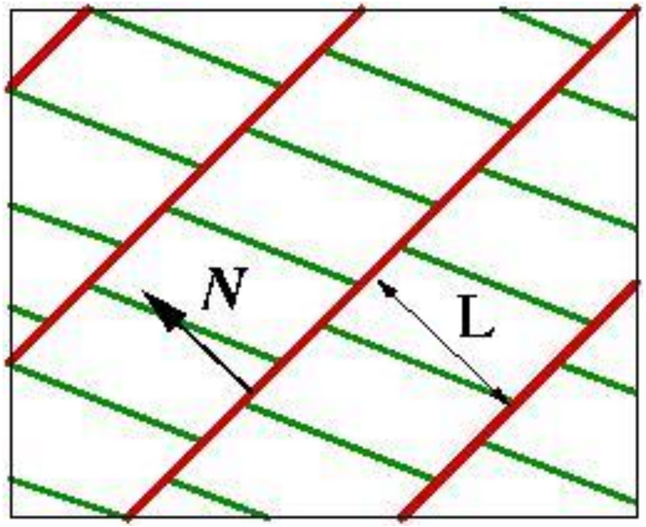
$$\boldsymbol{q} = -\boldsymbol{k}\frac{\rho_f q}{\mu}\nabla h$$

Permeability tensor expressed as a function of faults distribution

Brittle damage model: some definitions

- The model used is characterized by a homogeneous matrix with embedded nested families of equi-spaced cohesive faults.

- Each level k of cohesive faults is characterized by an orientation (defined by the normal $\mathbf{N}_k$ to the faults) and a spacing L_k.

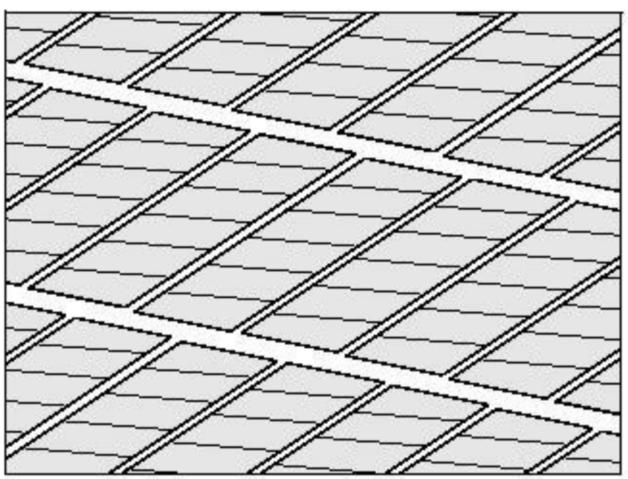


- L is a microstructural feature of the material that derives from optimality conditions on the system energy

Brittle damage model: kinematics of faults

- The material deforms due to both matrix deformation and fault development
- Additive decomposition of the macroscopic strain (small strain kinematics):

$$\boldsymbol{\varepsilon} = \mathrm{sym}\nabla \boldsymbol{u} = \boldsymbol{\varepsilon}^{\mathrm{m}} + \boldsymbol{\varepsilon}^{\mathrm{f}}$$



ε^m = matrix deformation

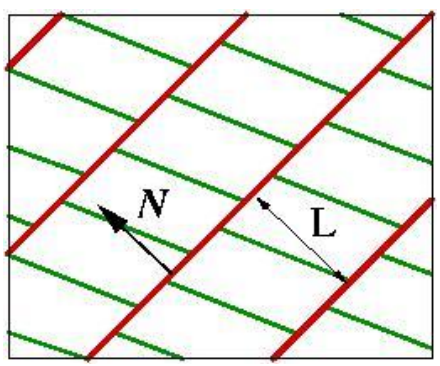
ε^f = deformation due to faults

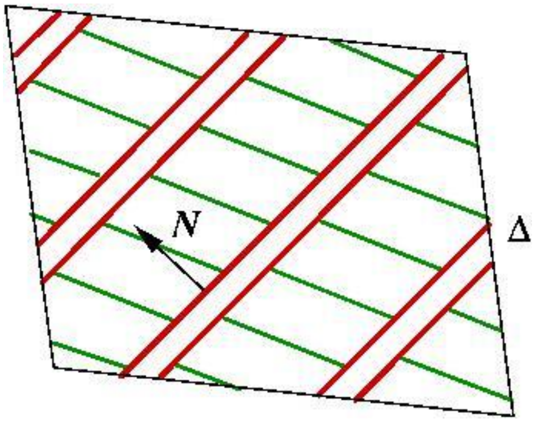
Brittle damage model: kinematics of faults

Straightforward determination of rock deformation due to the opening of a single fault family (spacing L and normal **N**).

- Take a segment d**x** that spans two material points and define the number of faults *n* traversed by the vector

$$n = \frac{1}{L} d\boldsymbol{x} \cdot \boldsymbol{N}$$



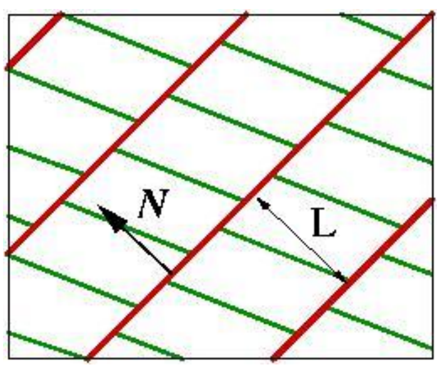


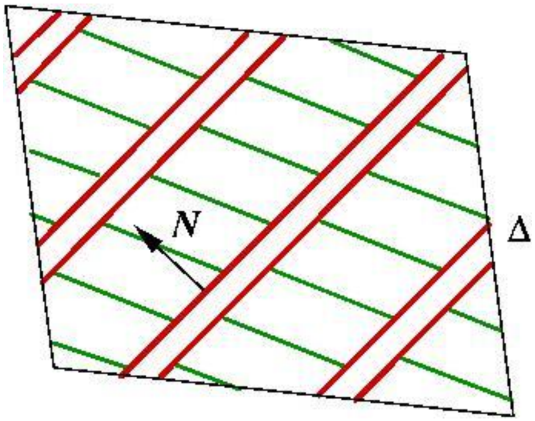
Brittle damage model: kinematics of faults

Straightforward determination of rock deformation due to the opening of a single fault family (spacing L and normal **N**).

- Suppose that the opening displacement **Δ** is applied to all the *n* faults and obtain the displacement

$$d\boldsymbol{u}^{\mathrm{f}} = n\boldsymbol{\Delta} = \frac{1}{L}(d\boldsymbol{x}\cdot\boldsymbol{N})\boldsymbol{\Delta} = \frac{1}{L}\boldsymbol{\Delta}\otimes\boldsymbol{N}\,d\boldsymbol{x} \equiv \nabla\boldsymbol{u}^{\mathrm{f}}d\boldsymbol{x}$$



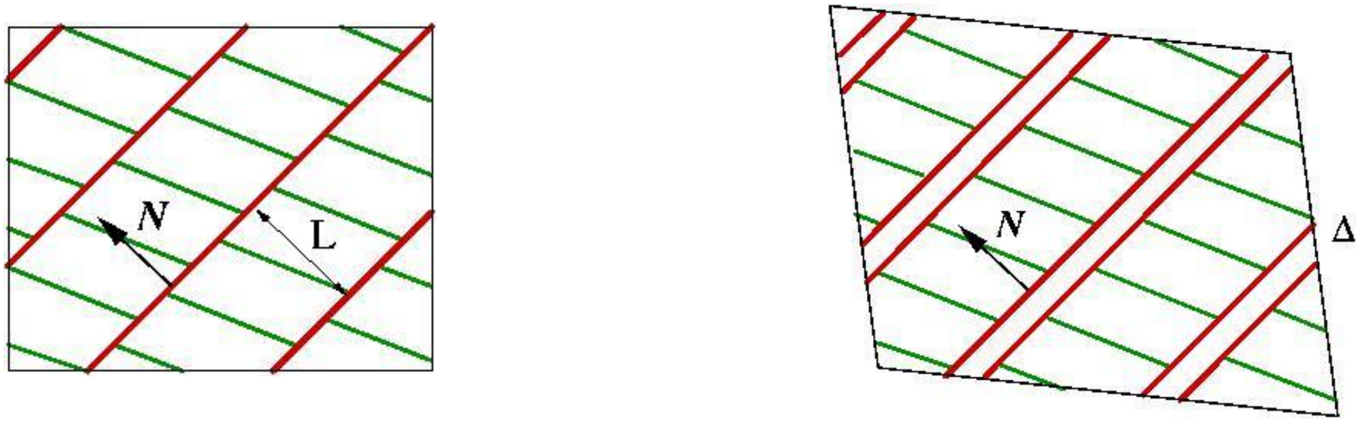


Brittle damage model: kinematics of faults

Straightforward determination of rock deformation due to the opening of a single fault family (spacing L and normal **N**).

- Derive the deformation component due to fault activity:

$$\boldsymbol{\varepsilon}^{\mathrm{f}} = \mathrm{sym}\nabla \boldsymbol{u}^{\mathrm{f}} = \frac{1}{2L}\left(\boldsymbol{\Delta} \otimes \boldsymbol{N} + \boldsymbol{N} \otimes \boldsymbol{\Delta}\right)$$

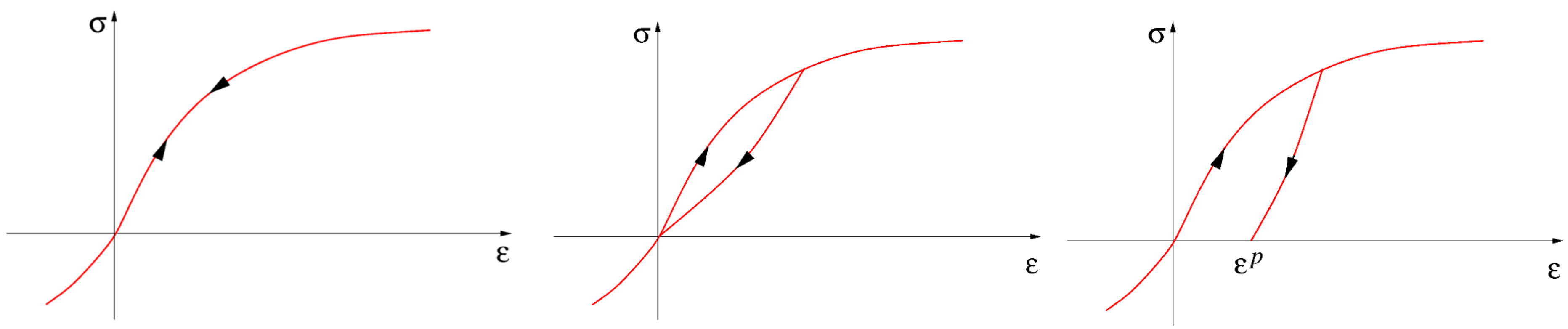


Brittle damage model: elasticity (or inelasticity) of the matrix

- Here we assume linear elastic behaviour for the underlying matrix → Cauchy stress tensor follows as

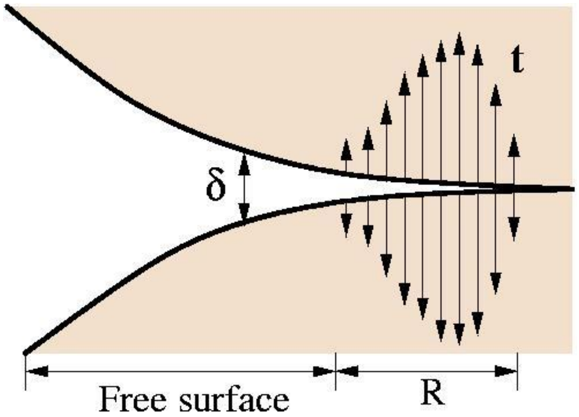
$$W^{\mathrm{m}} = \frac{1}{2}\boldsymbol{\varepsilon}^{\mathrm{m}\,T} \cdot \boldsymbol{D}\boldsymbol{\varepsilon}^{\mathrm{m}} \qquad \boldsymbol{\sigma} = \frac{\partial W^{\mathrm{m}}(\boldsymbol{\varepsilon}^{\mathrm{m}})}{\partial \boldsymbol{\varepsilon}^{\mathrm{m}}}$$

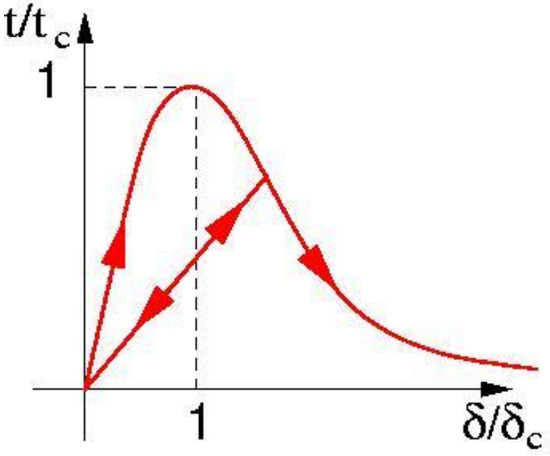
- Any other material model, accounting for plasticity, or viscosity, or other material behavior typical of soils, can be considered for the matrix



Brittle damage model: cohesive approach to fracture

Cohesive theories describe fracture evolution as the progressive separation of two surfaces. The displacement jump δ is resisted by tractions **t** along the cohesive zone R (ahead of the crack tip)





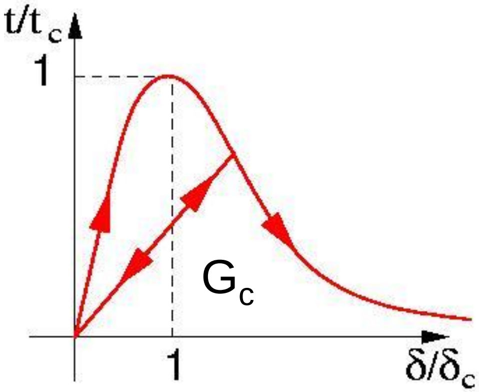
Cohesive law **t**(δ):

relation between tractions **t** acting on R and the displacement jump δ (separation between fracture surfaces)

Brittle damage model: cohesive approach to fracture

Simple uniaxial cohesive laws are defined by two parameters:

- the cohesive strength t_c of the material
- the critical energy release rate G_c

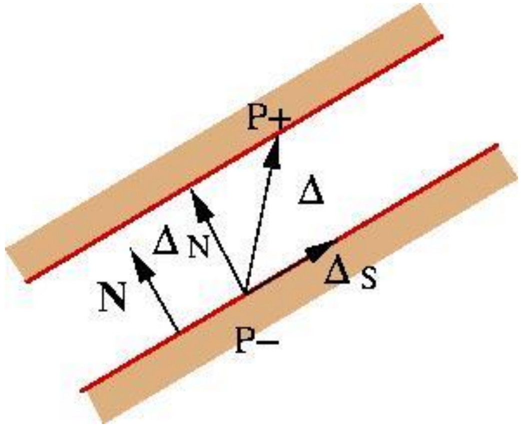


Extension to mixed mode of failure, irreversibility and 3D [Camacho and Ortiz, 1996; Ortiz and Pandolfi, 1999] → Effective opening displacement Δ

$$\Delta = \sqrt{{\Delta^N}^2 + \beta^2 {\Delta^S}^2}$$

$$\Delta^N = \mathbf{\Delta} \cdot \mathrm{N} > 0, \qquad \Delta^S = |\mathbf{\Delta} - \Delta^N \mathrm{N}|$$

β = material parameter

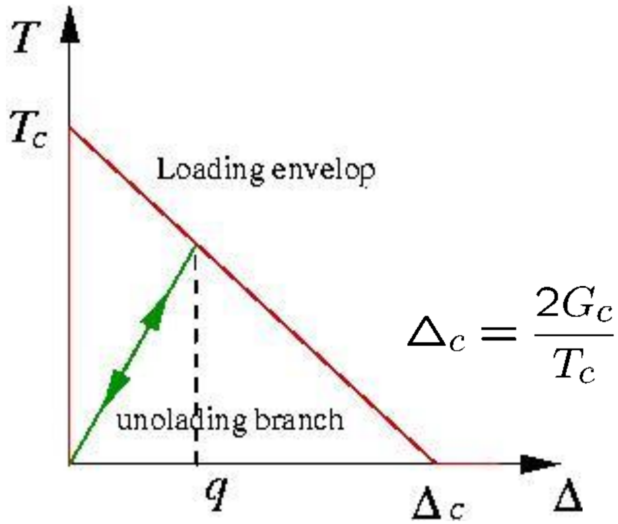


Brittle damage model: cohesive approach to fracture

Cohesive law linking the effective opening displacement Δ and the effective traction T via the effective cohesive energy ϕ

$$\Phi = \Phi(\Delta, \boldsymbol{q})$$

$$T = \frac{\partial \Phi(\Delta, \boldsymbol{q})}{\partial \Delta}$$



Irreversible linear decreasing cohesive law in terms of effective open displacement

G_c = enclosed area = critical energy release rate of the material

T_c = effective tensile resistance

Δ_c =critical opening displacement

Brittle damage model: cohesive approach to fracture

Cohesive law linking the effective opening displacement D and the effective traction T via the effective cohesive energy ϕ

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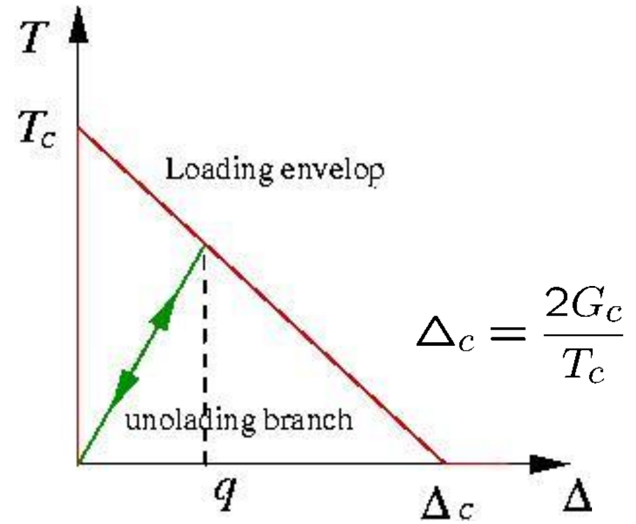

Irreversibility enforced by assuming unloading to the origin → one internal variable q equal to the maximum Δ: $q = \Delta_{max}$

$$\dot{q} = \begin{cases} \dot{\Delta}, & \text{if } \Delta = q \text{ and } \dot{\Delta} \geq 0 \text{ first loading} \\ 0, & \text{unloading/reloading} \end{cases}$$

Brittle damage model: frictional dissipation

Friction is an essential dissipation mechanism in geomaterials → upon the attainment of a critical opening displacement, faults loose cohesion and friction remains the only dissipation mechanism

Dual kinetic potential Ψ* per unit area: Coulomb friction

$$\Psi^*(\dot{\boldsymbol{\Delta}}; \varepsilon, \boldsymbol{\Delta}) = \mu \max\{0, \; -\boldsymbol{c} \cdot \boldsymbol{N}\} \; |\dot{\boldsymbol{\Delta}}|$$

- If faults undergo opening and are not in contact, then $\Psi^* = 0$
- If faults are closed and the contact tractions are compressive, the dissipation is proportional to the normal component of contact tractions **c**= **σN**
- $\mu = \tan\phi'$ is the friction angle of the material

Brittle damage model: fault propagation and orientation

- Undamaged material at time t_n (the state a t_n is known)
- Given the total deformation ε_{n+1} at time t_{n+1}
- Define an incremental work of deformation E, defined over the increment Δt

$$E_n(\boldsymbol{\varepsilon}^{\mathrm{m}}, \boldsymbol{\Delta}, q) = W^{\mathrm{m}}(\boldsymbol{\varepsilon}^{\mathrm{m}}) + \frac{1}{L}\Phi(\boldsymbol{\Delta}, q) + \frac{\Delta t}{L}\Psi^*\left(\frac{\boldsymbol{\Delta} - \boldsymbol{\Delta}_n}{\Delta t}; \boldsymbol{\varepsilon}, \boldsymbol{\Delta}\right)$$

Strain energy density per unit volume of the matrix

Cohesive energy density per unit surface of the faults

Frictional dissipation

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Brittle damage model: fault propagation and orientation

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An incremental strain energy function W follows from the solution of the constrained optimization problem:

$$W_n(\boldsymbol{\varepsilon}) = \inf_{\boldsymbol{\Delta}, q} E_n(\boldsymbol{\varepsilon}^{\mathrm{m}}, \boldsymbol{\Delta}, q)$$

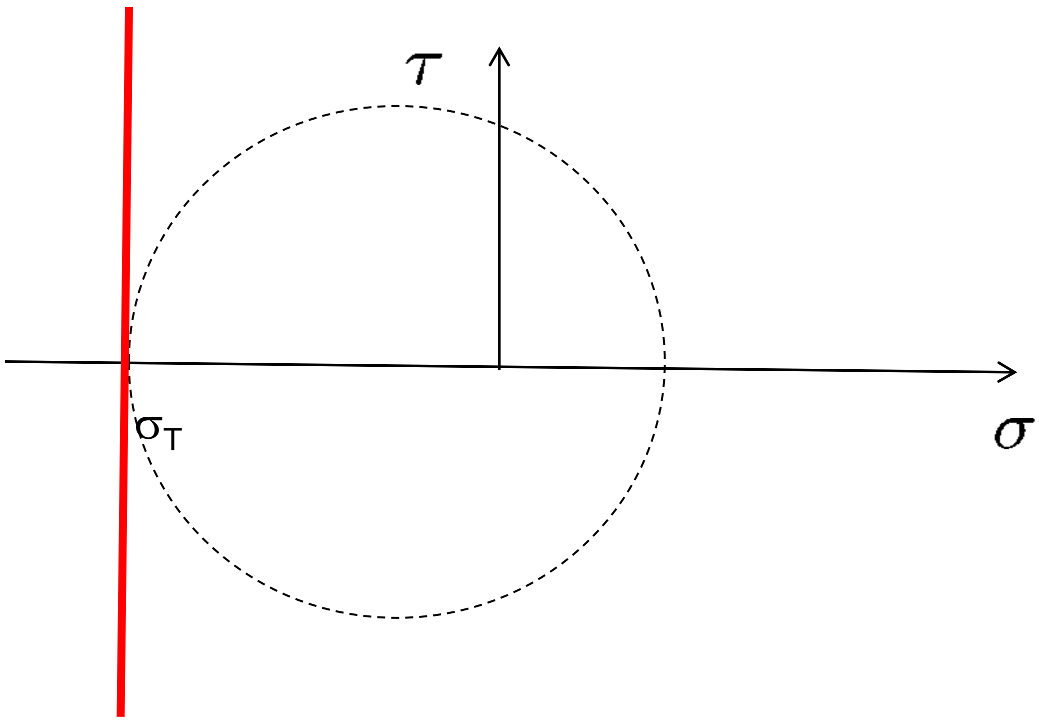
$$\boldsymbol{\Delta} \cdot \boldsymbol{N} \geq 0;$$ → Impenetrability of closing faults

$$q \geq q_n \,.$$ → Irreversibility of damage

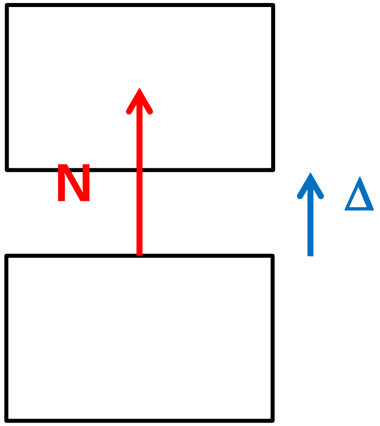
Brittle damage model: fault propagation and orientation

The corresponding eigenvalue problem identifies two different failure conditions:

- Failure in opening (Galileo-Rankine criterion)



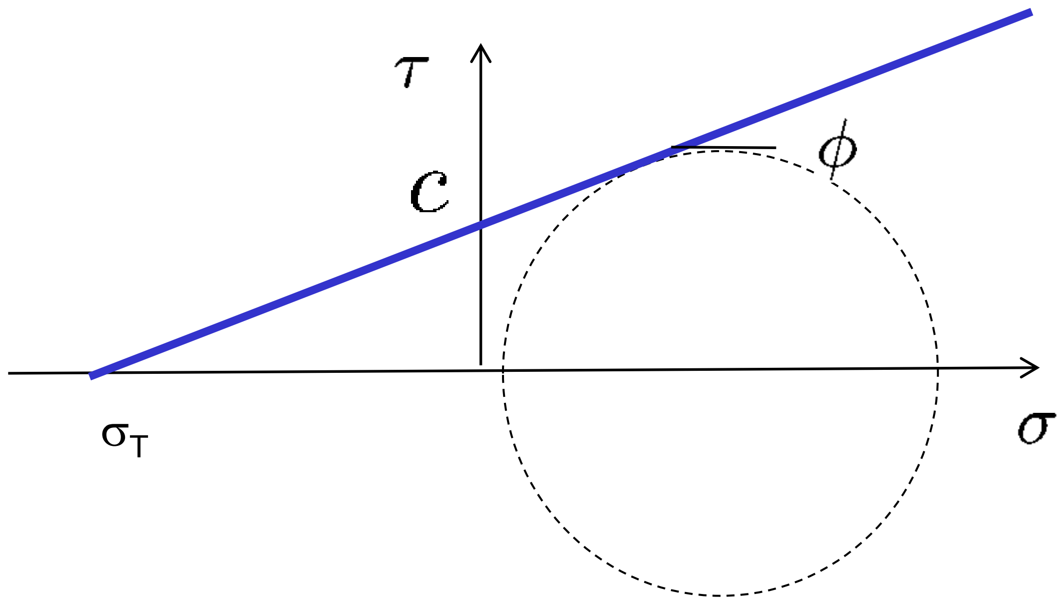
$$\Delta \cdot N > 0$$

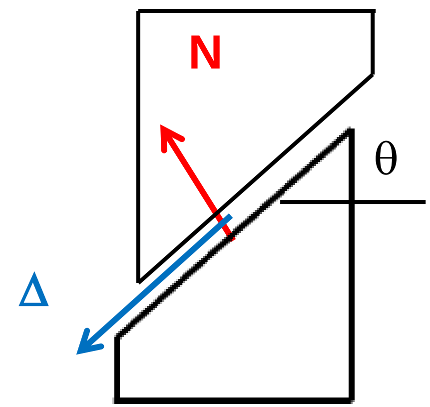


Brittle damage model: fault propagation and orientation

The corresponding eigenvalue problem identifies two different conditions of brittle materials

- Failure in sliding (Mohr-Coulomb criterion)



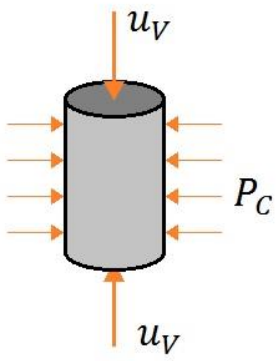


$$\theta = \pi/4 - \phi/2$$

$$\tau = \beta T_c + \sigma' \tan\varphi'$$

Model prediction: triaxial compression test with axial strain control

Sensitivity analysis: confining pressure

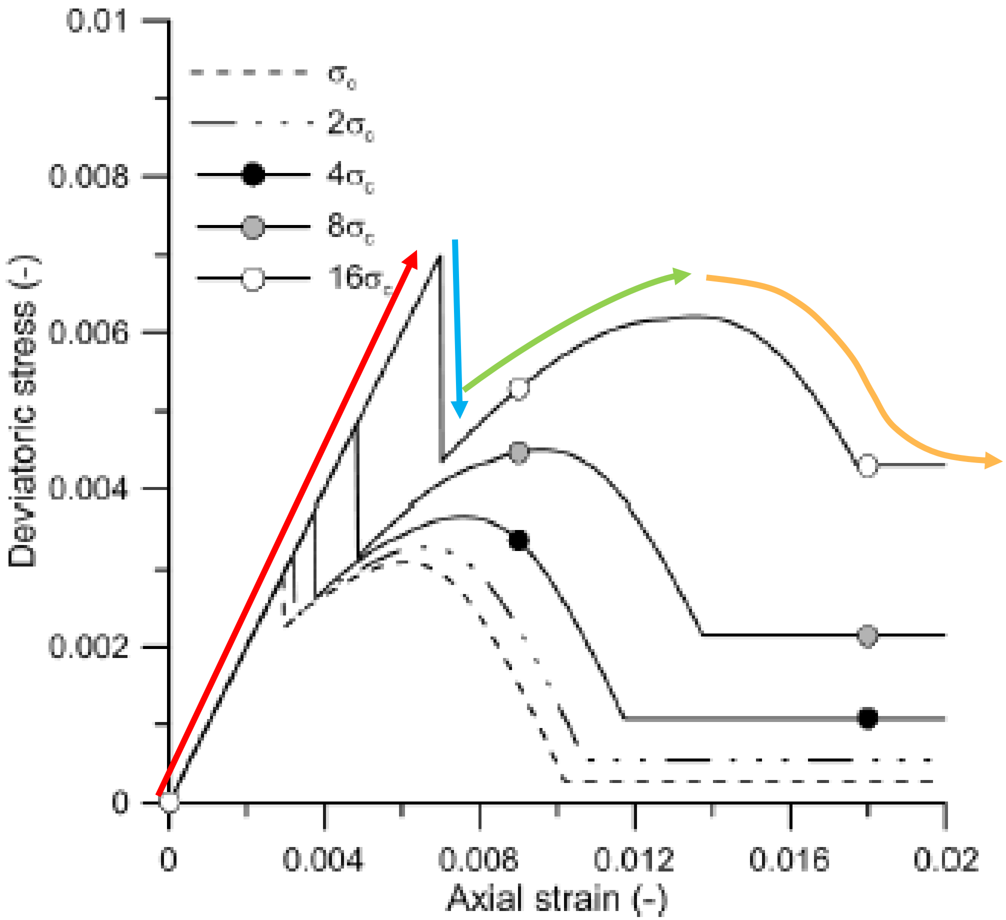


1) ELASTIC LOADING

2) STRUCTURAL BRITTLE FAILURE

3) COhESIVE STAGE

4) SOFTENING



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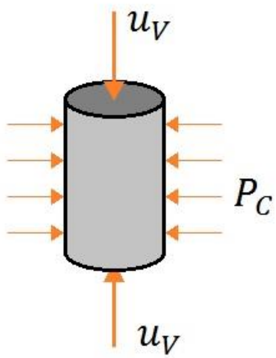

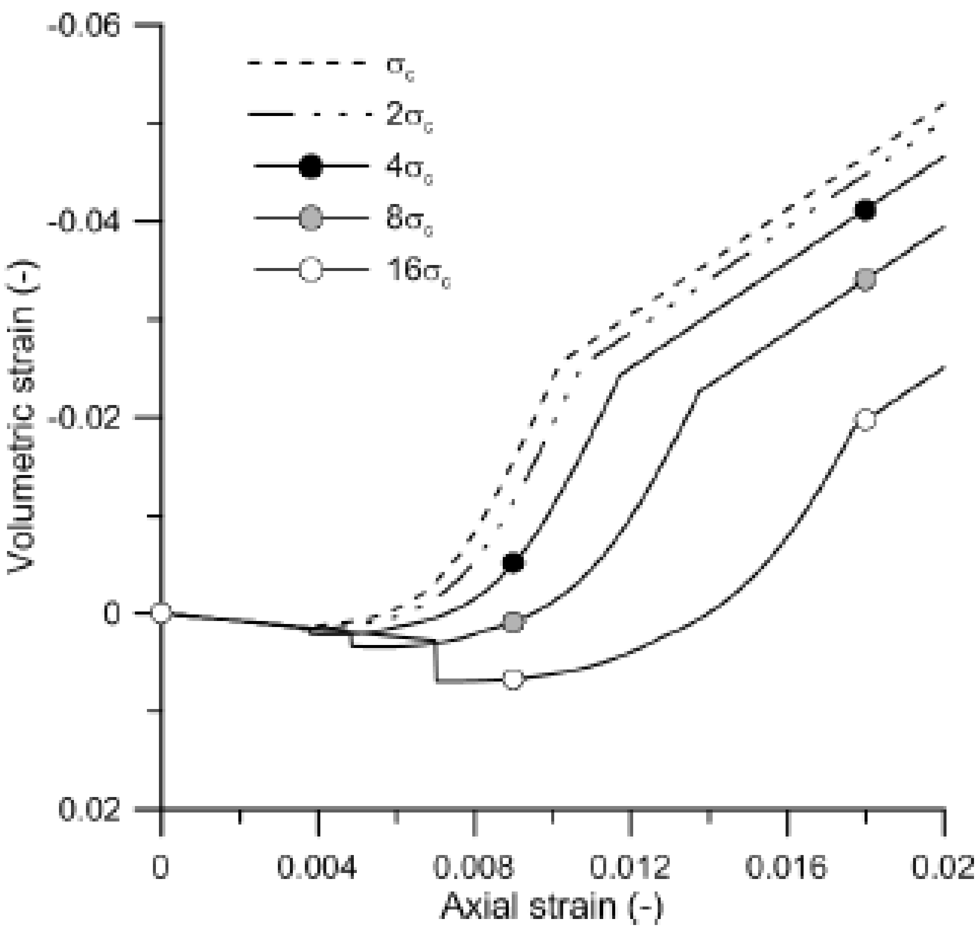

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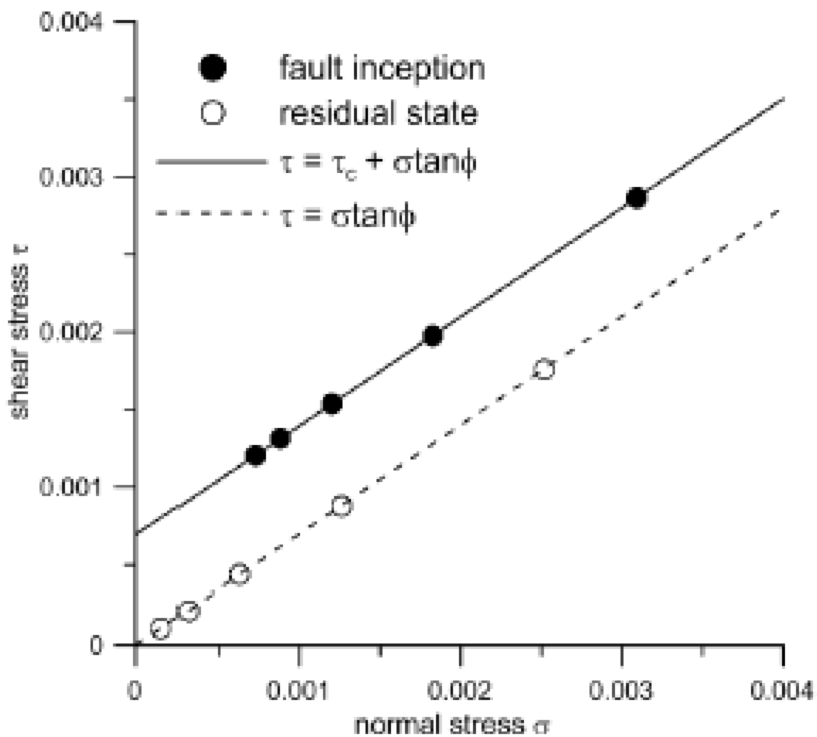
Model prediction: triaxial compression test with axial strain control

For increasing confining pressure:

- Increase in shear strength
- Fragile to ductile transition
- Reduction of the volumetric expansion



Built-in features:

- Peak strength envelope with cohesion
- Residual strength envelope without cohesion
- Dilatancy angle = friction angle

Porosity and permeability evolution

Total porosity n: sum of matrix porosity and change of porosity induced by the faults

$$n = n^{\mathrm{m}} + n^{\mathrm{f}}$$

Matrix porosity

Fault induced porosity

$$n^{\mathrm{m}} = \varepsilon^{\mathrm{m}}{}_{v} = \varepsilon^{\mathrm{m}}{}_{kk}$$

$$n^{\mathrm{f}} = \varepsilon^{\mathrm{f}}{}_{kk} = \frac{1}{L}\Delta_k N_k = \frac{\Delta_N}{L}$$

Porosity and permeability evolution

Total permeability k: sum of matrix permeability and permeability induced by the faults

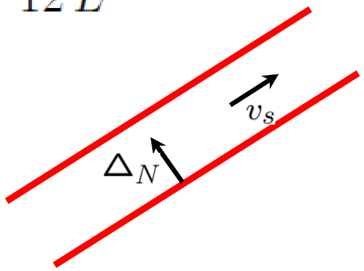
$$\boldsymbol{k} = \boldsymbol{k}^{\mathrm{m}} + \boldsymbol{k}^{\mathrm{f}}$$

Matrix permeability (Kozeny-Carman type)

$$\boldsymbol{k}^{\mathrm{m}} = k_{\mathrm{KC}} \boldsymbol{I} \,, \qquad k_{\mathrm{KC}} = C_{\mathrm{KC}} \frac{(n^{\mathrm{m}})^3}{(1 - n^{\mathrm{m}})^2}$$

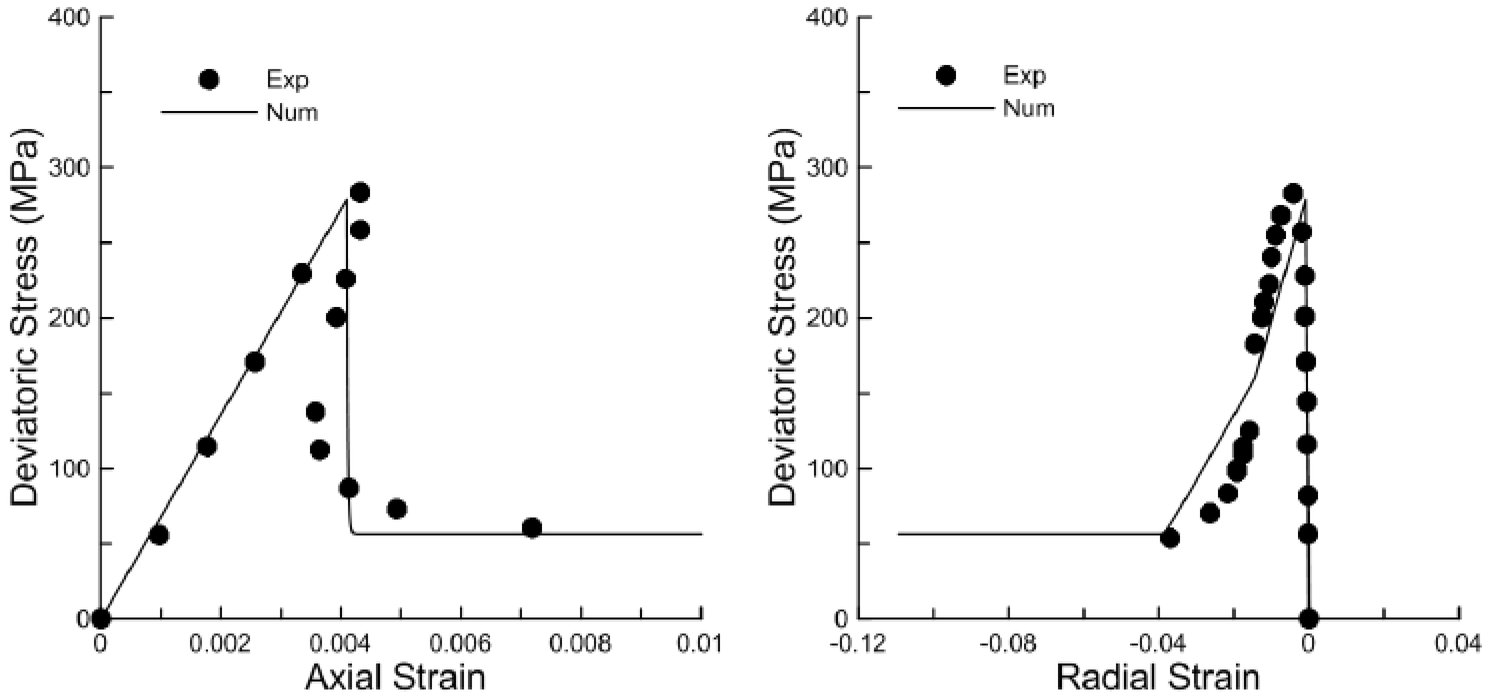
Fault induced permeability

$$\boldsymbol{k}^{\mathrm{f}} = \frac{\Delta_N^3}{12\, L} \left(\boldsymbol{I} - \boldsymbol{N} \otimes \boldsymbol{N} \right)$$

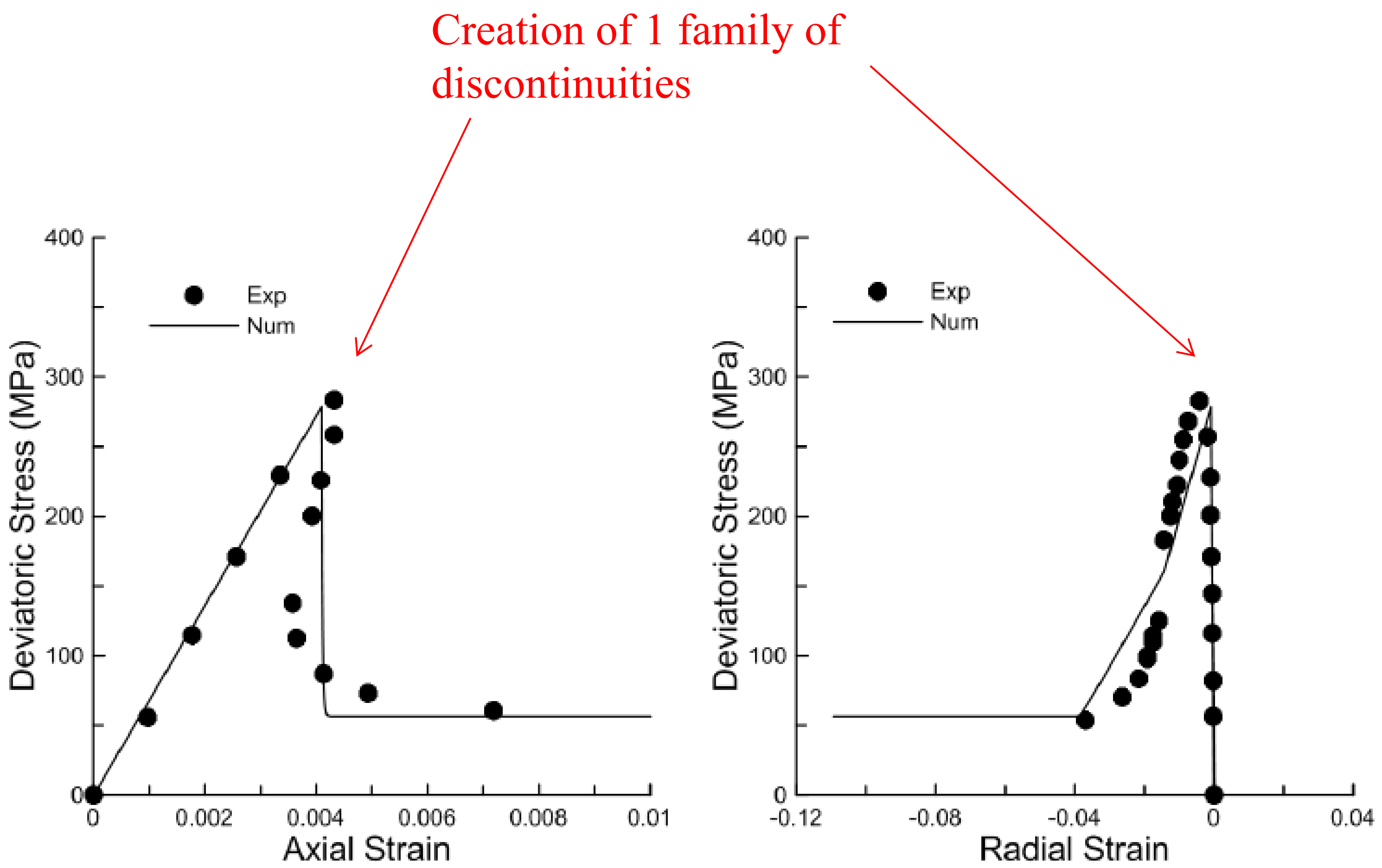


Model predictions: comparison with experimental data on Inada granite

Triaxial test with 5 MPa confinement – Inada granite (Kiyama et al, 1996)



Model predictions: comparison with experimental data on Inada granite



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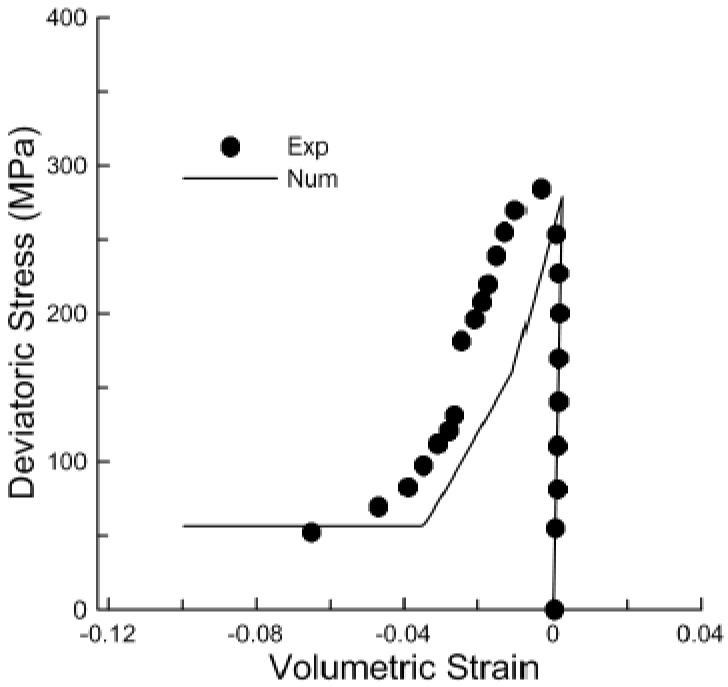

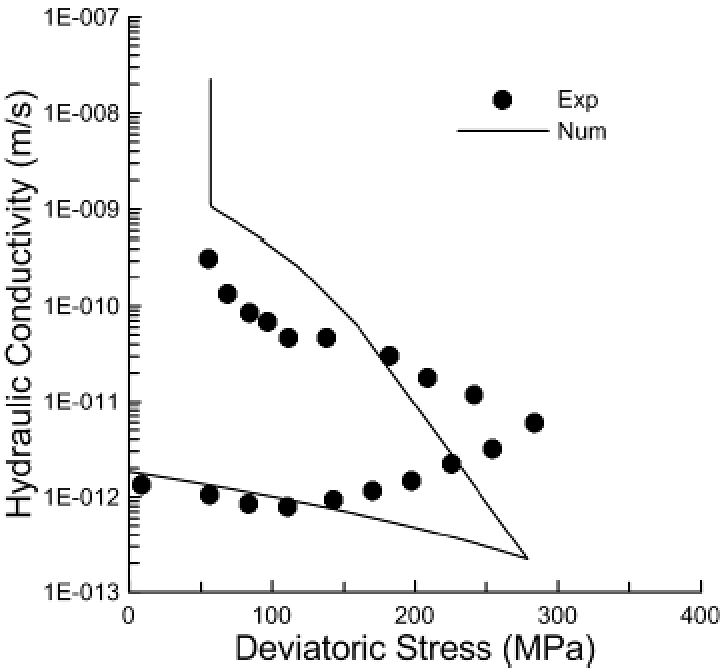

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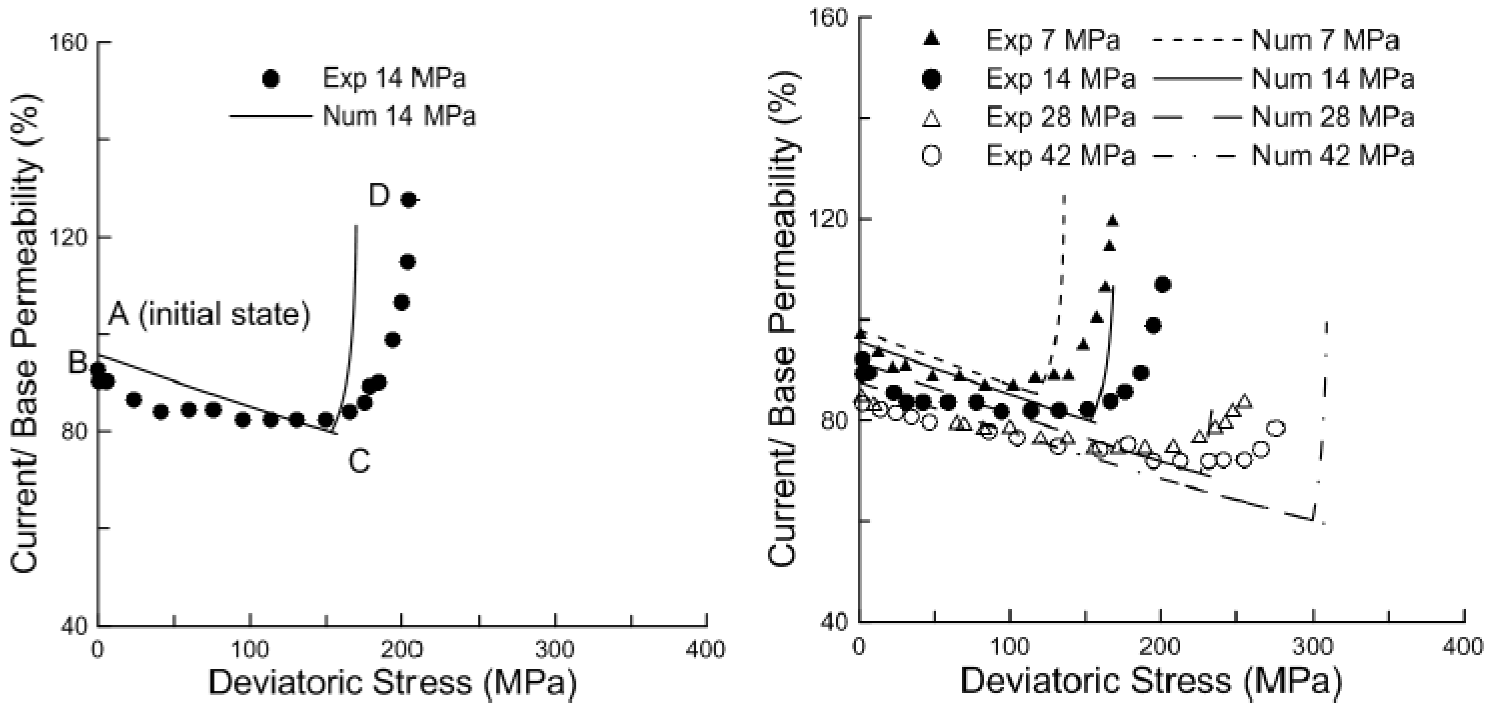
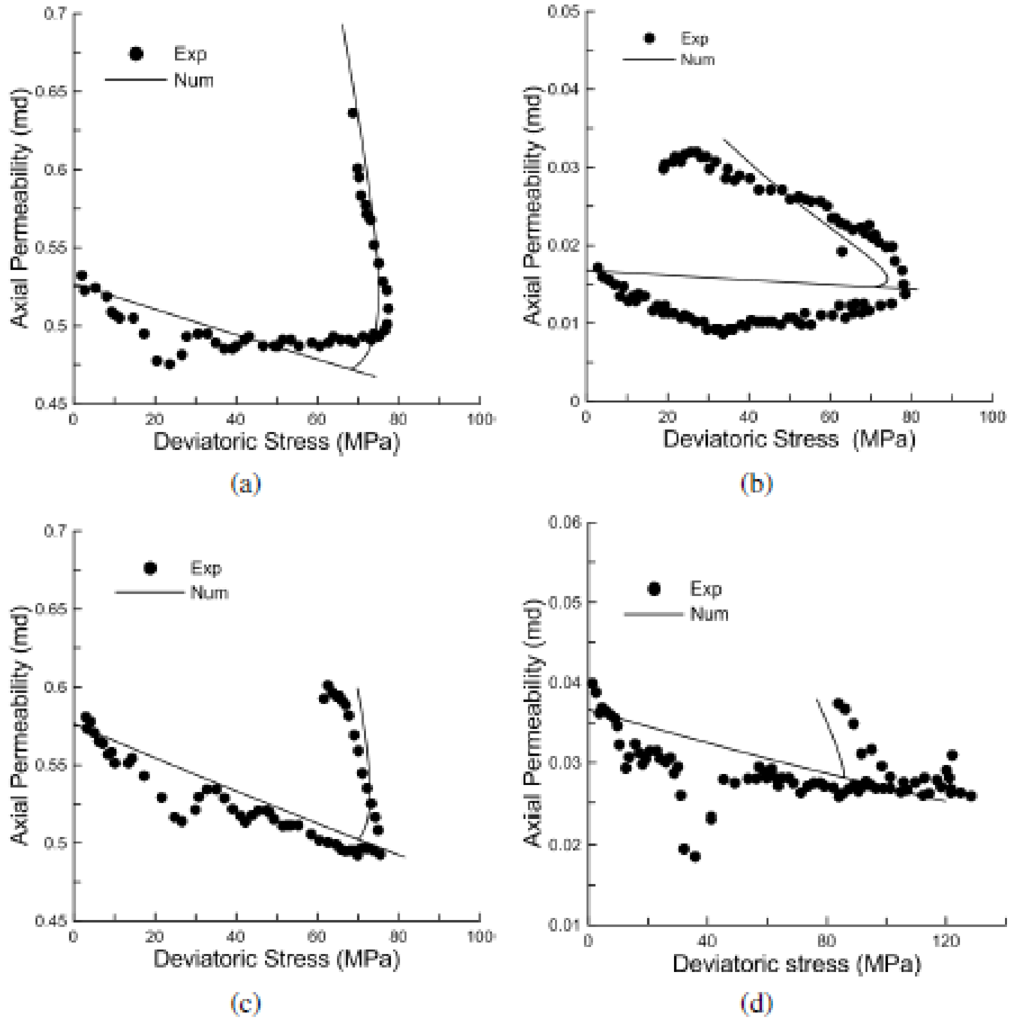
Model predictions: comparison with experimental data on a Darley Dale Sandstone



Triaxial tests at different confining pressures (Mordecai, 1970)

Model predictions: comparison with experimental data on a Permian Sandstone



Triaxial tests at different confining pressures – Permian sandstone (Heiland, 2003)

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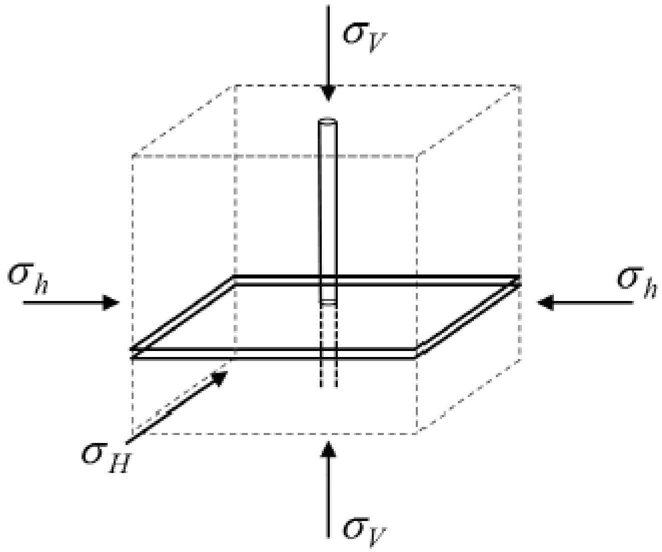
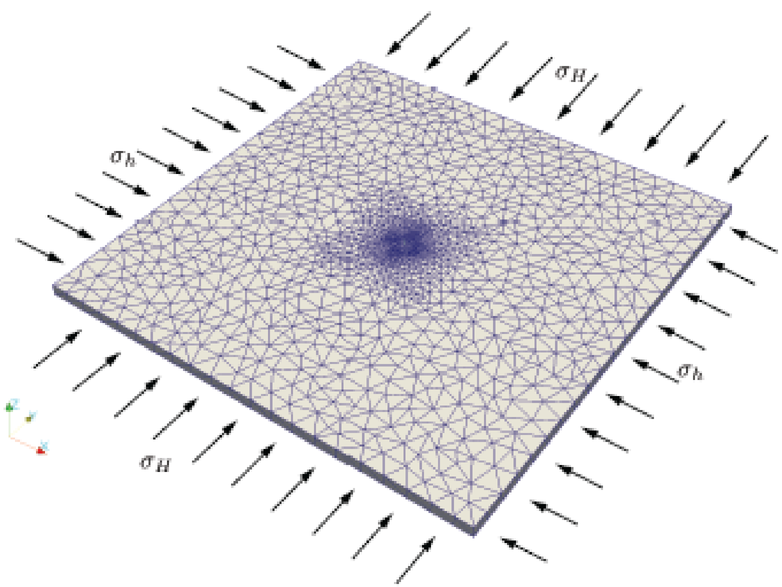
FEM application: borehole excavation

FEM 3D simulation: vertical borehole with far-field anisotropic stress state



σ_v = 20.6 MPa
σ_h = 17.2 MPa
σ_H = 68.8 MPa

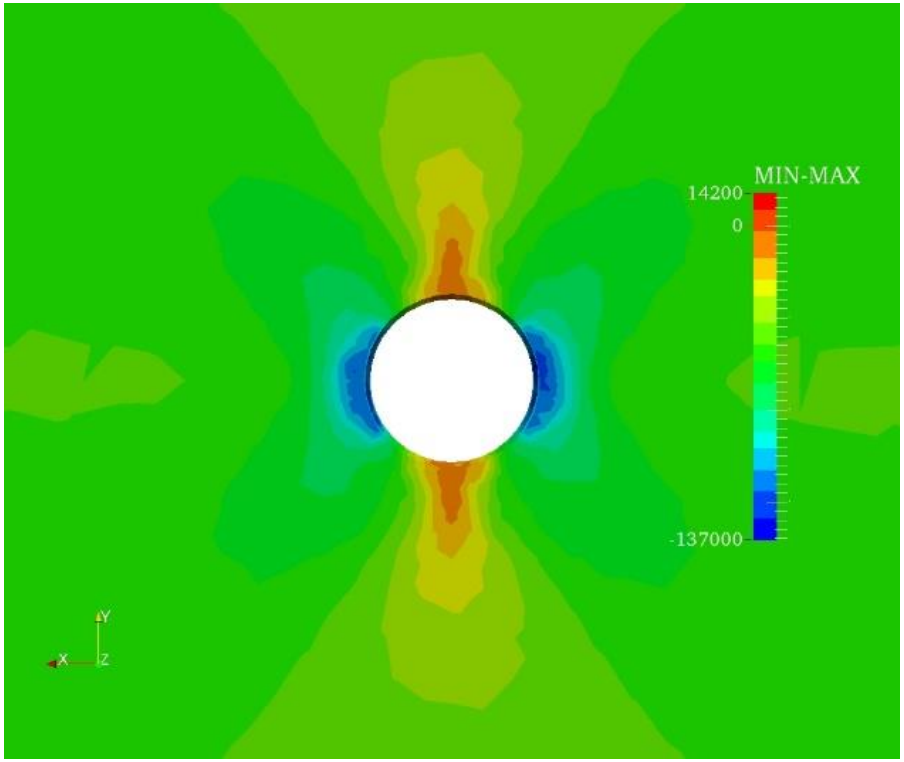


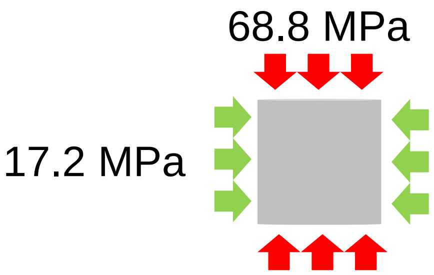
FEM application: borehole excavation

1) Application of the initial stress state

2) Borehole simulated by progressive deactivation of the finite elements pertaining to concentric cylinders, up to R = 2 m

Maximum shear stress → Breakout failure



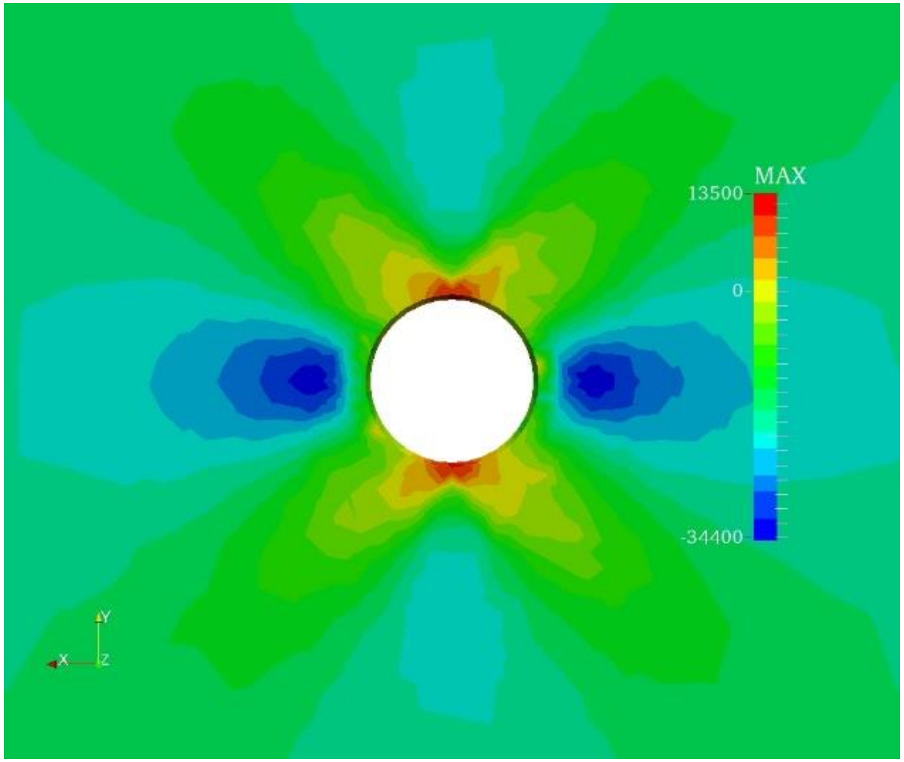


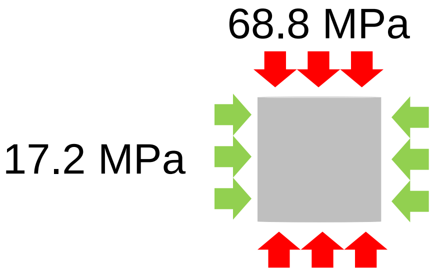
FEM application: borehole excavation

1) Application of the initial stress state

2) Borehole simulated by progressive deactivation of the finite elements pertaining to concentric cylinders, up to R=2 m

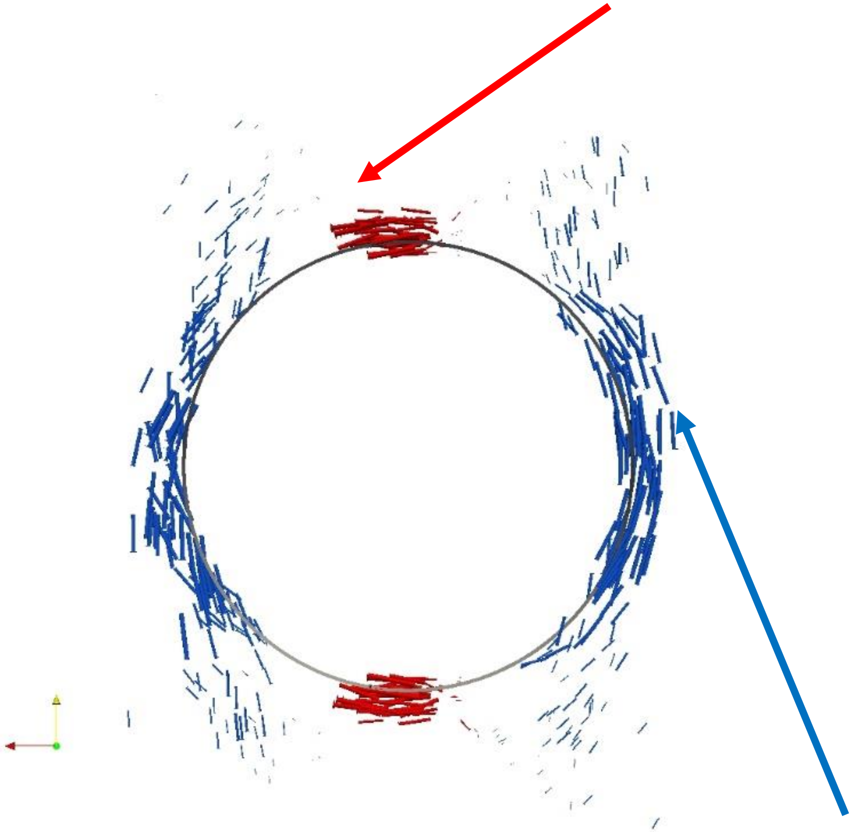
Minimum principal stress → Tensile failure



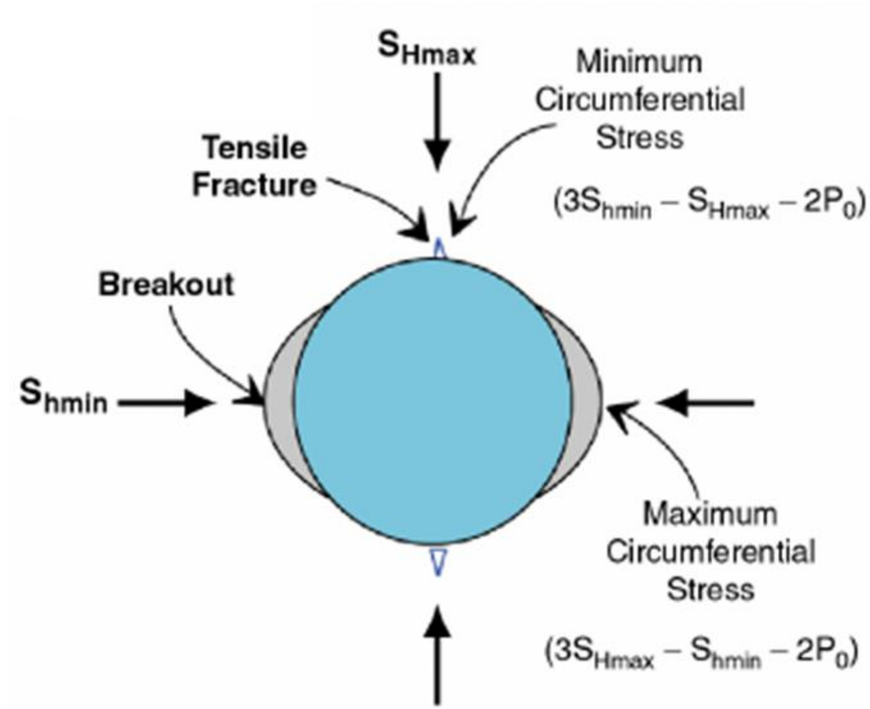


FEM application: borehole excavation

Tensile fractures



Borehole breakouts



Conclusions

- We propose a linearized distributed brittle damage model, derived from the original finite deformation model, to be used for modeling the development of multiscale cracks in natural rocks
- Faults show a cohesive behavior in opening and frictional dissipation in sliding
- The particular structure of the faults allows for the analytical definition of natural and damage-induced porosity and permeability
- Limited number of material parameters
- We are applying the model to simulate damage induced by excavations and fracking processes