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From discrete to continuum approach of Boundary Value Problems in Geomechanics : FEMxDEM integrated approach



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CNRS UMR 5521

GRENOBLE INP

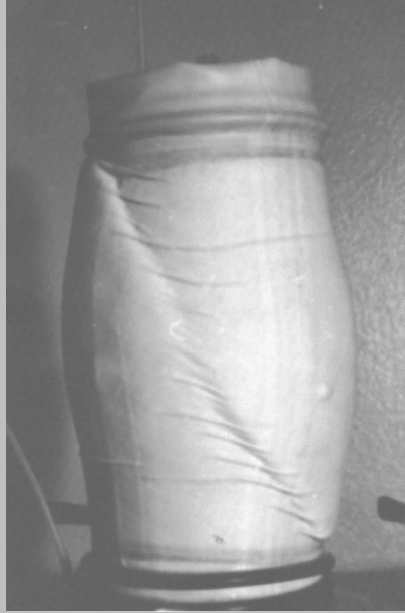
UJF GRENOBLE I

Modern Trends in Geomechanics 2016– Assisi – 16-18 May 2016

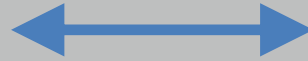
Outline

1. Introduction : principle
2. Micro-scale (DEM) Model
3. Multi-scale Coupling Method
4. FEM-DEM simulations
5. 2nd gradient : motivation, methods and results
6. Cpu cost issue : parallelization solution
7. Conclusions & Perspectives

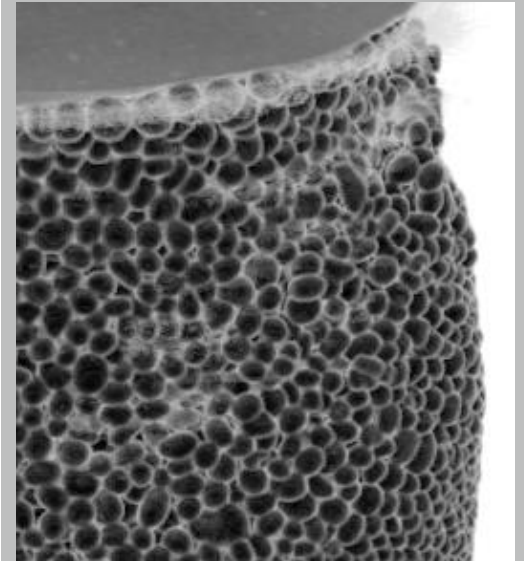
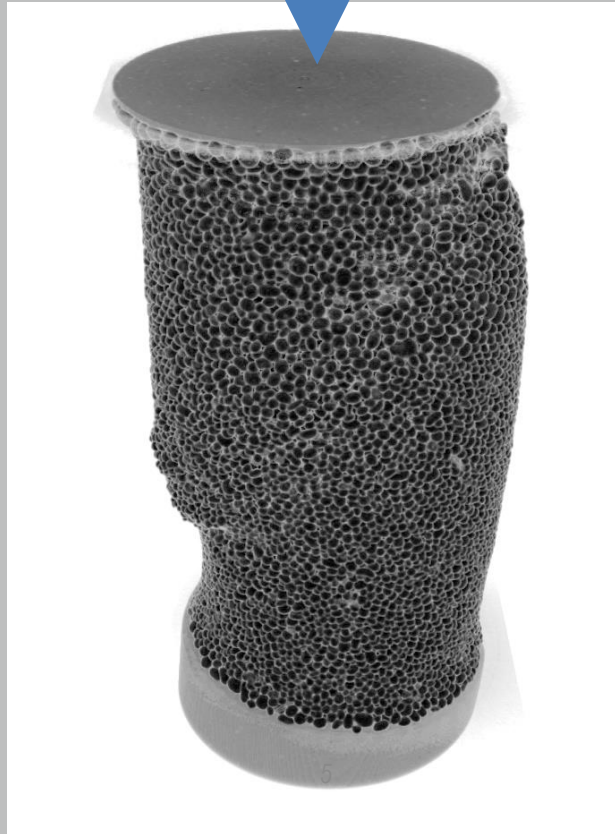
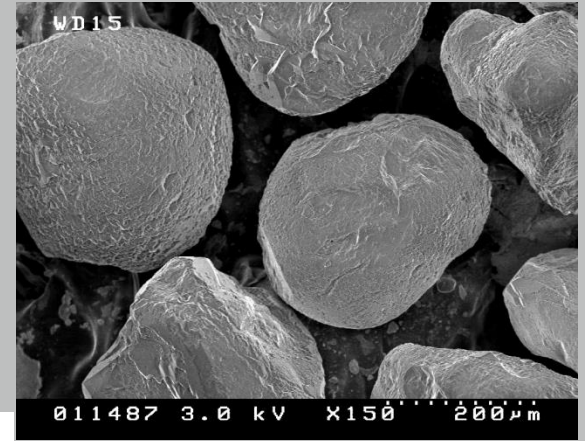
Introduction : bridging scales in Geomechanics :



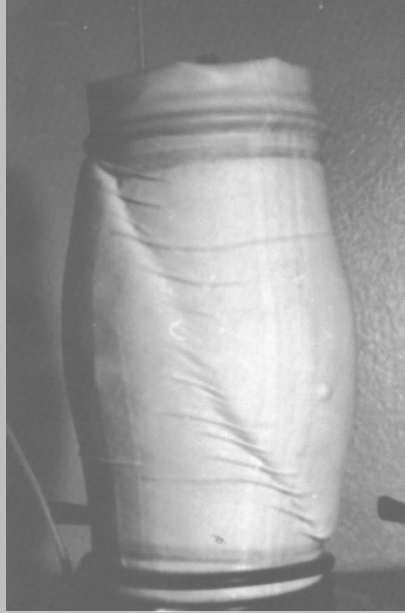
in experiments ...



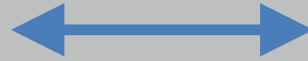
High resolution
XR tomography (> 2000)



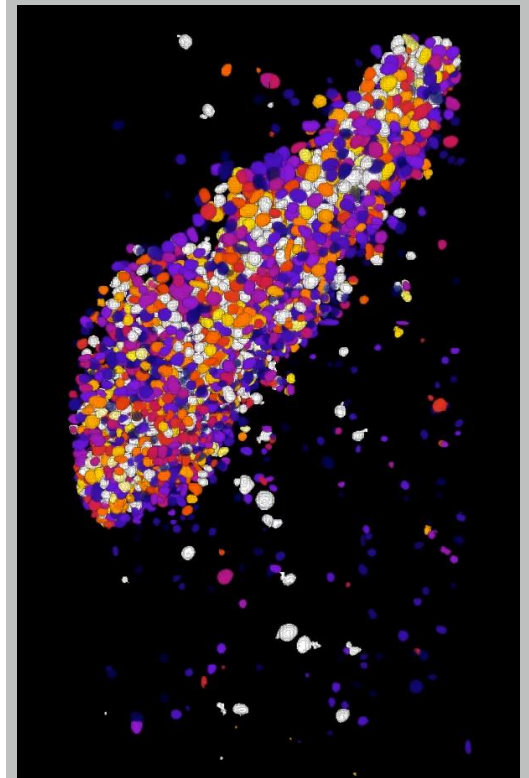
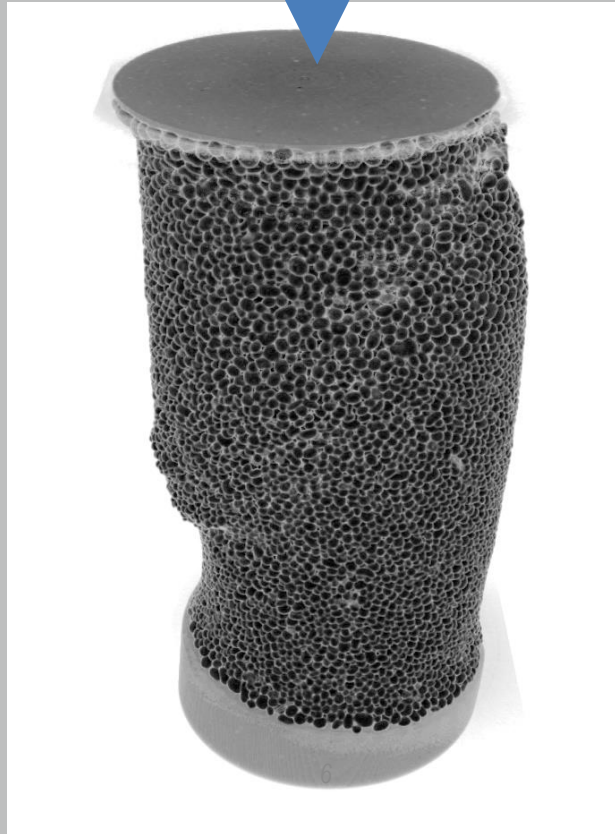
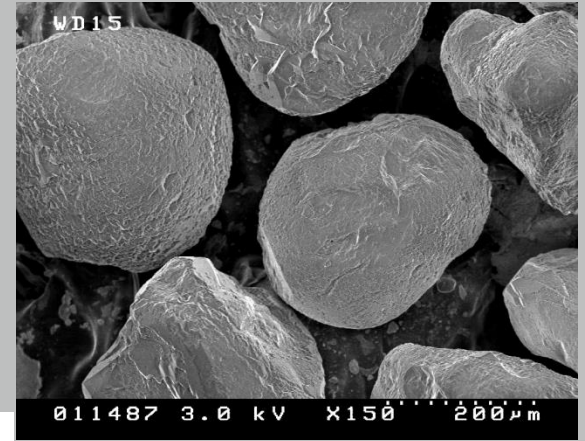
Introduction : bridging scales in Geomechanics :



in experiments ...



High resolution
XR tomography (> 2008)



In experiments :
X-Ray μ tomography
allows to catch both
the big picture and
the fine details in a
single shot

bridging scales in Geomechanics : modelling



A continuum media
or
an assembly of particles ?

Continuum : FEM	Particles : DEM
<p>☺ well suited to Real scale problem</p> <p>☹ simply <i>disregard</i> the discrete nature of granular media : instead, accounts for the observed consequences of this <i>discrete nature</i> through <i>formal</i> constitutive equations</p> <p>> always lacking generality, because the general behaviour is too complex</p>	<p>☺ Reproduces « naturally » the complex behaviour of grains assembly : cyclic response, anisotropy, strain path dependency, strain softening</p> <p>☹ Computation time depends on the number of grains -> high CPU costs</p> <p>> limitation to “small” problems</p>
<p>→ Coupling FEM-DEM ☺ ☺</p>	



Principle

Introducing a two-scale numerical homogenization approach by FEM - DEM

A rather recent trend in geomaterials, a few teams in the world :

(not all the papers of each team,
may be not all the teams, also !)

2003 Kaneko K, Terada K et al. (Japan)

2004 Miehe & Dettmar (Germany)

2009 Meyer et al. 2009 (Germany)

2010 Nitka, Desrues et al. (3SR,Grenoble, France)

2013, 2014,... Guo, Zhao (Hong-Kong)

2014 Nguyen, Desrues et al. (3SR,Grenoble, France)

2015 Liu, WaiChing et al. (USA)

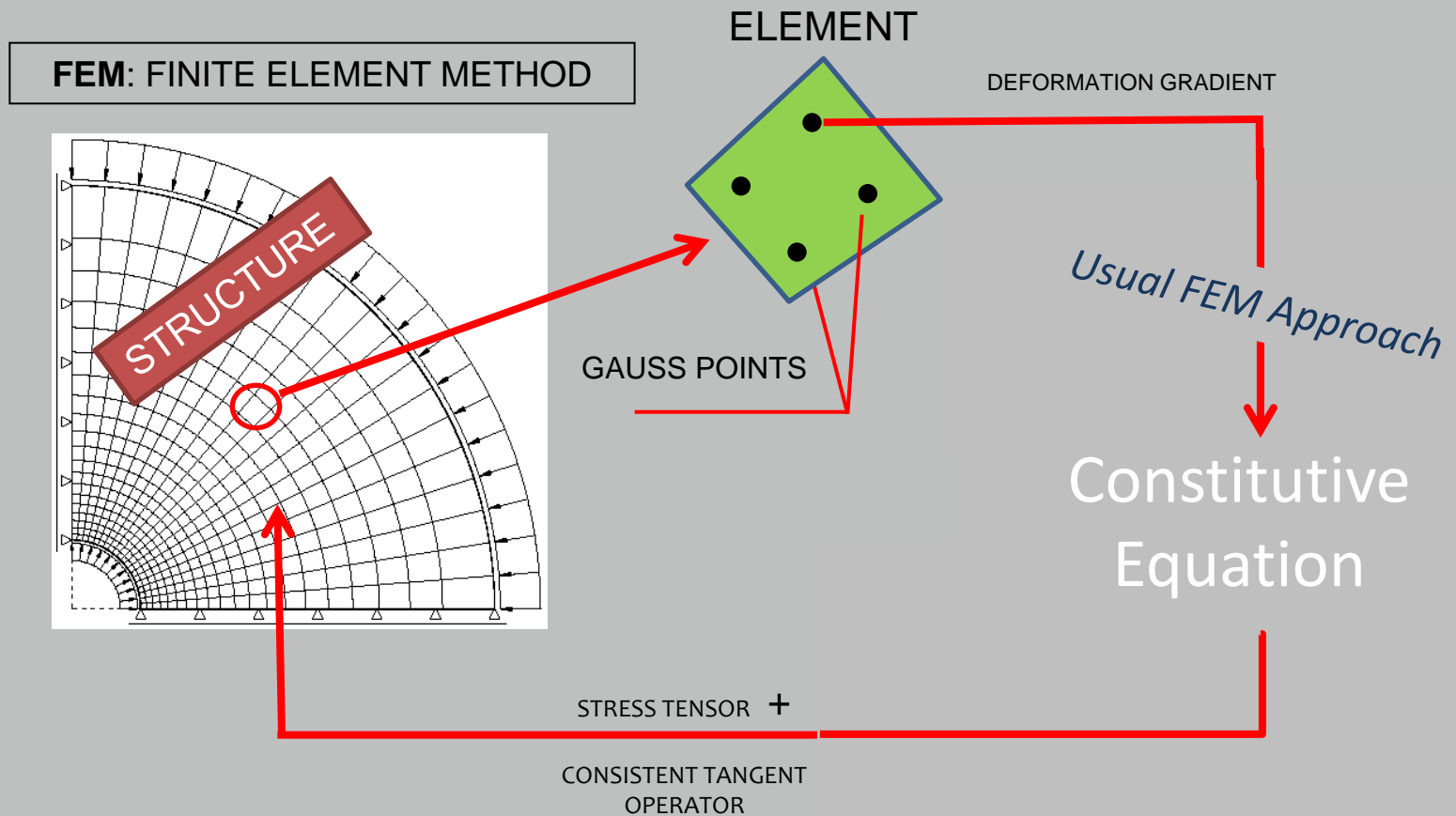
2016 Shahin, Desrues et al. (3SR,Grenoble, France)

...



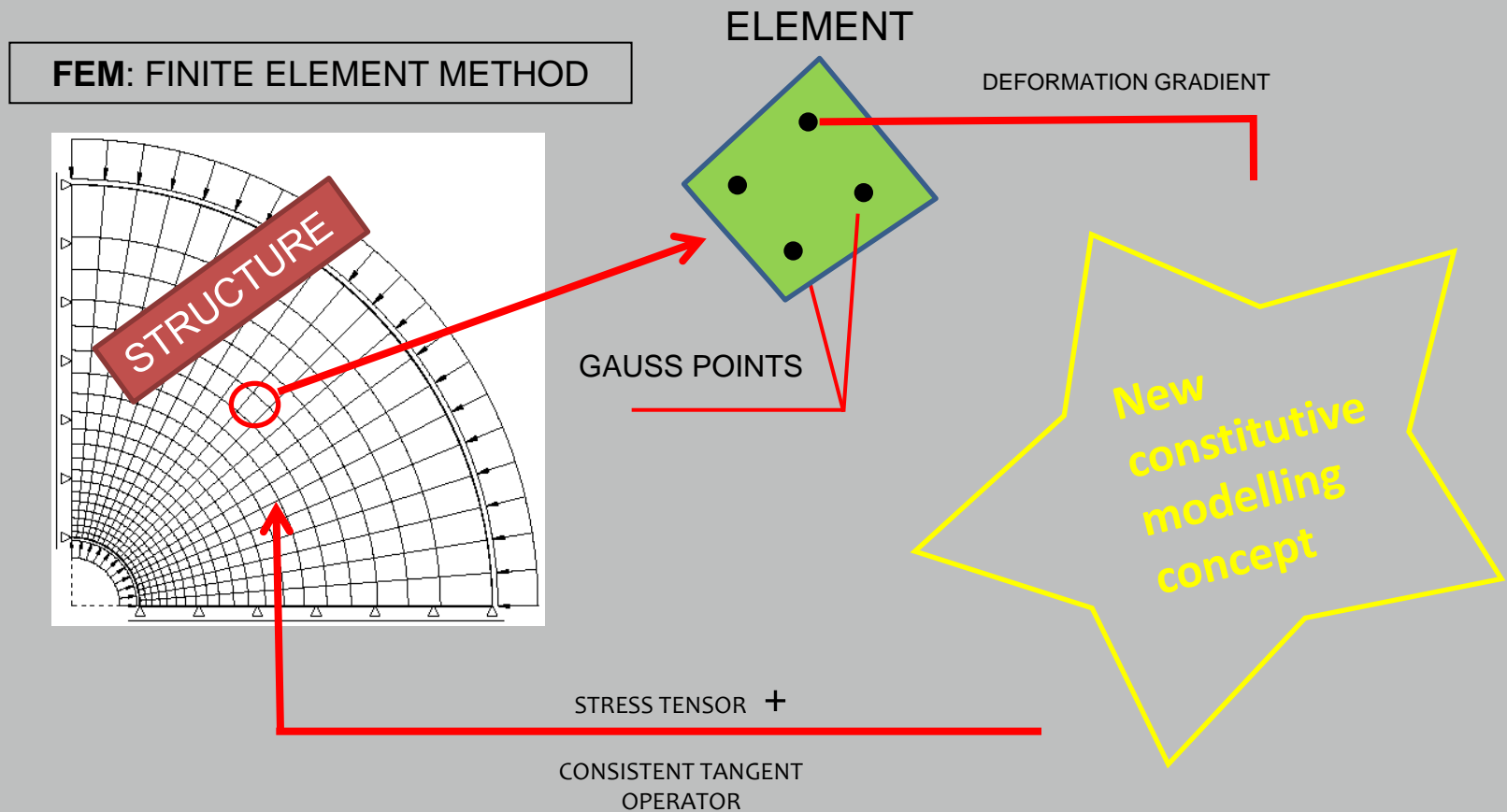
Principle

Introducing a two-scale numerical homogenization approach by FEM - DEM



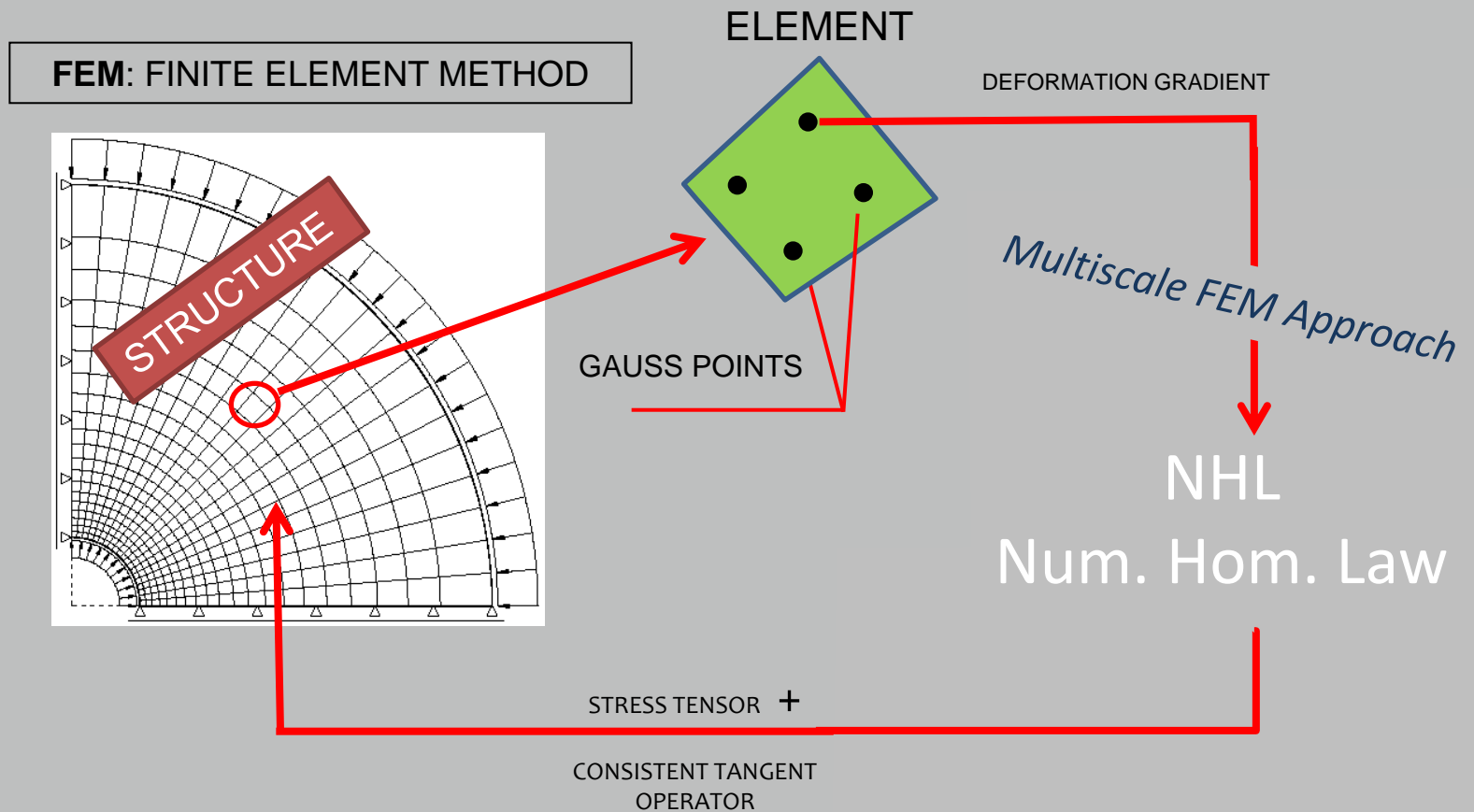
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Introducing a two-scale numerical homogenization approach by FEM - DEM



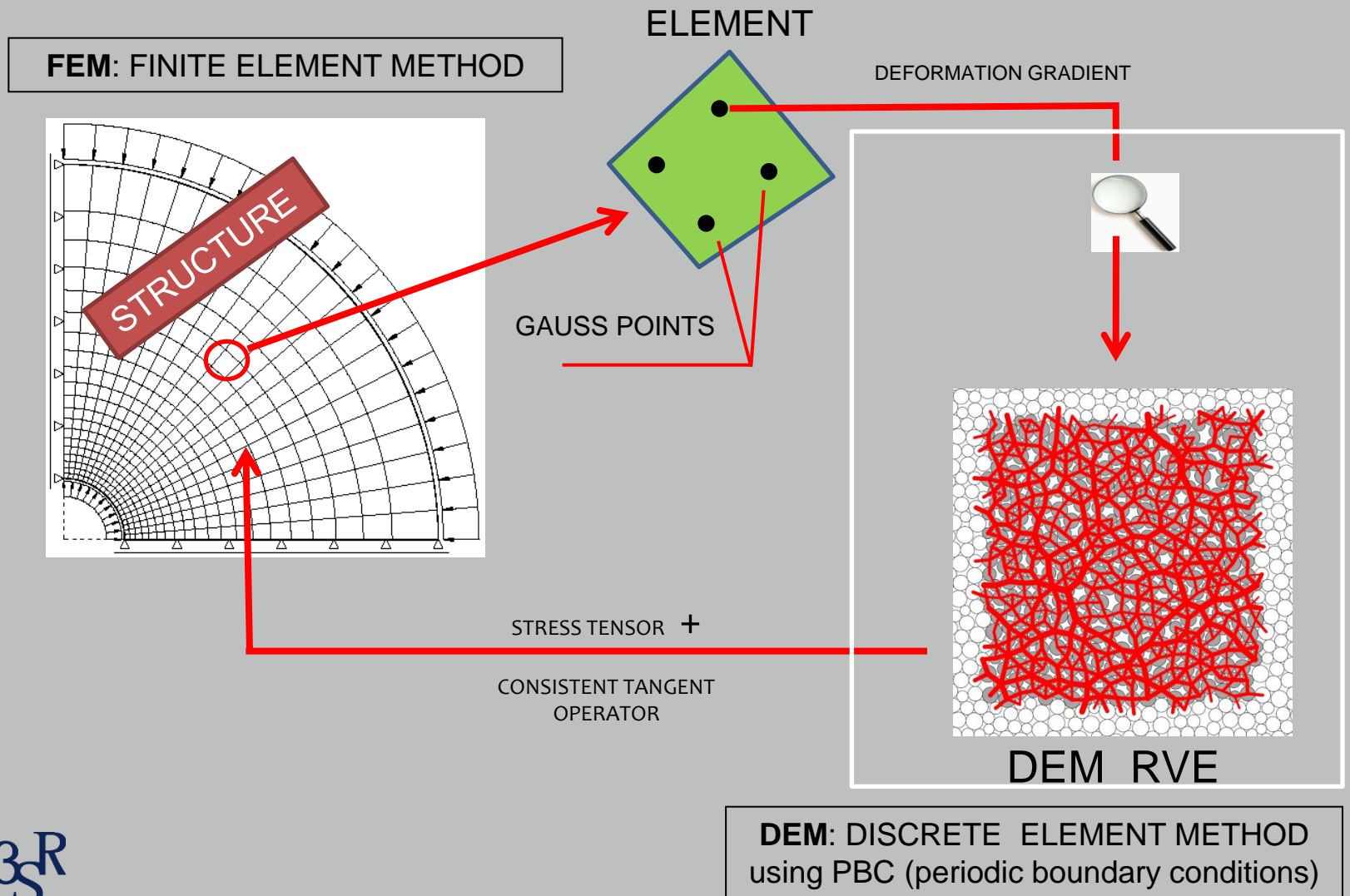
Principle

Introducing a two-scale numerical homogenization approach by FEM - DEM



Principle

A two-scale numerical homogenization approach by FEM - DEM

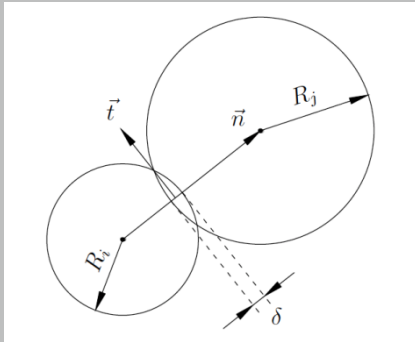


Outline

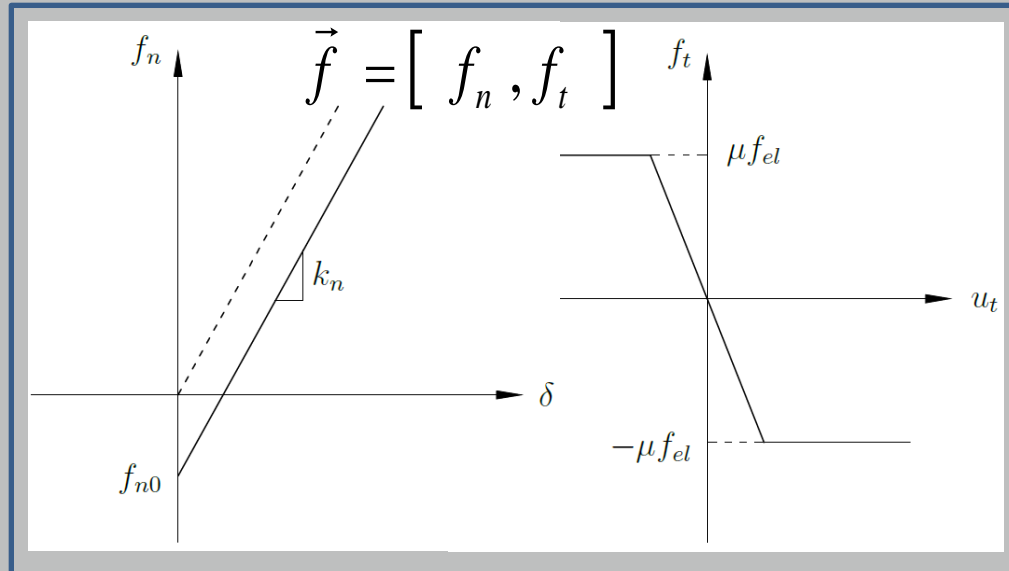
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Micro-scale Model

Contact laws *



Discrete Element Method
(Soft contact dynamics type,
Cundall & Strack 1979)
with bi-Periodic **Boundary**
Conditions



- Normal repulsive contact force

$$f_{el} = k_n \cdot \delta$$

$$\begin{cases} \delta > 0 & \text{Contact present} \\ \delta = 0 & \text{No contact} \end{cases}$$

- Tangential contact force

$$\delta f_t = k_t \cdot \delta u_t$$

- Coulomb condition

$$\|f_t\| \leq \mu \cdot f_{el}$$

- Cohesion

$$f_n = f_{el} + f_{n0}$$

f_{n0} : cohesive force

$$f_{n0} = p^* \cdot \sigma_0 \quad p^* = 1, 2, \dots$$

Macroscopic Stress tensor :

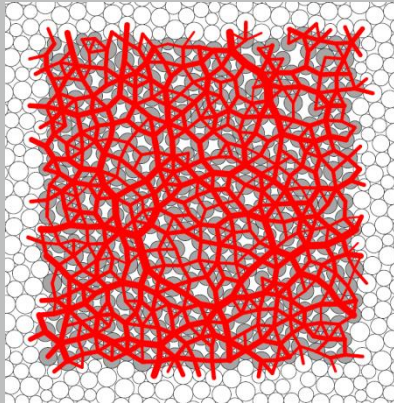
$$\sigma_{ij} = \frac{1}{S} \cdot \sum_{k=1}^{N_C} f_i^k \cdot l_j^k$$

* : (e.g. Gilabert et al., 2007)

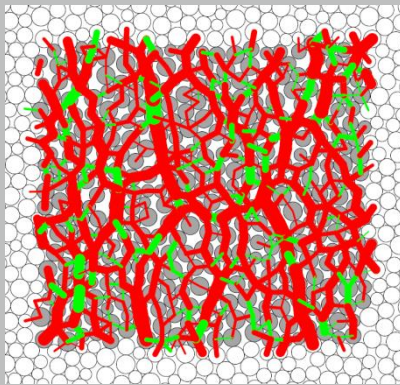


Micro-scale Model

Biaxial test (DEM with PBC): REV contains 400 particles



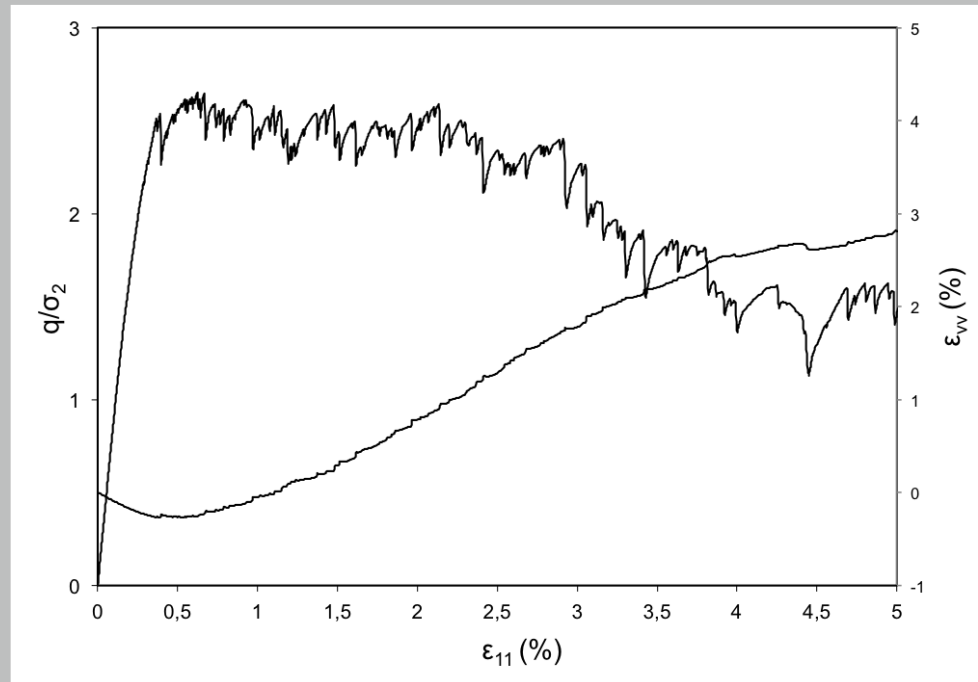
Initial configuration



at 3% of axial strain (ϵ_{11})

— f_c effective

— $f_c = 0$



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Principle

What do we need ? a **FEM code** + a **DEM code** + a **bridging procedure**

▶ **FEM code :**

the choice made has been to use the large multi-purpose FEM code Lagamine¹, Liège University (ULg). Also implemented in FlagShyp²

▶ **DEM code :** an as-compact-as-possible DEM kernel !

-> in-house 3SR-Grenoble DEM code, Geomechanics team.
strong requirement : quasi-perfect static equilibrium
at the end of each DEM step.

▶ **Bridge :**

direct incorporation of the DEM code as a constitutive law in the FEM code
(convenient for sequential programming, or OpenMP parallel programming)



1 – Lagamine, Liège University ULg

2 – FlagShyp Software, Bonet and Wood, Swansea UK 2012

Outline

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3. Multi-scale Coupling Method
- 4. FEM-DEM at work : examples of simulations :**
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Two examples of failure in real geomaterials

1. Triaxial test :

ideally, should be *homogeneous*, but ...

in the lab, observation :

localised deformation

Triaxial test on Hostun sand specimen, JL Colliat, 3SR Grenoble 1986



2. Borehole or gallery stability problem ,

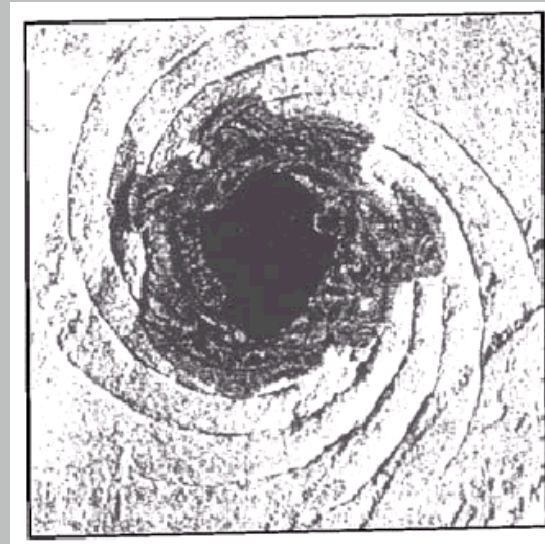
(can be studied as a hollow cylinder under differential pressure)

(analogous to a borehole or a gallery)

heterogeneous by essence

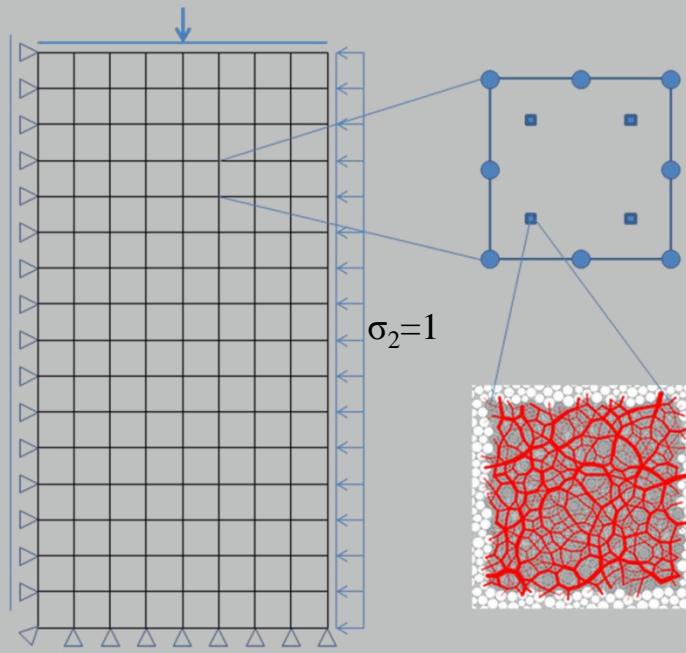
in the field, observation :

localised deformation



van den Hoek, P.J., Smit, D.-J., Kooijman, A.P., de Bree, P., Kenter, C.J., Khodaverdian, M., 1994. Size dependency of hollow-cylinder stability. Eurock, vol. 94. Balkema, Rotterdam.

Multiscale Computations: Numerical results



DEM parameters

$$\kappa = k_n / \sigma_0 = 1000$$

$$k_n / k_t = 1$$

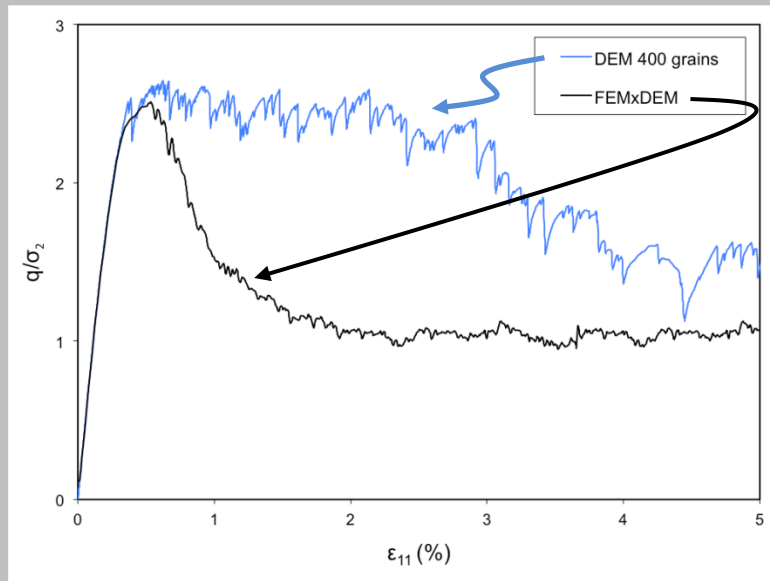
$$\mu = 0.5$$

$$p^* = \frac{f_c}{a \cdot \sigma_0} = 1$$

FEM x DEM simulation of a biaxial compression test

- Macro: discretization by 128 finite elements Q8
- Micro : REV contains 400 grains

Strain Softening and Strain localization : FEM x DEM response

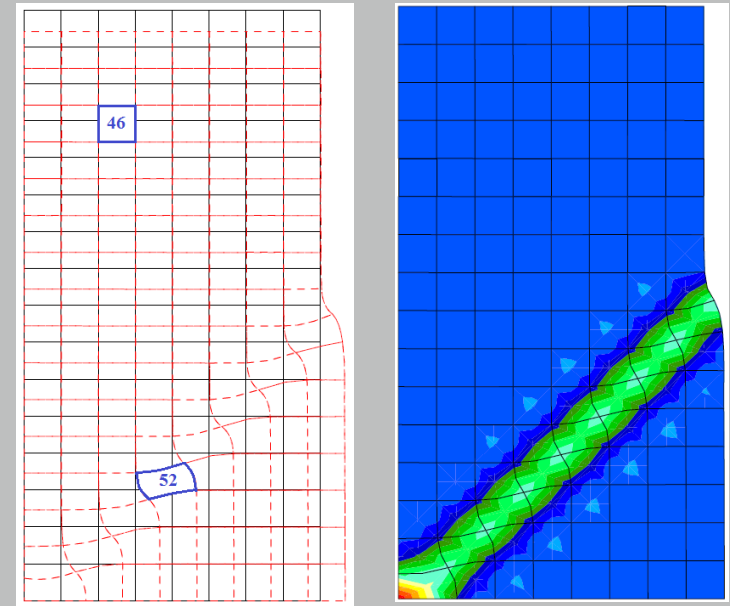
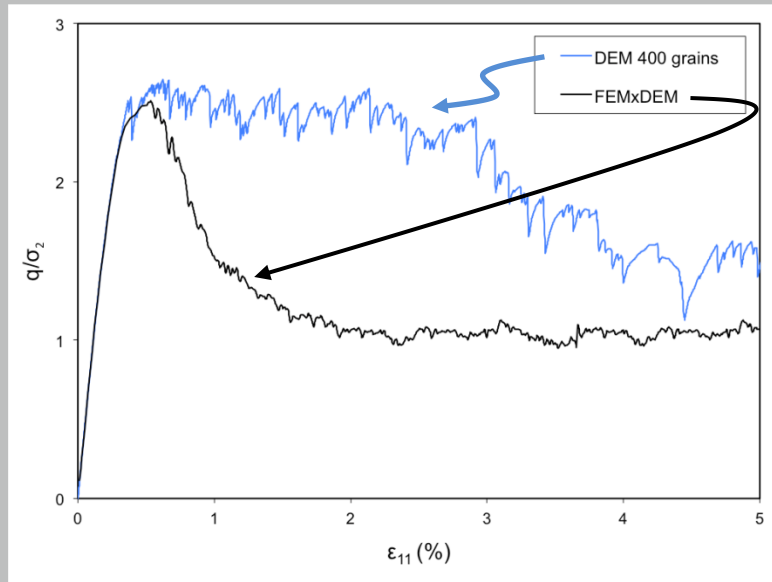


?? The response of the specimen modelled as a structure by FEMxDEM differs considerably from the pure DEM response ??

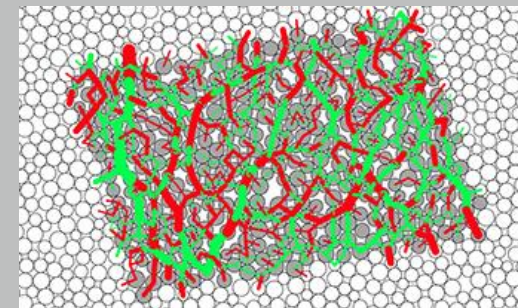
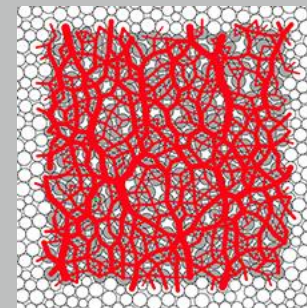
→ This is due to Strain localization in the structure



Strain Softening and Strain localization : FEM x DEM response



Deformed structure and second invariant of strain tensor



Element 46

Element 52

Deformed REV



NHL-DEM performances :

- ▶ Strain softening and strain localisation

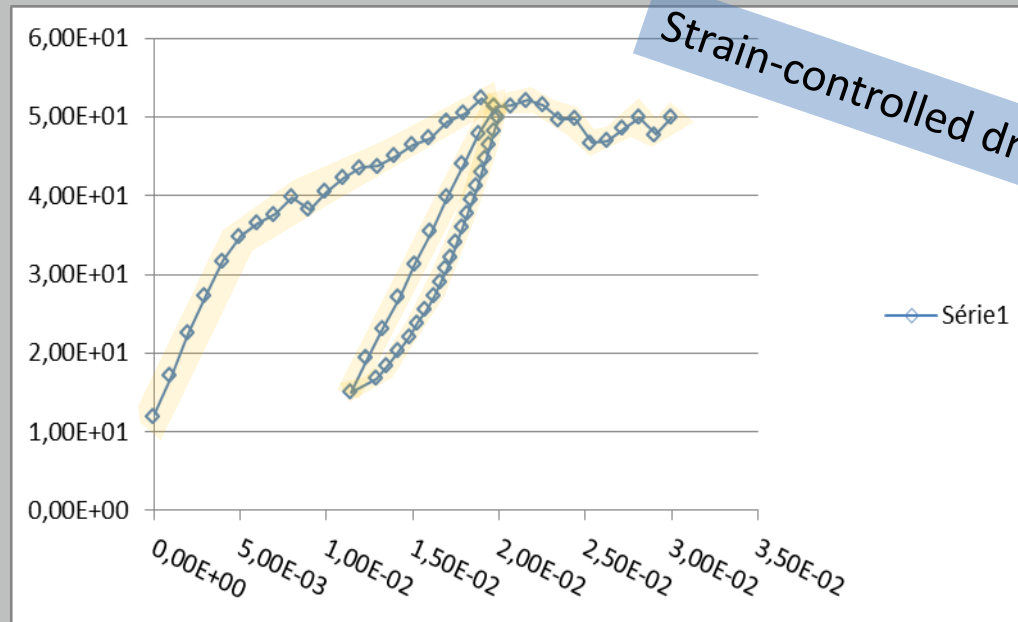


NHL-DEM performances :

- ▶ Strain softening and strain localisation
- ▶ **Cyclic response**
- ▶ Anisotropy
- ▶ Principal stress rotations

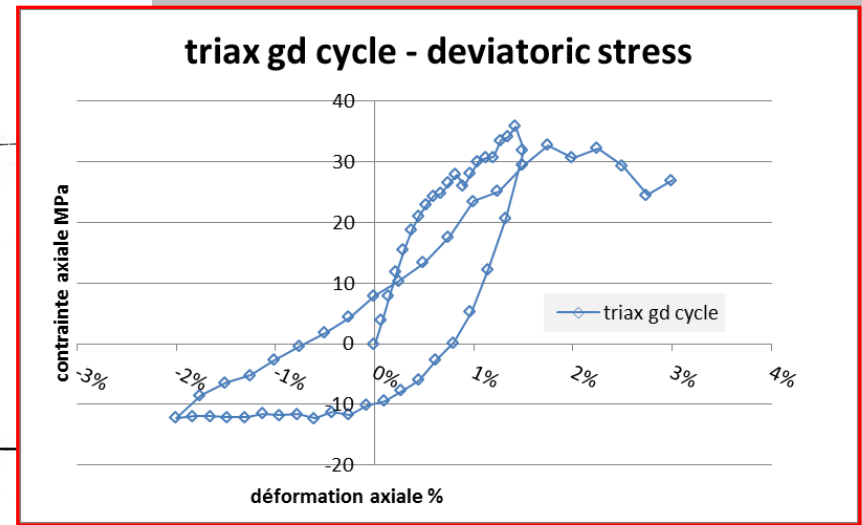
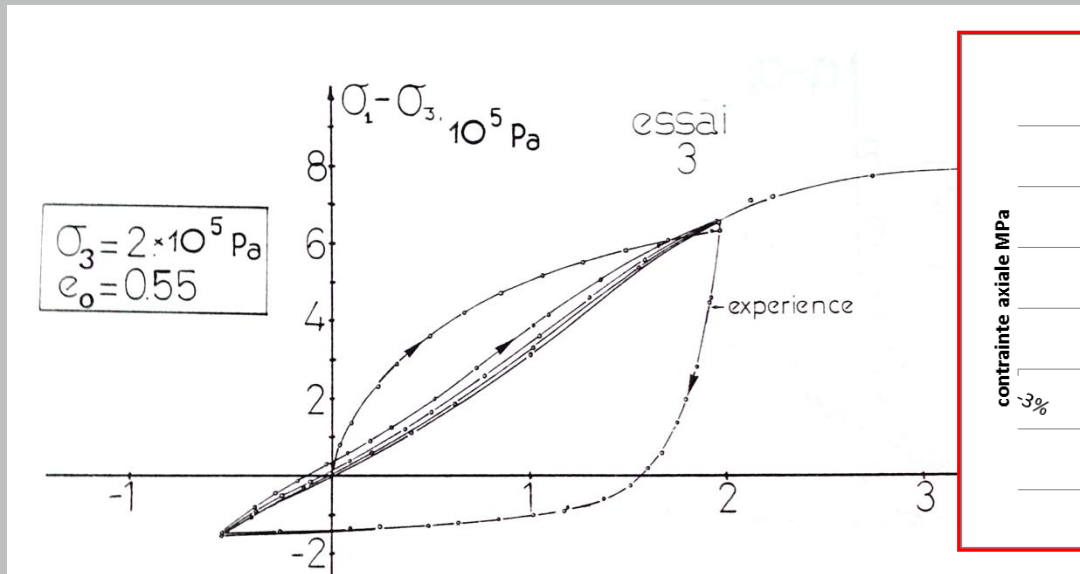


NHL-DEM performances : cycles



- ▶ Simple loading-unloading-reloading :
- ▶ The RVE state variables (grain's position, contacts and contact forces) retain all the information necessary to predict :
 - ▶ progressive stiffness degradation upon continuous loading,
 - ▶ then quasi-but-not-totally elastic unloading,
 - ▶ then elastic reloading
 - ▶ up to re-entering the plastic regime

NHL-DEM performances : compression-extension cycles



I. Thanopoulos (1981) "Contribution à l'étude du comportement cyclique des milieux pulvérulents",
Thèse de Docteur-Ingénieur, Grenoble University

- ▶ Large compression-extension cycles :
Not impossible to model with formal CE,
...but difficult
- ▶ NHL-DEM provide a **reasonably good** response
...without any special development

NHL-DEM performances :

- ▶ Strain softening and strain localisation
- ▶ **Cyclic response**
- ▶ Anisotropy
- ▶ Principal stress rotations



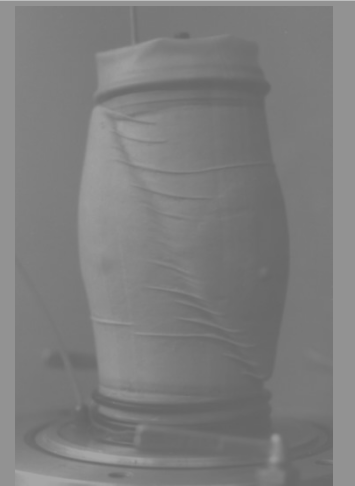
1. **Triaxial test :**

ideally, should be *homogeneous*, but ...

in the lab, observation :

localised deformation

Triaxial test on Hostun sand specimen, JL Colliat, 3SR
Grenoble 1986



Second example :

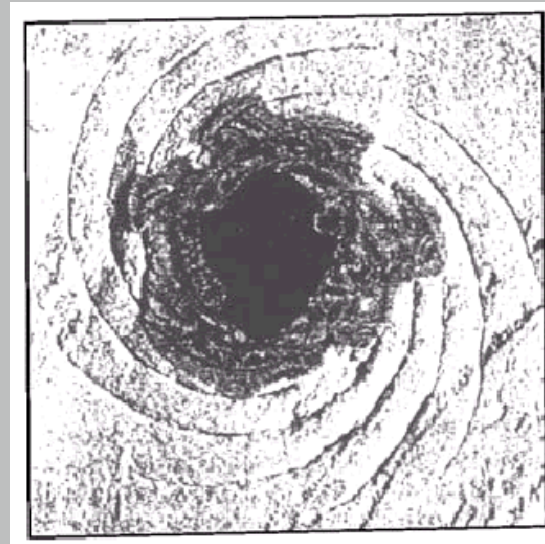
2. **Hollow cylinder under differential pressure**

(analogous to a borehole or a gallery)

heterogeneous by essence

in the field, observation :

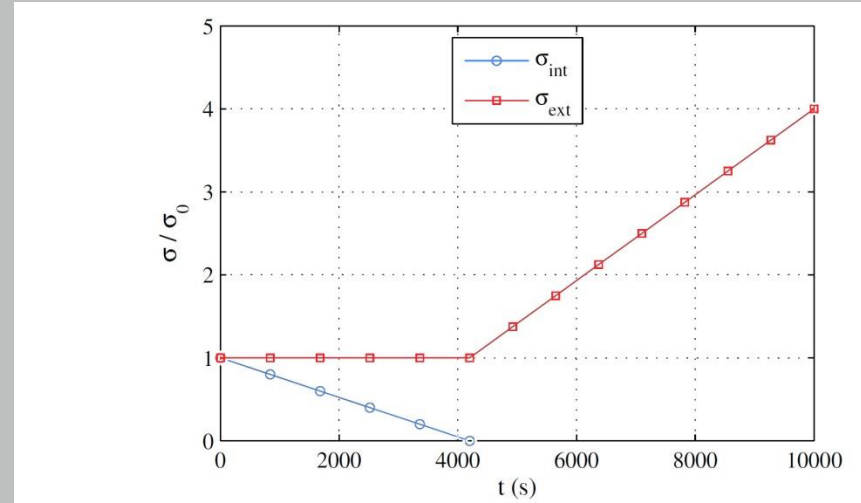
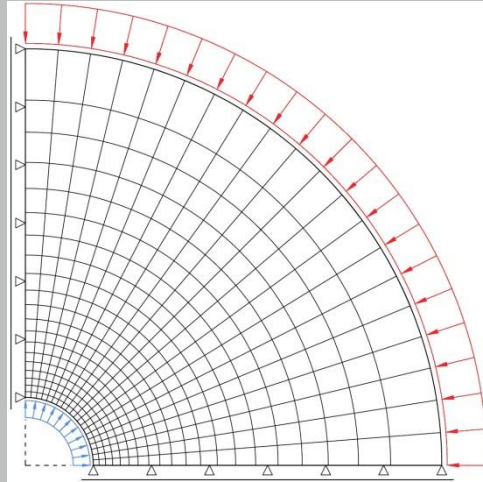
localised deformation



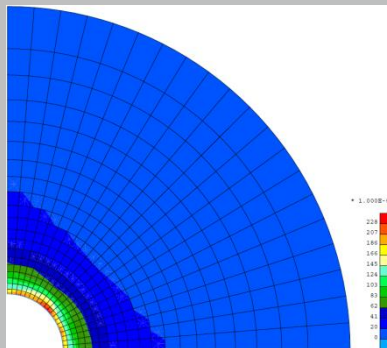
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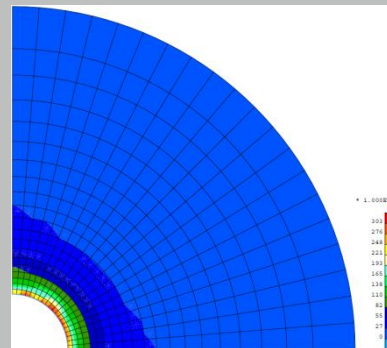
Multiscale Computations: Hollow cylinder (drilling), Strain localization



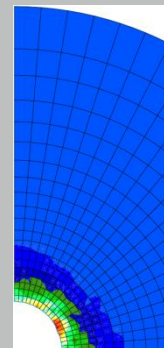
Deformed structure and second invariant
of strain tensor



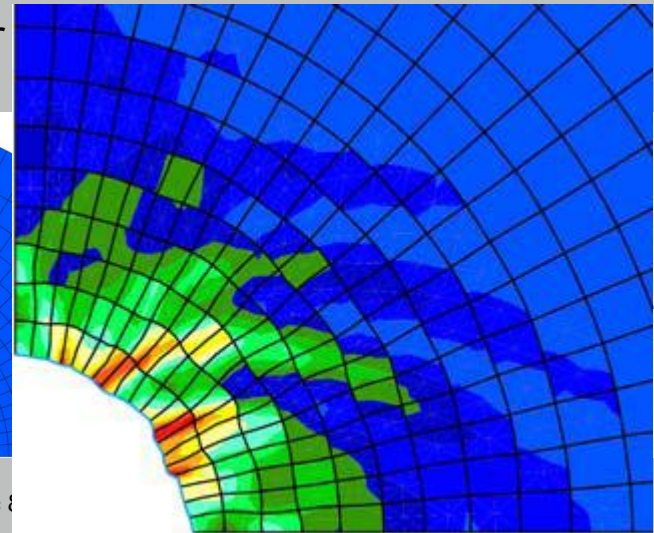
t = 3700
 $\epsilon_{equ \max} = 228 \cdot 10^{-5}$



t = 4000
 $\epsilon_{equ \max} = 300 \cdot 10^{-5}$



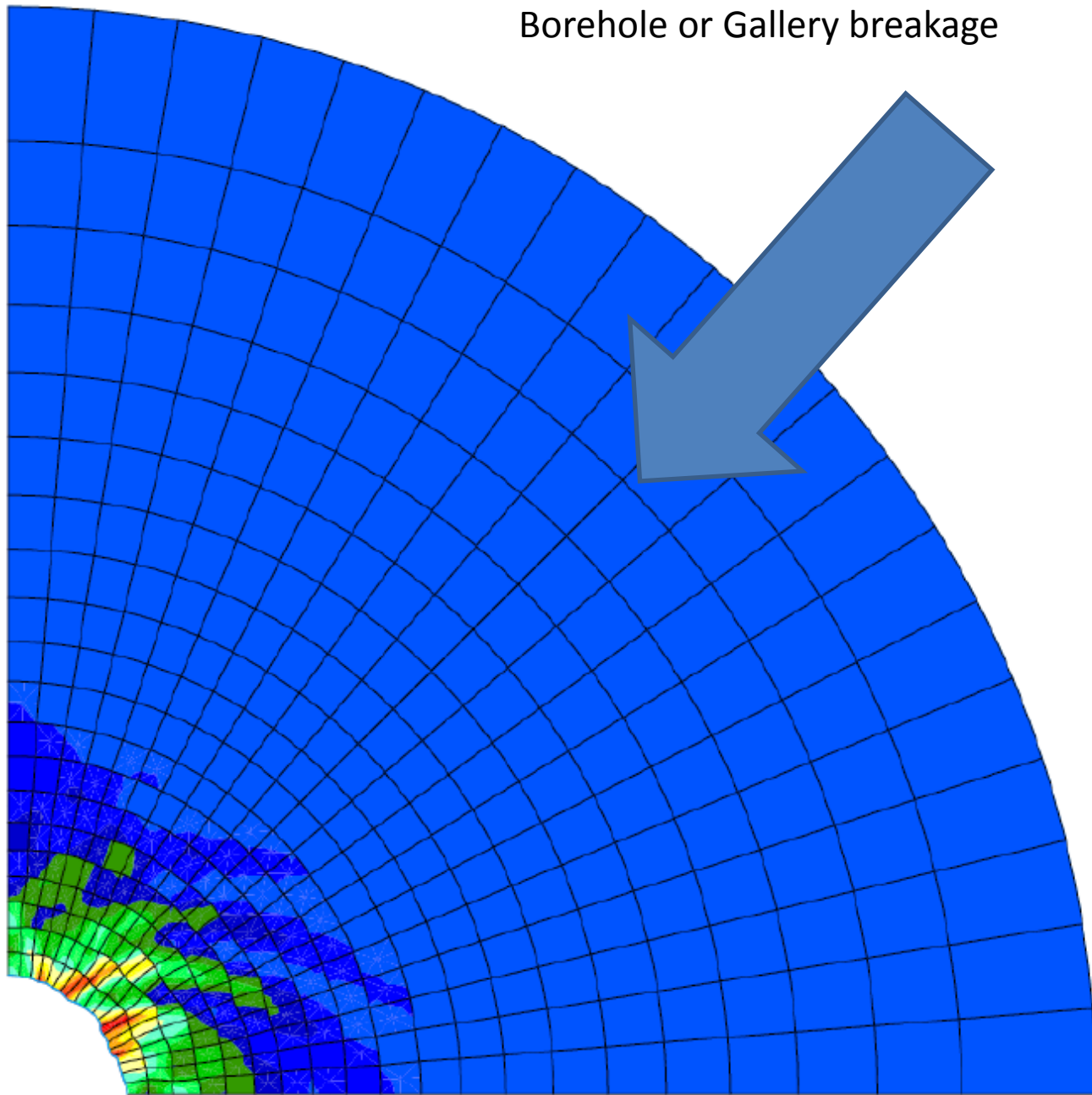
t = 4500
 $\epsilon_{equ \max} = 124 \cdot 10^{-5}$



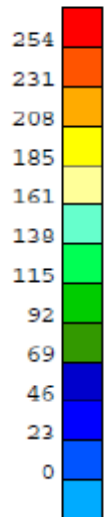
t = 8000
 $\epsilon_{equ \max} = 254 \cdot 10^{-5}$



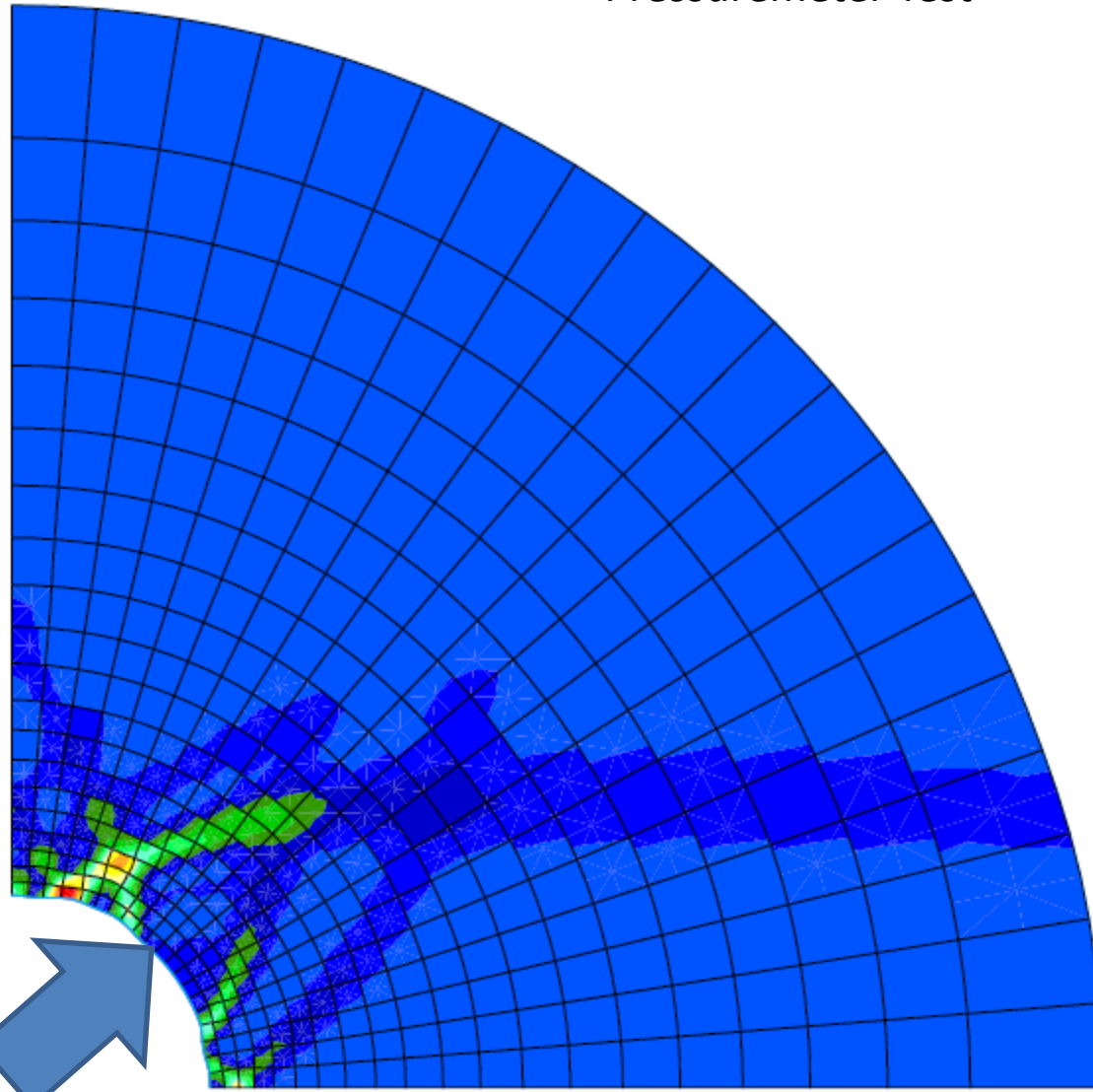
Borehole or Gallery breakage



* 1.000E-03



Pressuremeter Test

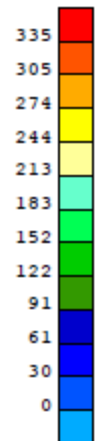


COURBE DE E-EQ
 TIME DMULCUM
 5.626E+03 1.00

DELT= 0.305E-01
 X 0.100E+04
 TMIN= 0.00
 TMAX= 0.364
 DANS STRUCTURE DEFORMEE: ITYPE=
 (DEPL= 1.00)

VUE EN PLAN X Y

* 1.000E-03



	MIN	MAX
X	0.000	4.360
Y	0.000	4.325
Z	0.000	0.000

SELECTION DES ELEMENTS
 TOUS

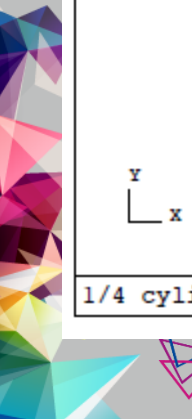


DESFIN 9.4 17/03/2014

1/4 cylindre creux q8 - maillage progressive pl

tknguyen

Cynlindre_creux_q8



Hollow cylinder inflation as a pressuremeter test

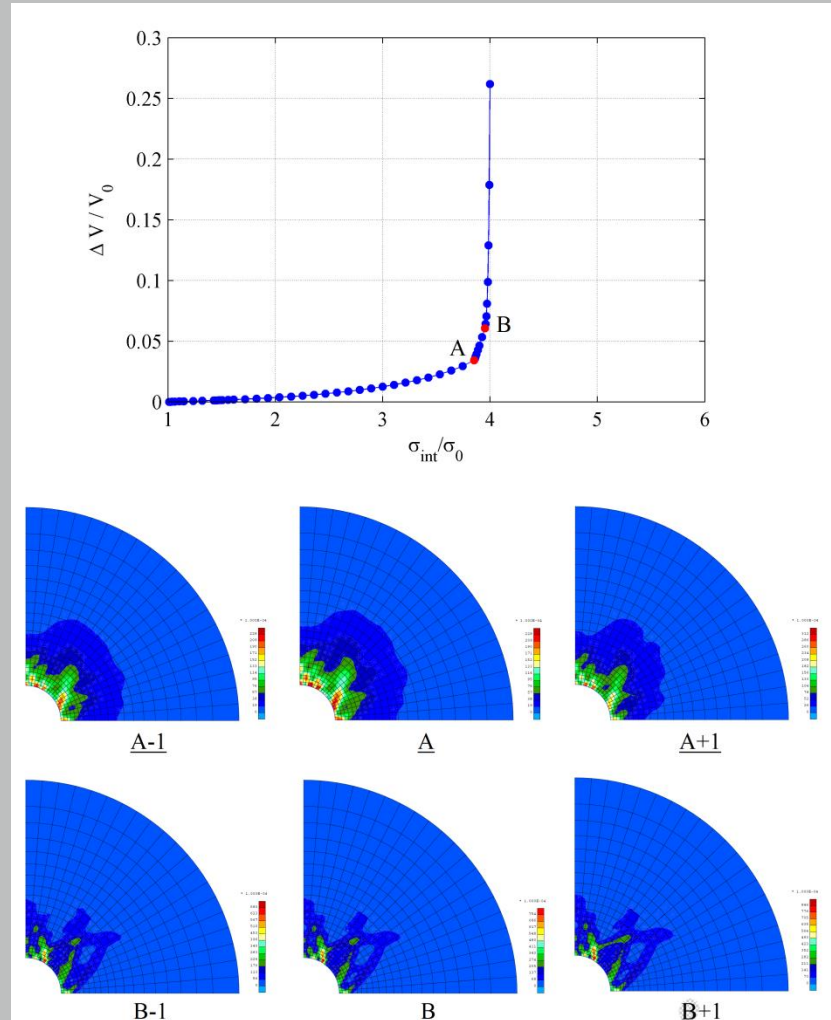
Cavity volume vs Pressure curve :

Typical features of the **pressuremeter log curve** :

Initial quasi linear phase,
(*pressuremeter module*)

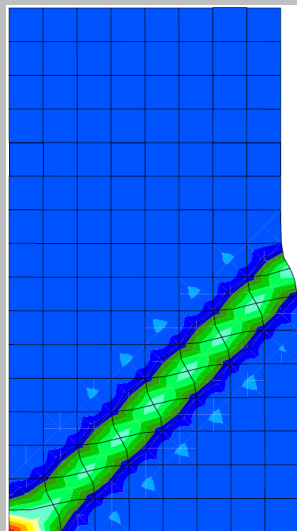
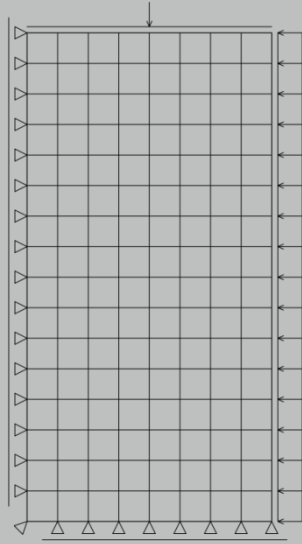
Then transition to a vertical asymptot (so
called limit pressure)

Localisation effect : quasi discontinuous
transition from the quasi linear phase to
the final vertical branch (point A)

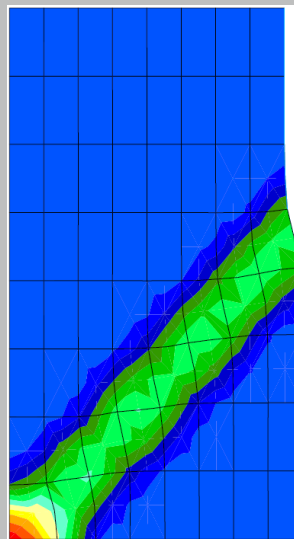
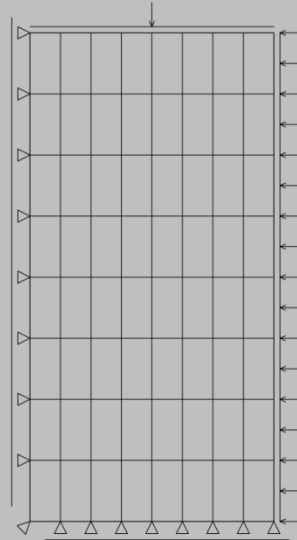


Multiscale Computations: different meshes

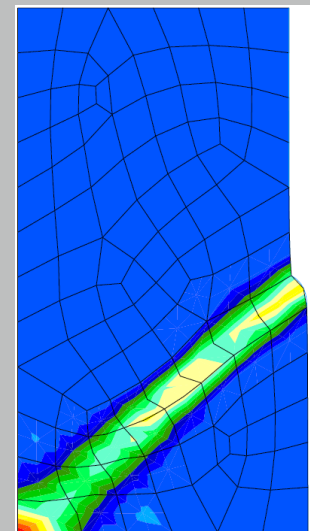
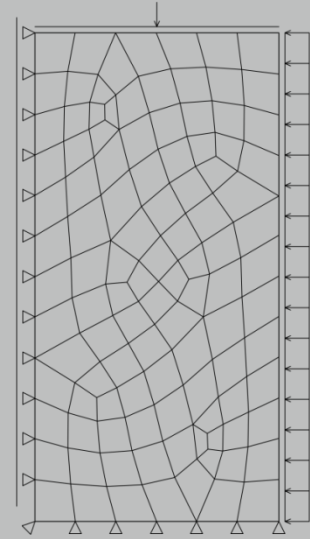
!!! Mesh dependency (as usual in FEM) : issues and solutions



128 elements



64 elements



106 elements



(partial) Conclusions

First CONCLUSION :

- We have presented an **integrated Two-scale numerical approach** for granular materials: combining FEM (at macro scale) and DEM (at micro scale).
 - Illustration by two examples of BVP :
 - a biaxial compression test and
 - a hollow cylinder (analogy of underground excavations and drilling)
 - Strain localization was observed in both cases.
 - Mesh dependency confirmed.
- 2nd **gradient regularization** allows to restore mesh *independency*
- **Parallelisation (OpenMP / MPI)** allows to mitigate the CPU cost issue :
 - Parallelisation of the code (element loop) using **OpenMP** has showed to be very effective : scalability about 80%, but shared memory → limited number of processors
 - Parallelisation using **MPI** even more effective since a priori no limit is set to the number of processors : excellent scalability as well, improving with the size of the micro problem

PERSPECTIVES

- 3D approach



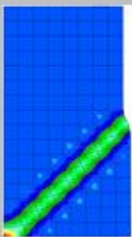
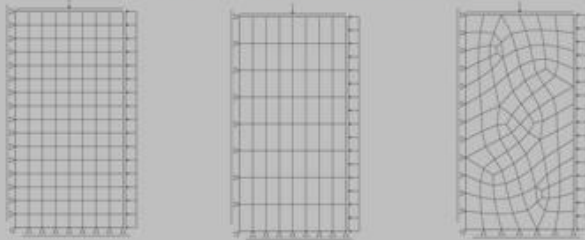
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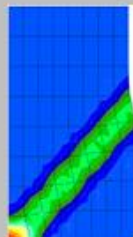
Mesh dependency problem

Multiscale Computations: different meshes

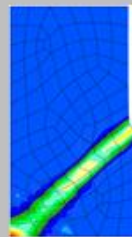
Mesh dependency as usual in FEM : issues and solutions



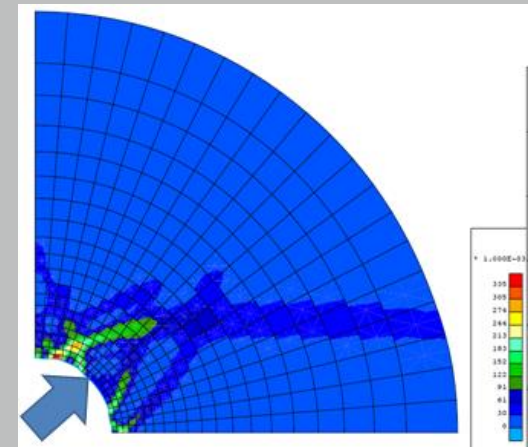
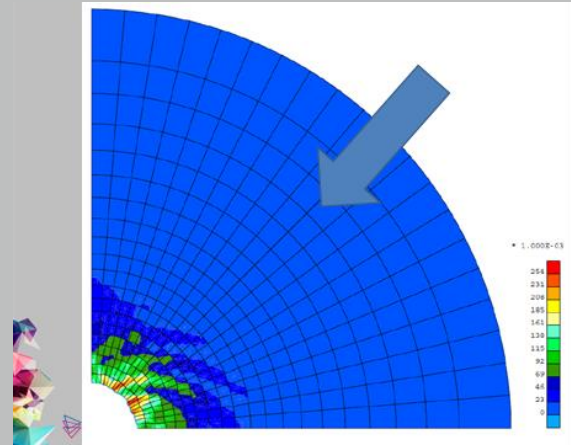
128 elements



64 elements



106 elements



Same problem in borehole / pressuremeter

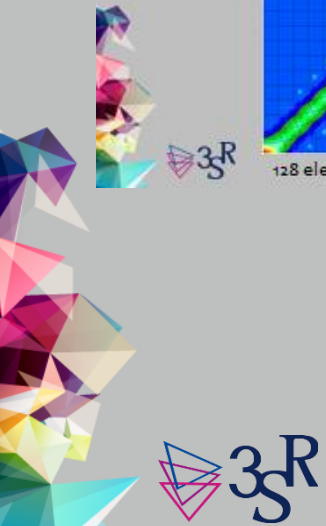
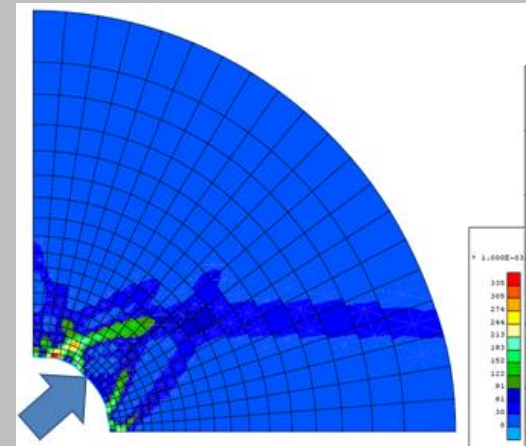
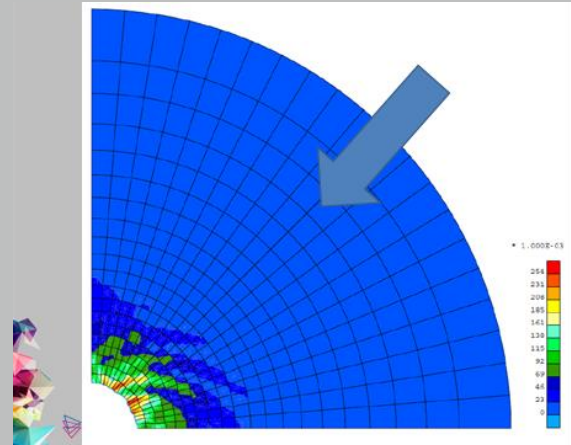
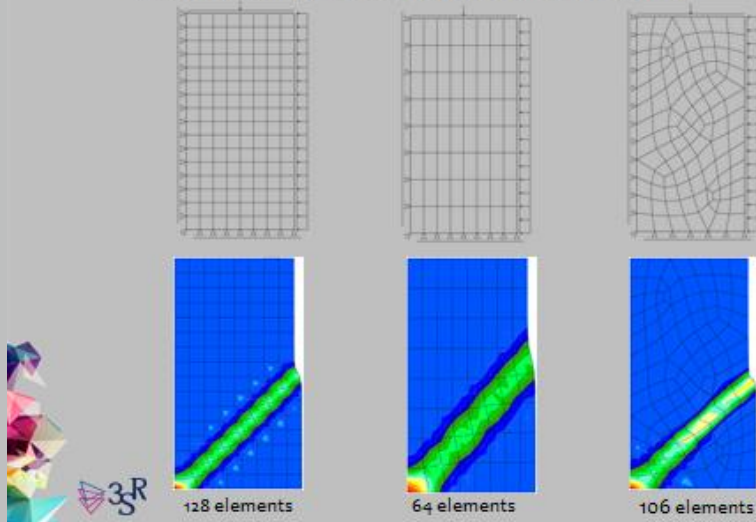
Mesh dependency problem

Solution ?

: regularization of the bvp

Multiscale Computations: different meshes

Mesh dependency as usual in FEM : issues and solutions



2nd gradient regularisation

- ▶ What is it ? ... a brief *aperçu* in 4 slides

Second gradient regularisation
after Chambon R. et al. (1) & Bésuelle P. (2)

- Media with microstructure - enriched kinematics
- macrokinematics
 - \mathbf{u} is the (macro) displacement field
 - \mathbf{F}_i is the macro displacement gradient
 - $\mathbf{F}_i = \frac{\partial \mathbf{u}}{\partial \mathbf{x}_i}$
 - \mathbf{D}_i is the macro strain:
 - $\mathbf{D}_i = \frac{1}{2}(\mathbf{F}_i + \mathbf{F}_i^T)$
 - \mathbf{R}_i is the macro rotation:
 - $\mathbf{R}_i = \frac{1}{2}(\mathbf{F}_i - \mathbf{F}_i^T)$
- enrichment : microkinematics
 - \mathbf{u}_μ is the microkinematic gradient:
 - $\mathbf{d}_\mu = \frac{1}{2}(\mathbf{F}_\mu + \mathbf{F}_\mu^T)$
 - \mathbf{e}_μ is the microstrain:
 - $\mathbf{e}_\mu = \frac{1}{2}(\mathbf{F}_\mu - \mathbf{F}_\mu^T)$
 - \mathbf{h}_μ is the (micro) second gradient:
 - $\mathbf{h}_\mu = \frac{\partial \mathbf{e}_\mu}{\partial \mathbf{x}_\mu}$: Local second gradient
- The internal virtual work

$$W^{int} = \int_{\Omega} w^* \, d\mathbf{e} = \int_{\Omega} (\sigma_{ij} D_{ij}^* + \tau_{ijk} (F_{ij}^* - F_{ij}^T) + \chi_{ijk} h_{ijk}^*) \, d\mathbf{r}$$

1 - Chambon R., Gurtin M., 1991, Int. Journal of Solids and Structures, vol 28, no 18-17, pp 4823-47
2 - Bésuelle P. et al., 2002, Journal of Mathematical Physics and Dynamics, vol. 1, no 7, pp 222-61

30

Second gradient regularisation
after Chambon R. et al. (1) & Bésuelle P. (2)

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- The internal virtual work: Additional kinematical constraint: $\mathbf{f}_{ij} = \mathbf{F}_{ij}$

$$W^{int} = \int_{\Omega} w^* \, d\mathbf{e} = \int_{\Omega} (\sigma_{ij} D_{ij}^* + \tau_{ijk} (F_{ij}^* - F_{ij}^T) + \chi_{ijk} h_{ijk}^*) \, d\mathbf{r}$$

1 - Chambon R., Gurtin M., 1991, Int. Journal of Solids and Structures, vol 28, no 18-17, pp 4823-47
2 - Bésuelle P. et al., 2002, Journal of Mathematical Physics and Dynamics, vol. 1, no 7, pp 222-61

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Second gradient regularisation (cont'd)
after Chambon R. et al. (1) & Bésuelle P. (2)

- FEM : Introducing Lagrange multipliers to enforce the constraint $\mathbf{f}_{ij} = \mathbf{F}_{ij}$:

$$\int_{\Omega} (\sigma_{ij} \frac{\partial u_i^*}{\partial x_j} + \chi_{ijk} \frac{\partial u_{i\mu}^*}{\partial x_\mu} - \tau_{ijk} (F_{ij}^* - F_{ij}^T) + \chi_{ijk} h_{ijk}^*) \, d\mathbf{r} - \int_{\Omega} \lambda_{ij} (\frac{\partial u_i^*}{\partial x_j} - F_{ij}^*) \, d\mathbf{r} = 0$$

1 - Chambon R., Gurtin M., 1991, Int. Journal of Solids and Structures, vol 28, no 18-17, pp 4823-47
2 - Bésuelle P. et al., 2002, Journal of Mathematical Physics and Dynamics, vol. 1, no 7, pp 222-61

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Second gradient regularisation (cont'd)

- A 2nd gradient model for FE/μDEM double scale analysis

Constitutive equations:

- Classical part : LAM/DEM
- 2nd gradient part : only 11 param: \mathbf{C}

1 - Chambon R., Gurtin M., 1991, Int. Journal of Solids and Structures, vol 28, no 18-17, pp 4823-47
2 - Bésuelle P. et al., 2002, Journal of Mathematical Physics and Dynamics, vol. 1, no 7, pp 222-61

33



Second gradient regularisation

after Chambon R. et al. (1) & Bésuelle P. (2)

- ▶ Media with microstructure : enriched kinematics
- ▶ macrokinematics

- u_i is the (macro) displacement field
- F_{ij} is the macro displacement gradient

$$F_{ij} = \frac{\partial u_i}{\partial x_j}$$

- D_{ij} is the macro strain:

$$D_{ij} = \frac{1}{2}(F_{ij} + F_{ji})$$

- R_{ij} is the macro rotation:

$$R_{ij} = \frac{1}{2}(F_{ij} - F_{ji})$$

- ▶ enrichment : microkinematics

- f_{ij} is the microkinematic gradient.
- d_{ij} is the microstrain:

$$d_{ij} = \frac{1}{2}(f_{ij} + f_{ji})$$

- r_{ij} is the microrotation:

$$r_{ij} = \frac{1}{2}(f_{ij} - f_{ji})$$

- h_{ijk} is the (micro) second gradient:

$$h_{ijk} = \frac{\partial f_{ij}}{\partial x_k} \quad \text{: Local second gradient}$$

- ▶ The virtual internal work :

$$W^{*i} = \int_{\Omega} w^* \, dv = \int_{\Omega} (\sigma_{ij} D_{ij}^* + \tau_{ij} (f_{ij}^* - F_{ij}^*) + \chi_{ijk} h_{ijk}^*) \, dv$$

Second gradient regularisation

after Chambon R. et al. (1) & Bésuelle P. (2)

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- ▶ macrokinematics

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- ▶ The virtual internal work :

Additional kinematical constraint : $f_{ij} = F_{ij}$

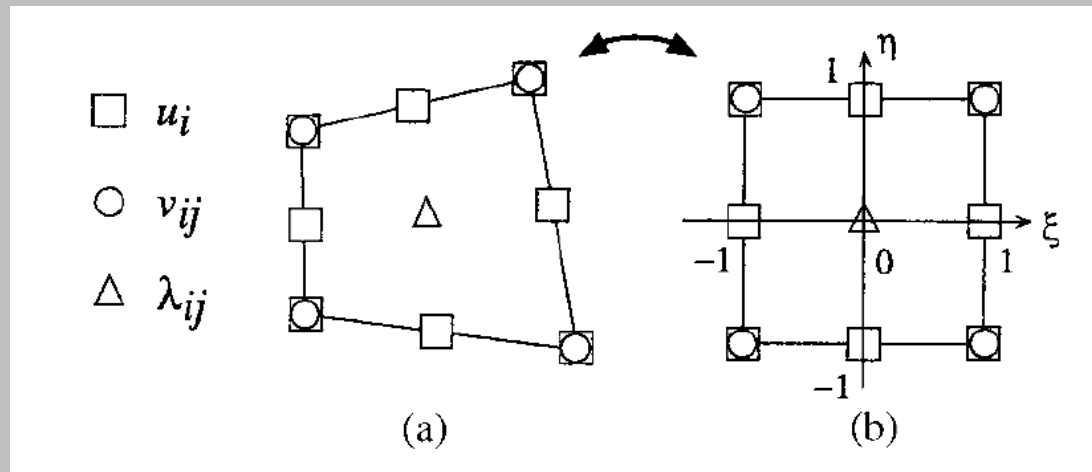
$$W^{*i} = \int_{\Omega} w^* dv = \int_{\Omega} (\sigma_{ij} D_{ij}^* + \tau_{ij} (f_{ij}^* - F_{ij}^*) + \chi_{ijk} h_{ijk}^*) dv$$

Second gradient regularisation (cont'd)

after Chambon R. et al. (1) & Bésuelle P. (2)

- ▶ FEM : introducing Lagrange multipliers to enforce the condition $f_{ij} = F_{ij}$:

$$\int_{\Omega^t} \left(\sigma_{ij}^t \frac{\partial u_i^*}{\partial x_j^t} + \chi_{ijk}^t \frac{\partial v_{ij}^*}{\partial x_k^t} \right) d\Omega^t - \int_{\Omega^t} \lambda_{ij}^t \left(\frac{\partial u_i^*}{\partial x_j^t} - v_{ij}^* \right) d\Omega^t - \bar{P}_e^* = 0$$

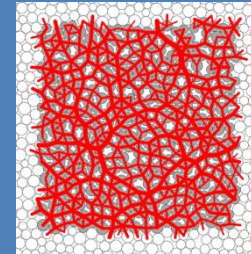


Second gradient regularisation (cont'd)

- ▶ A 2nd gradient model for FEM-DEM double scale analysis

▶ Constitutive equations :

- ▶ Classical part : LHN_DEM

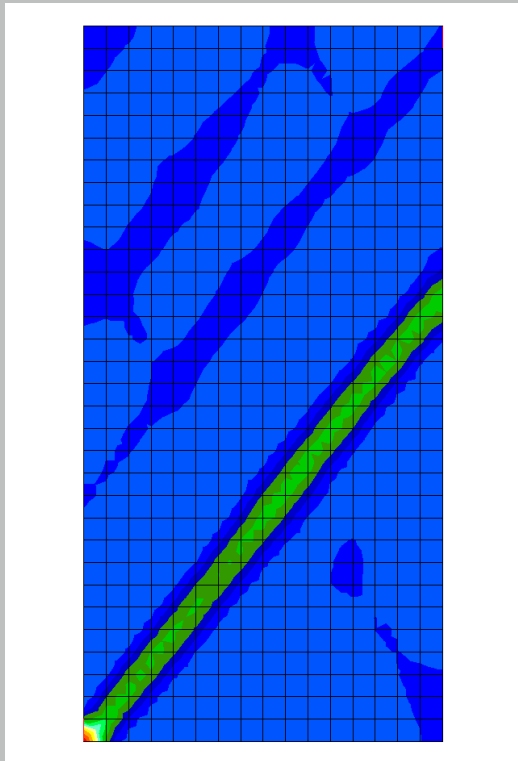


- ▶ 2nd gradient part : only 1 parameter **D**

$$\begin{bmatrix} \nabla \chi_{111} \\ \nabla \chi_{112} \\ \nabla \chi_{121} \\ \nabla \chi_{122} \\ \nabla \chi_{211} \\ \nabla \chi_{212} \\ \nabla \chi_{221} \\ \nabla \chi_{222} \end{bmatrix} = \begin{bmatrix} D & 0 & 0 & 0 & 0 & \frac{D}{2} & \frac{D}{2} & 0 \\ 0 & \frac{D}{2} & \frac{D}{2} & 0 & -\frac{D}{2} & 0 & 0 & \frac{D}{2} \\ 0 & \frac{D}{2} & \frac{D}{2} & 0 & -\frac{D}{2} & 0 & 0 & \frac{D}{2} \\ 0 & 0 & 0 & D & 0 & -\frac{D}{2} & -\frac{D}{2} & 0 \\ 0 & -\frac{D}{2} & -\frac{D}{2} & 0 & D & 0 & 0 & 0 \\ \frac{D}{2} & 0 & 0 & -\frac{D}{2} & 0 & \frac{D}{2} & \frac{D}{2} & 0 \\ \frac{D}{2} & 0 & 0 & -\frac{D}{2} & 0 & \frac{D}{2} & \frac{D}{2} & 0 \\ 0 & \frac{D}{2} & \frac{D}{2} & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{\partial v_{11}}{\partial x_1} \\ \frac{\partial v_{11}}{\partial x_2} \\ \frac{\partial v_{12}}{\partial x_1} \\ \frac{\partial v_{12}}{\partial x_2} \\ \frac{\partial v_{21}}{\partial x_1} \\ \frac{\partial v_{21}}{\partial x_2} \\ \frac{\partial v_{22}}{\partial x_1} \\ \frac{\partial v_{22}}{\partial x_2} \end{bmatrix}$$

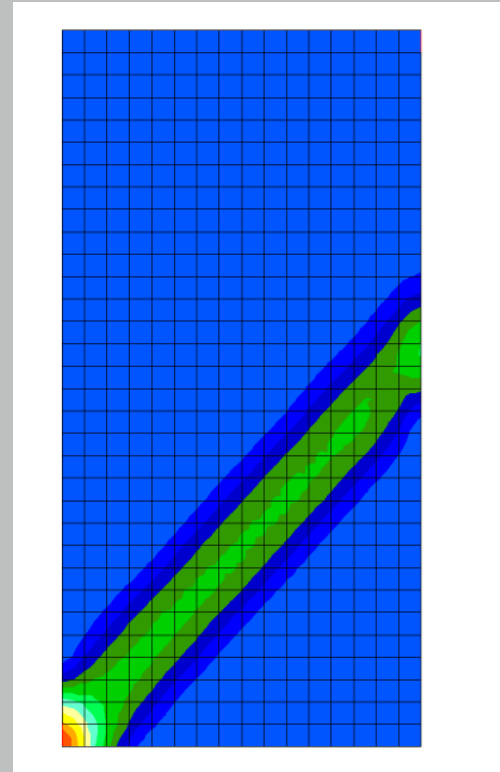
Restoring mesh independency with 2nd gradient

▶ No 2nd gradient



512 FE x 400 DE

▶ With 2nd gradient

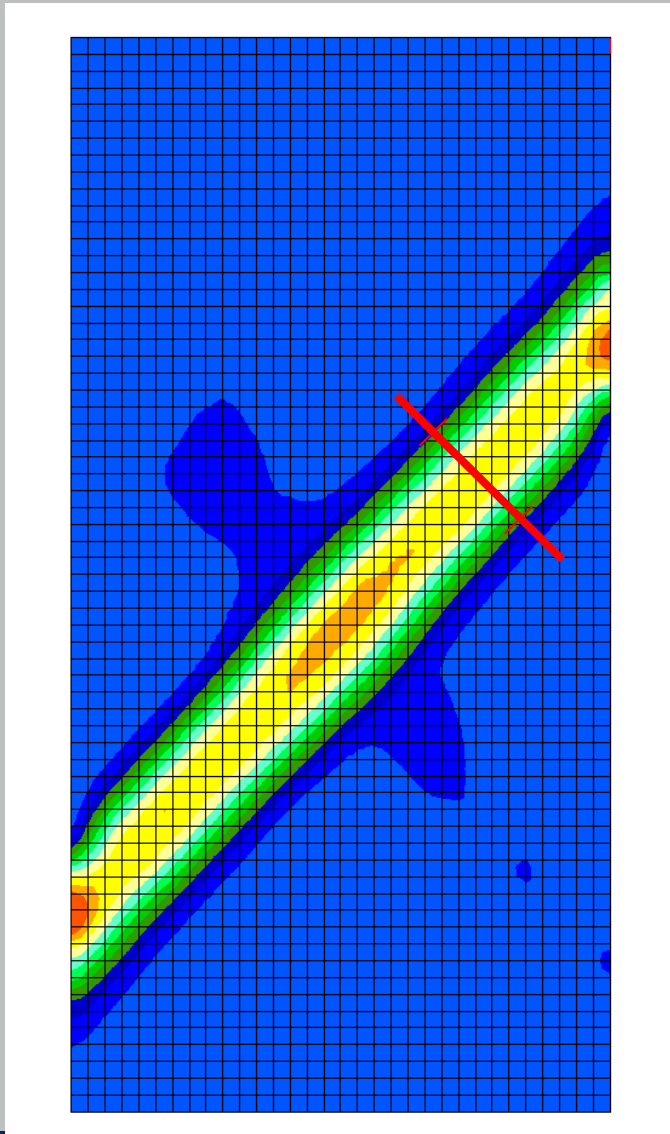
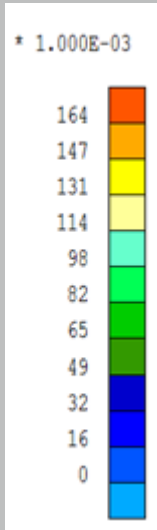


512 FE x 400 DE

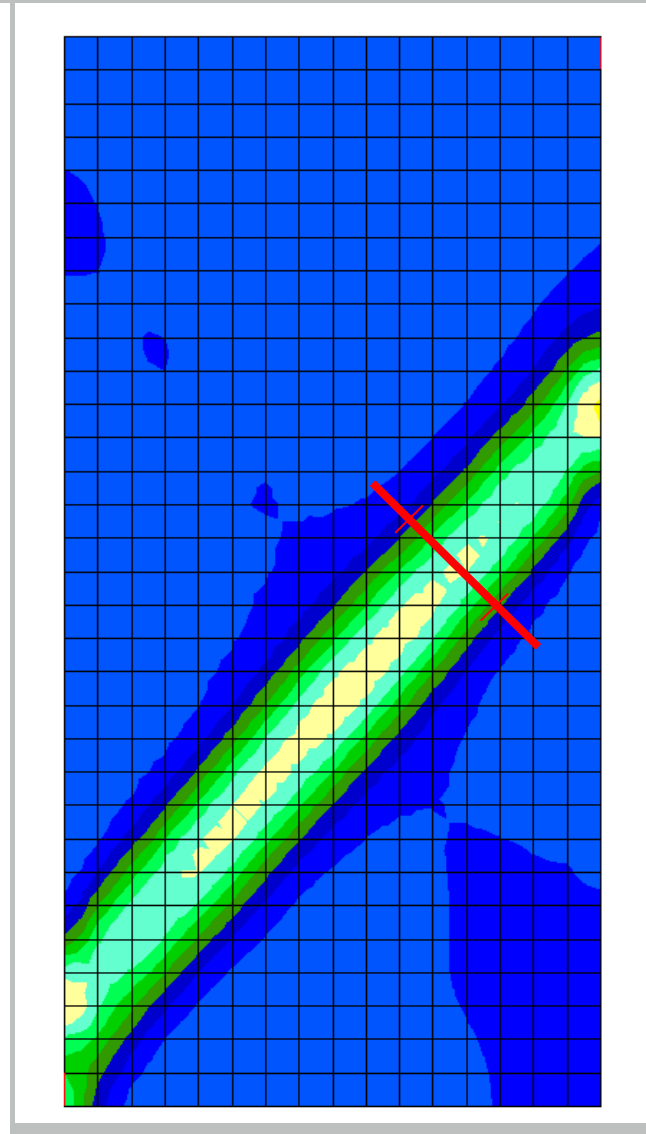


Second Gradient $D=0,64E-2$

Axial strain = $2 \cdot 10^{-2}$



2048FEMx400DEM

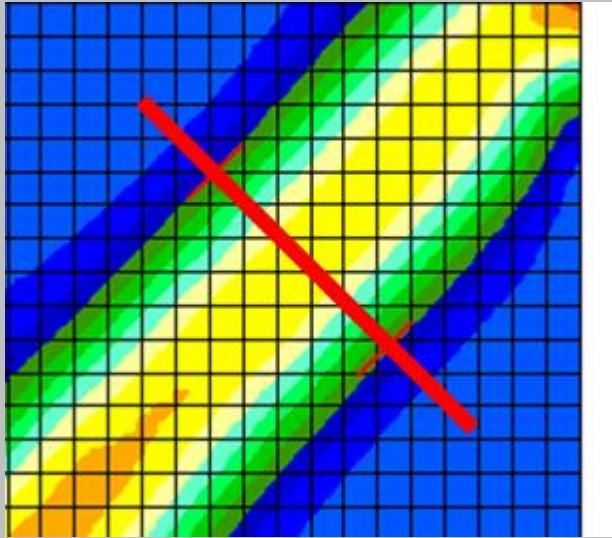


512FEMx400DEM

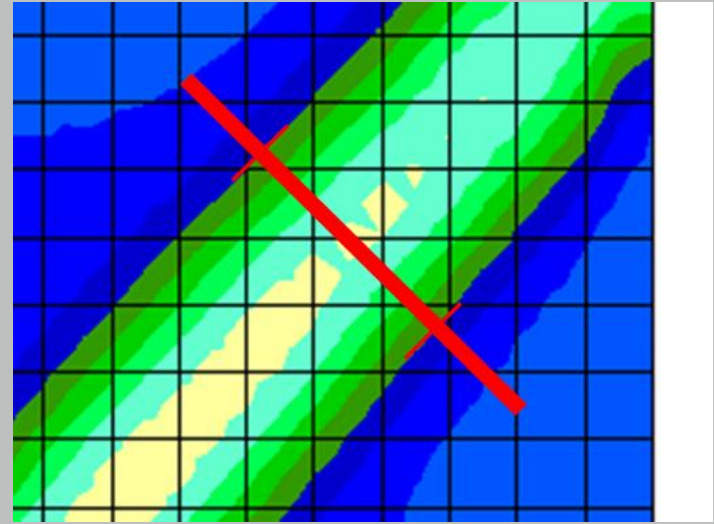


Restoring mesh independency with 2nd gradient

Second Gradient $D=0,64E-2$



2048 FE x 400 DE

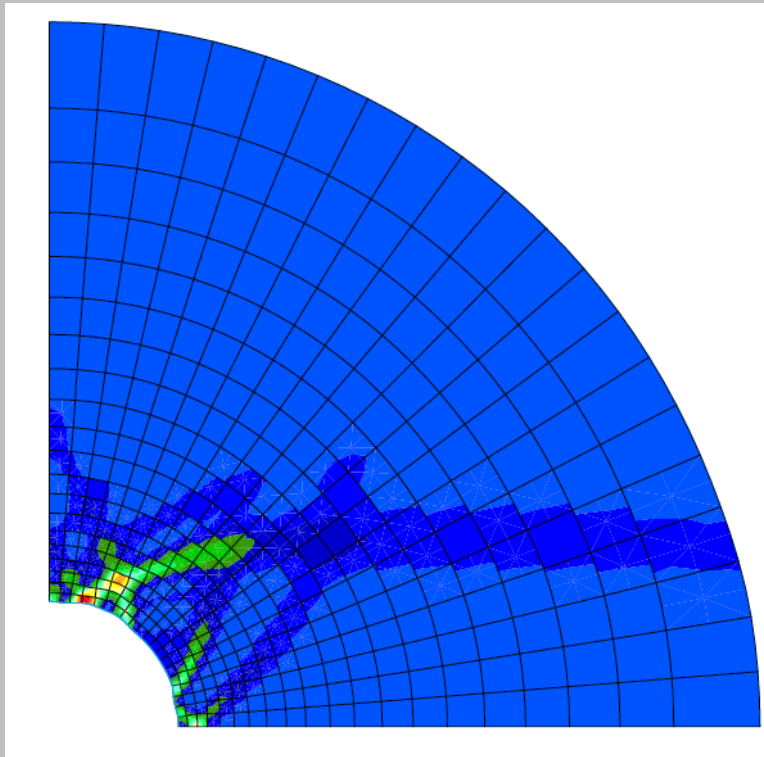


512 FE x 400 DE



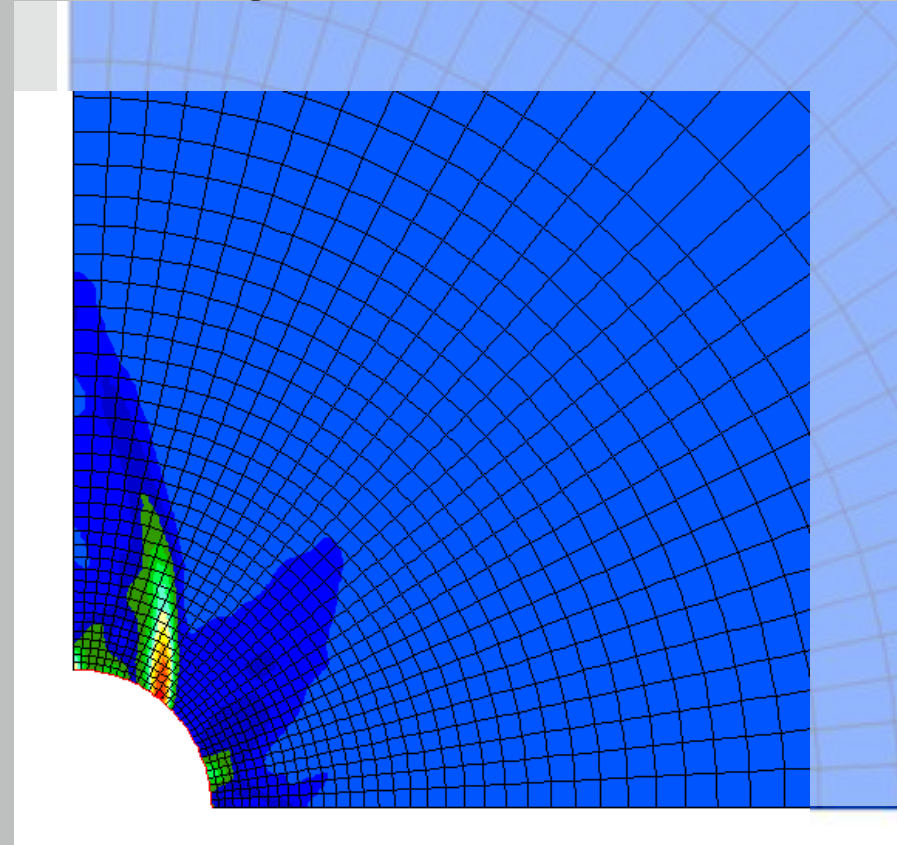
Restoring mesh independency with 2nd gradient

▶ No 2nd gradient



400 FE x 400 DE

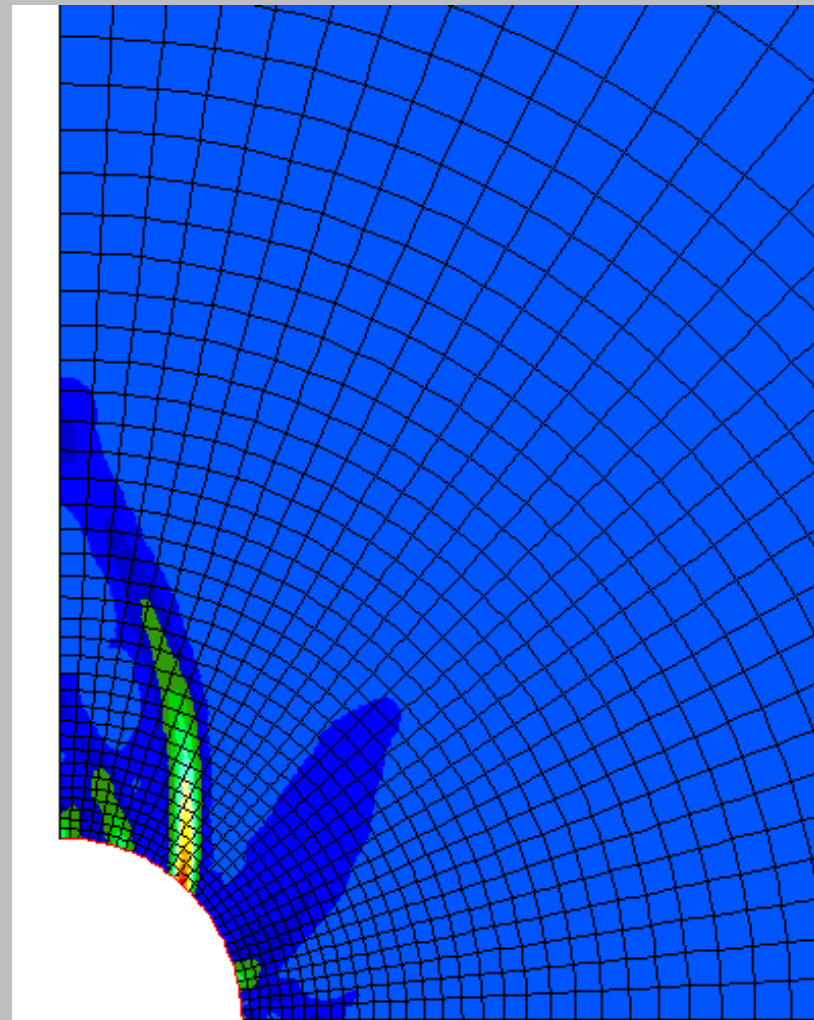
▶ With 2nd gradient



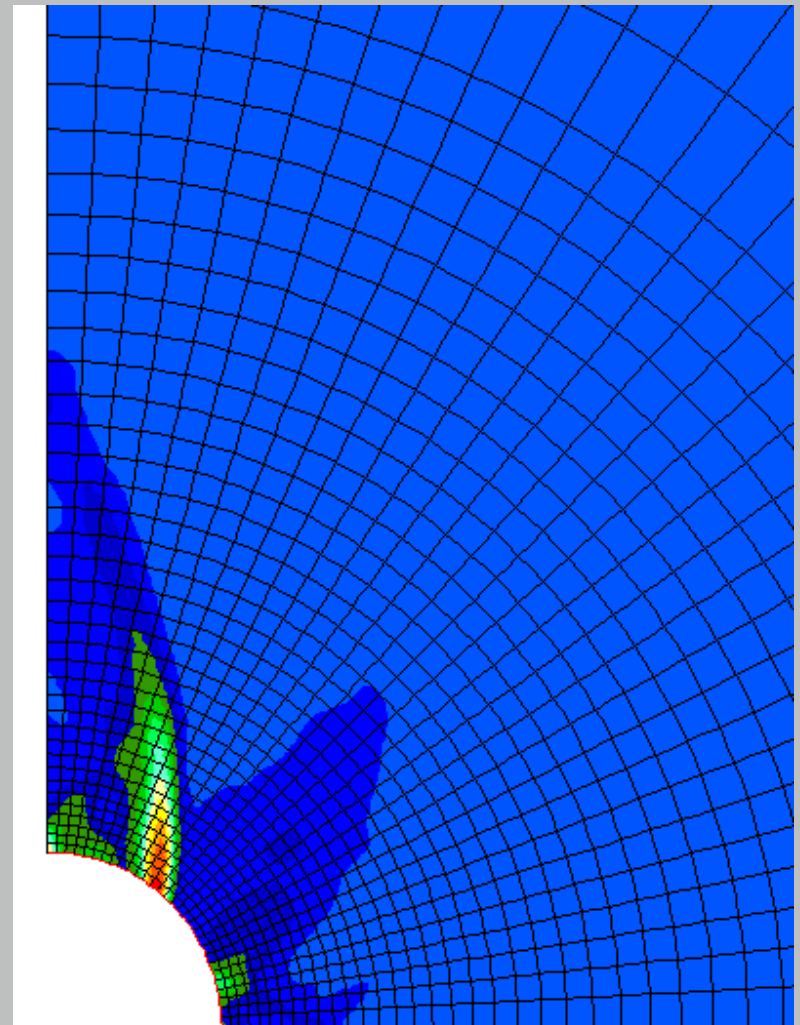
1230 FE x 400 DE



1290FEMx400DEM - increase internal pressure



Second Gradient $D=5,00E-2$



Second Gradient $D=1,00E-1$ (x2)



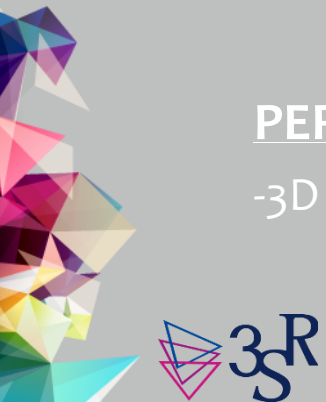
(partial) Conclusions

Second CONCLUSION

- We have presented an *integrated Two-scale numerical approach* for granular materials: combining FEM (at macro scale) and DEM (at micro scale).
 - Illustration by two examples of BVP :
 - a biaxial compression test and
 - a hollow cylinder (analogy of underground excavations and drilling)
 - Strain localization was observed in both cases.
 - Mesh dependency confirmed.
- **2nd gradient regularization** allows to restore mesh *independency*
- **Parallelisation (OpenMP / MPI)** allows to mitigate the CPU cost issue :
 - Parallelisation of the code (element loop) using **OpenMP** has showed to be very effective : scalability about 80%, but shared memory → limited number of processors
 - Parallelisation using **MPI** even more effective since a priori no limit is set to the number of processors : excellent scalability as well, improving with the size of the micro problem

PERSPECTIVES

- 3D approach



Outline

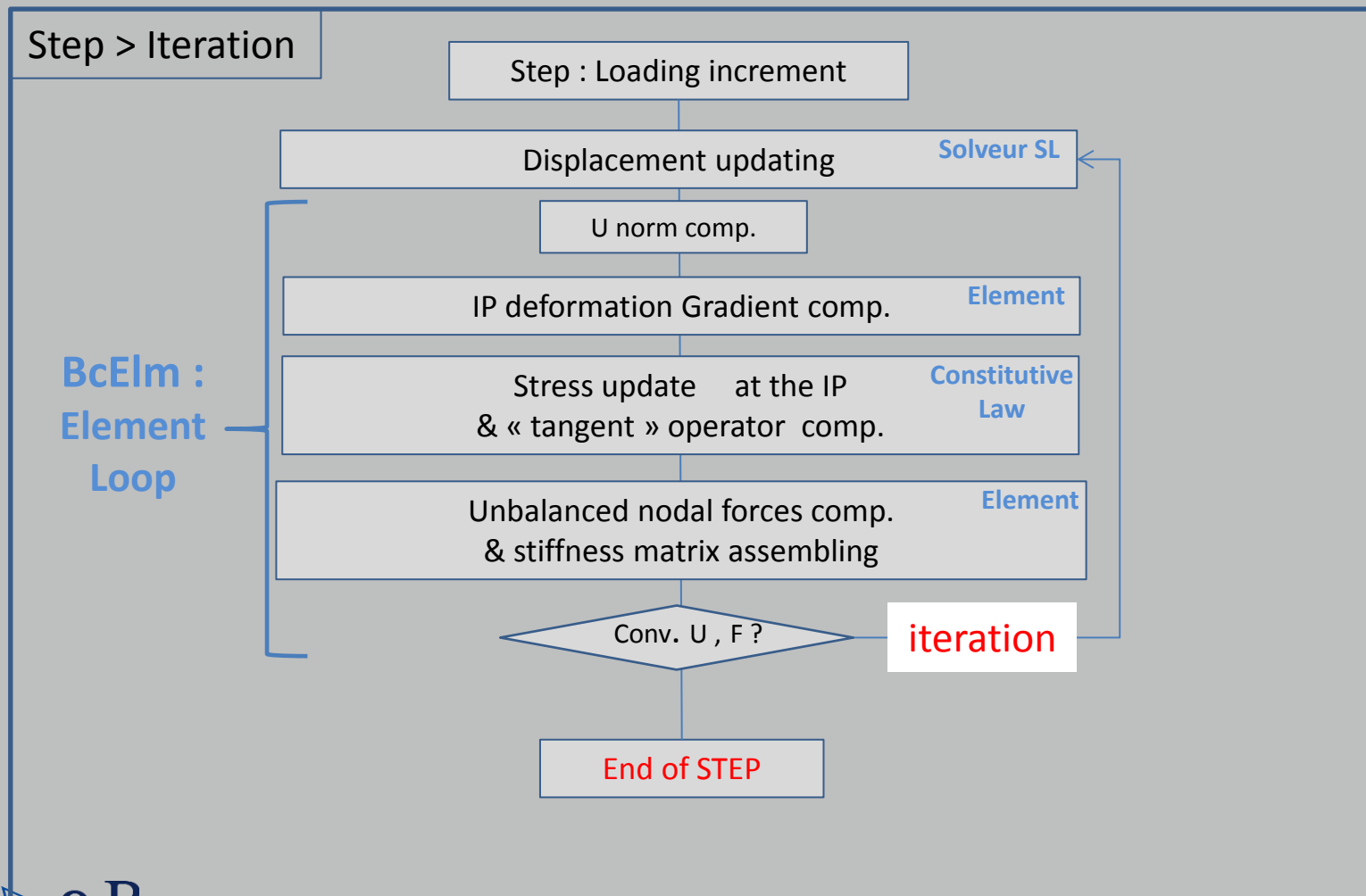
1. Introduction : principle
2. Micro-scale (DEM) Model
3. Multi-scale Coupling Method
4. FEM-DEM simulations
5. 2nd gradient : motivation, methods and results
- 6. CPU cost issue : parallelization solution**
7. Conclusions & Perspectives

Computational efficiency : the CPU cost Issue

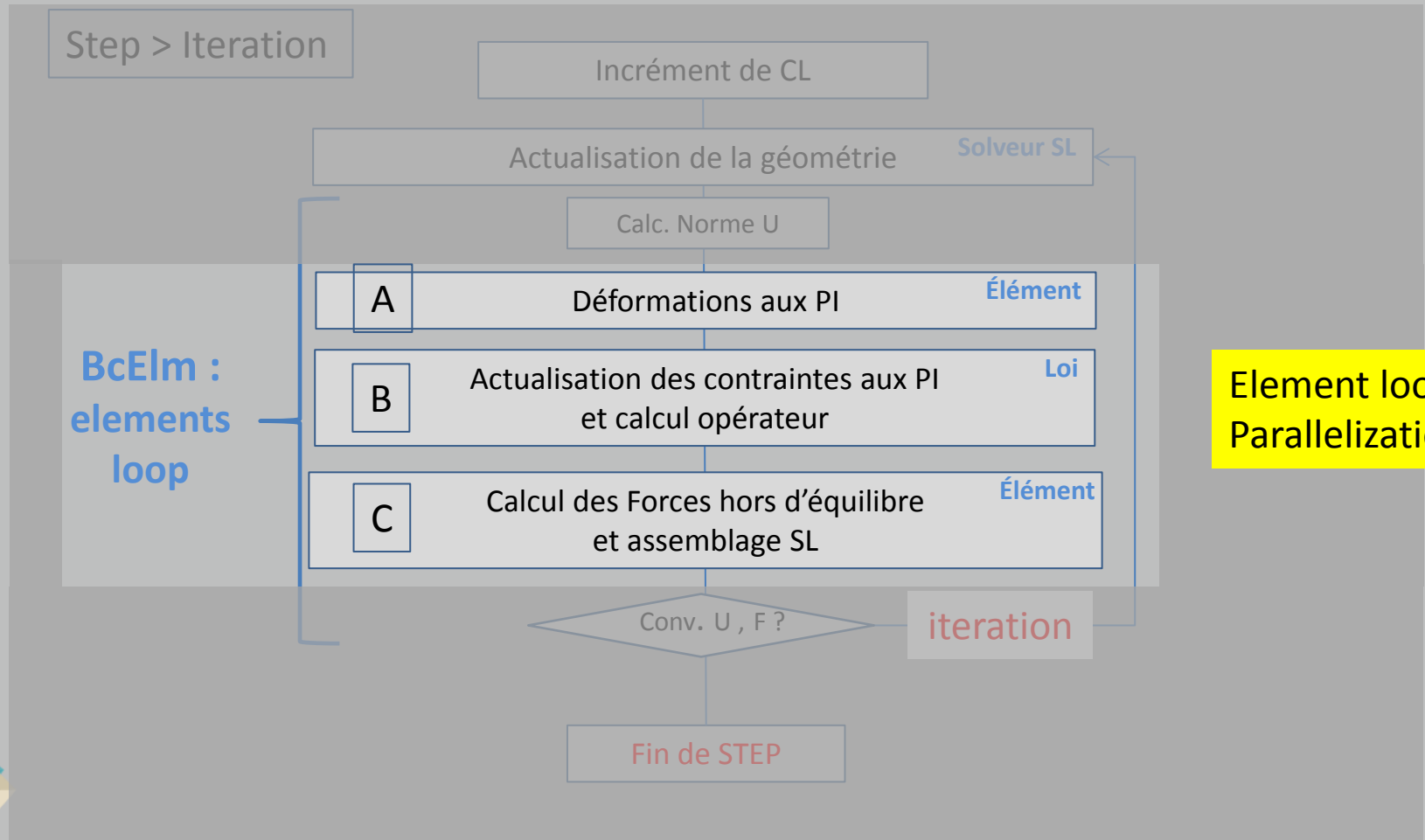
- ▶ Overcoming the CPU cost issue is a key point for practical use of double scale FEMxDEM analysis
- ▶ Enhancing the computational efficiency rely on several improvement tracks :
 - ▶ Choice of the Newton Raphson iteration operator
 - ▶ (2nd gradient regularization)
 - ▶ Parallelization



FEM code workflow (nothing new here)

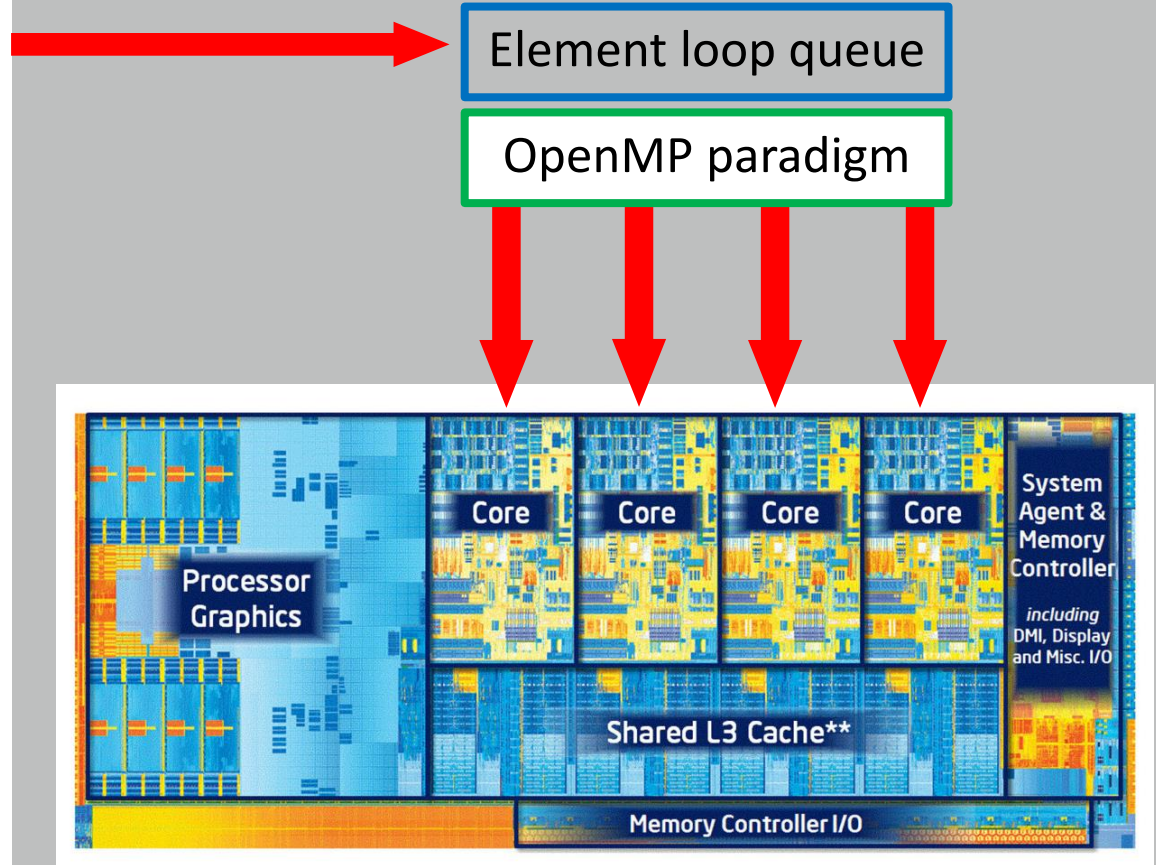
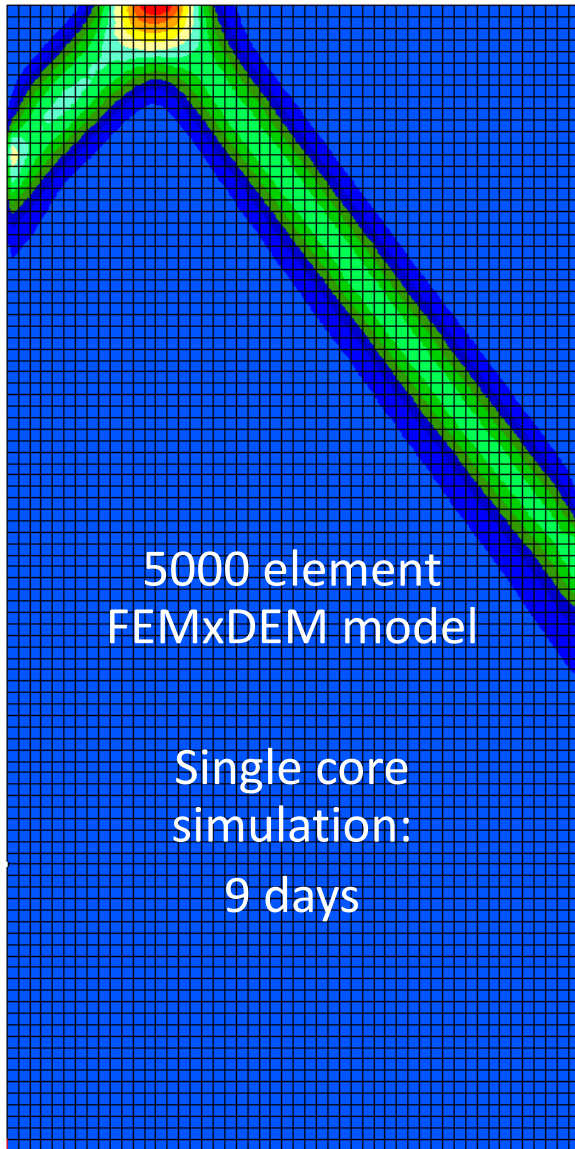


Element loop workflow

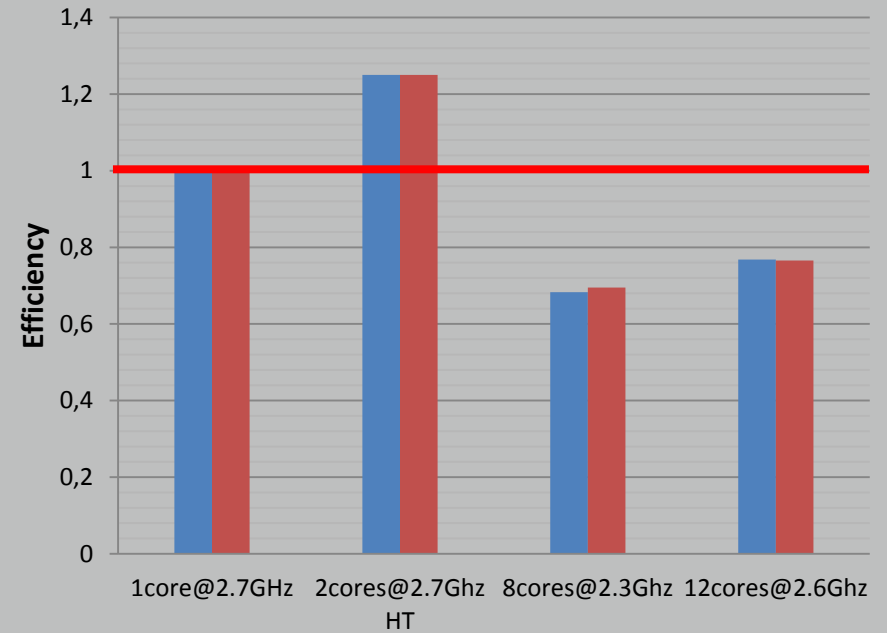
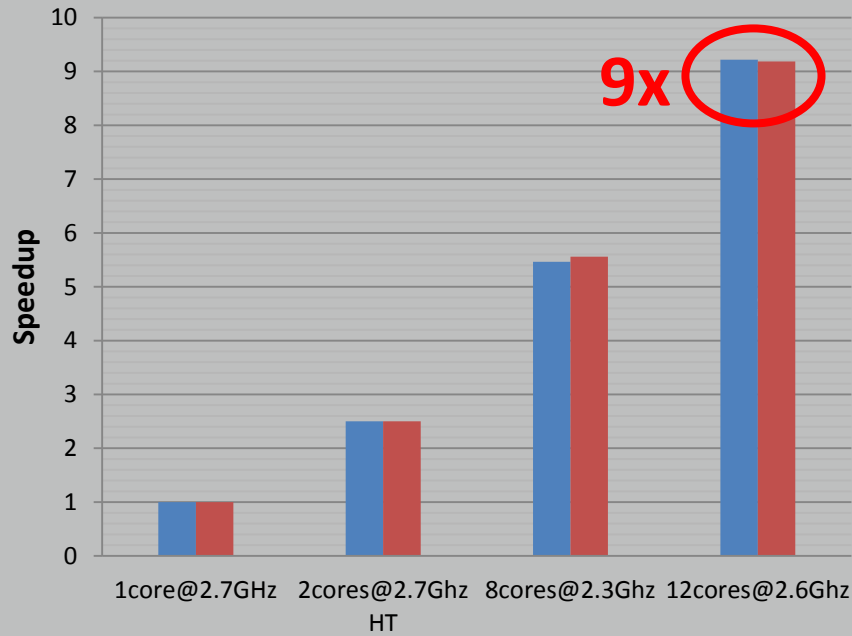


Element loop parallelisation

- Double scale → element intensive model.
- Parallelisation using an OpenMP : **shared memory model**
- Results: speedup up to 9 using a 12 core machine.
- Other approaches: massive parallelization (MPI).



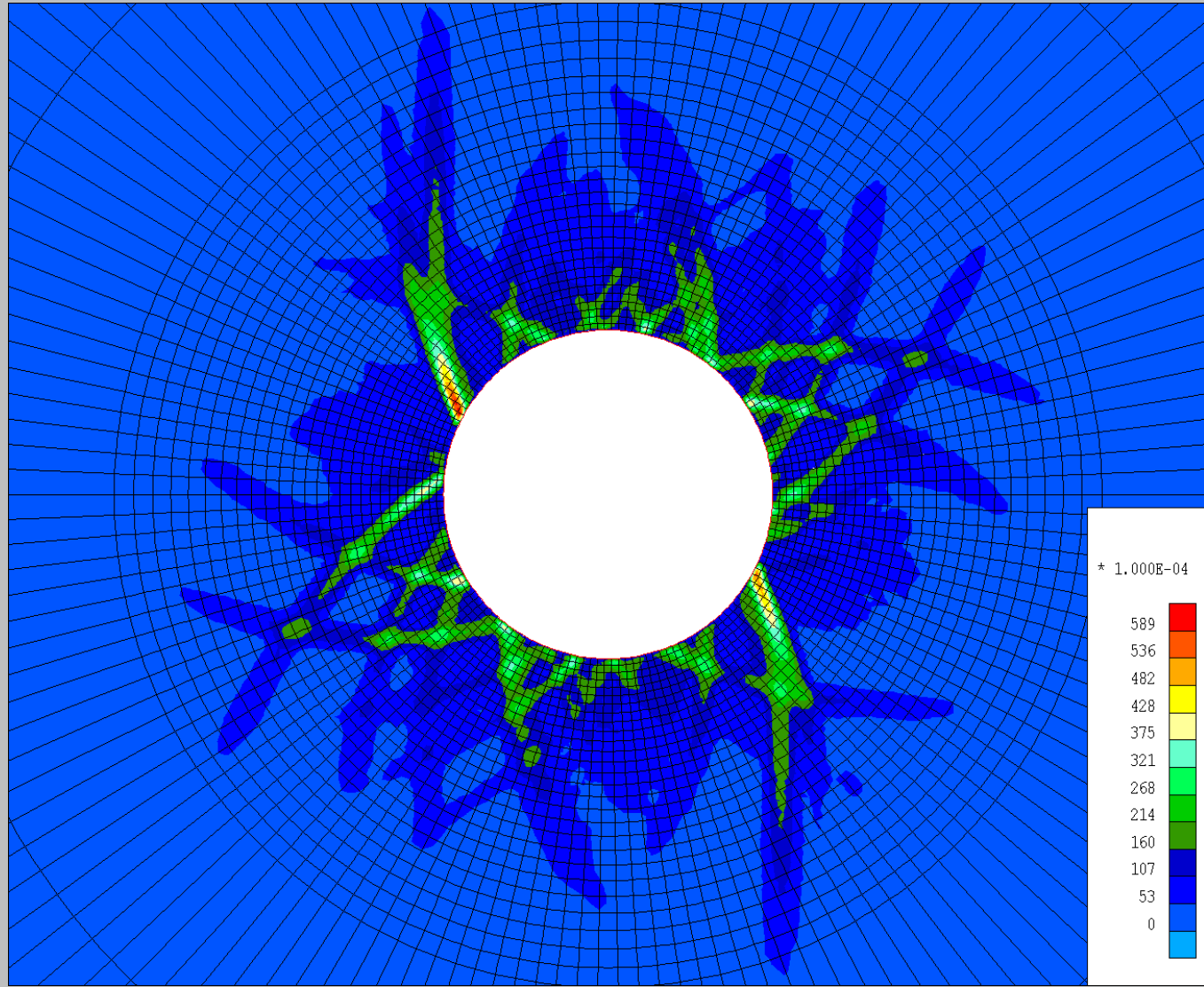
Element loop parallelisation, performance



512 FEM x 400 DEM
2048 FEM x 100 DEM

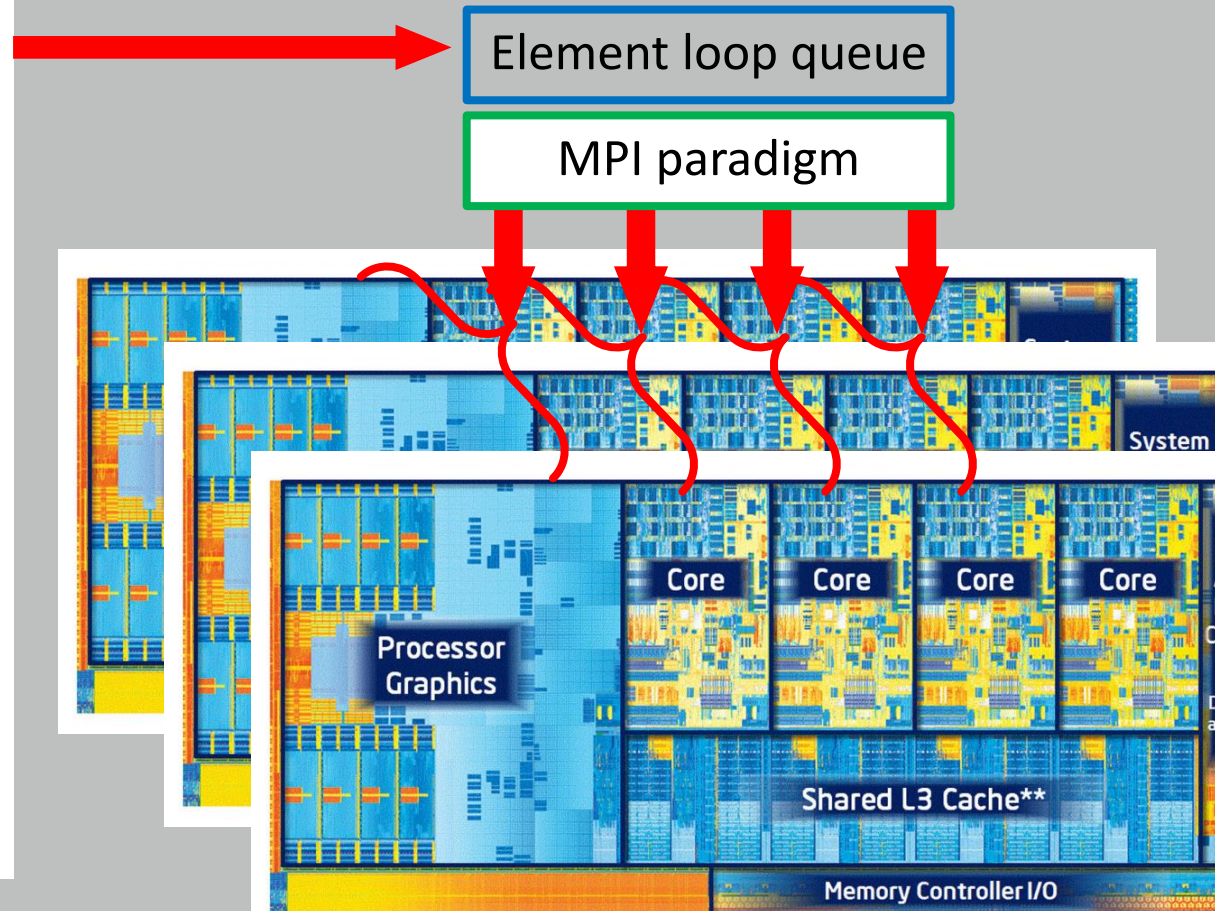
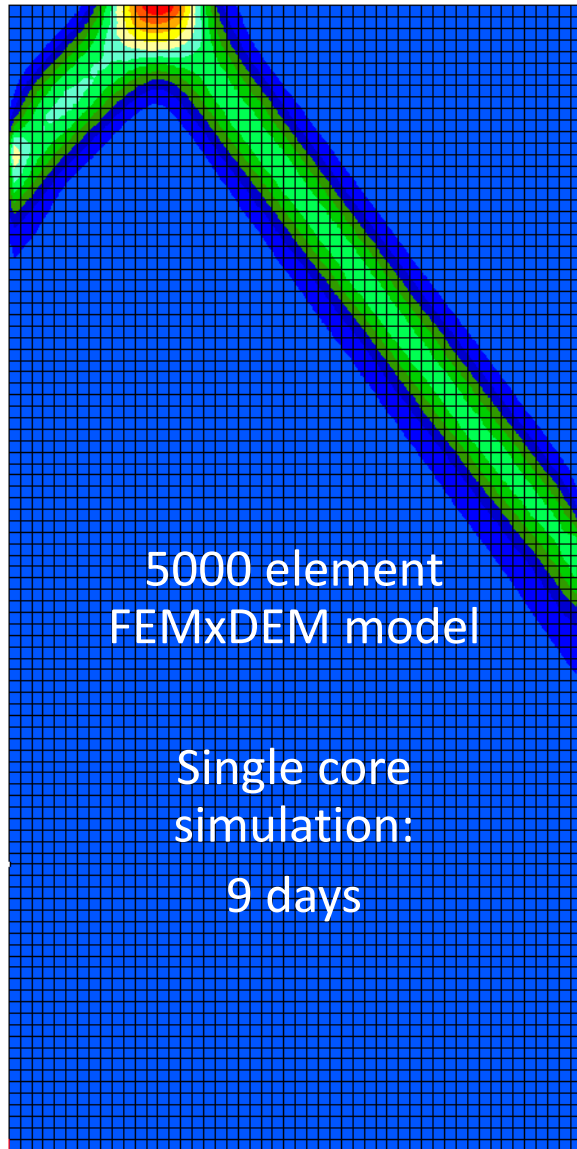


Parallelisation : Computations with rather refined meshes become possible



Element loop parallelisation : want more processors ? Use MPI

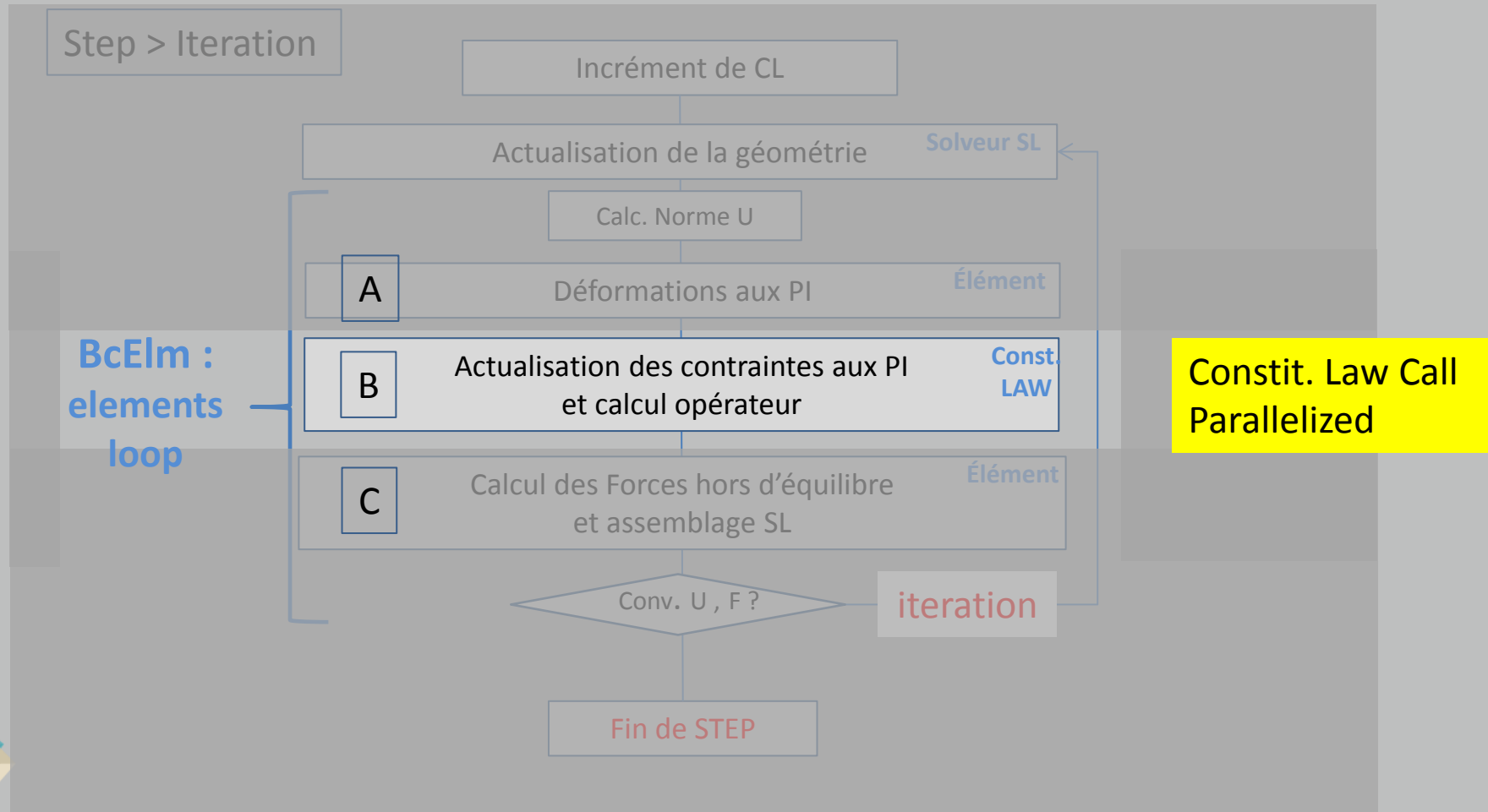
- Double scale → element intensive model.
- Parallelisation using an OpenMP paradigm.
- Results: speedup up to 9 using a 12-core machine.
- Other approaches: massive parallelization (MPI) : distributed memory model



MPI = Message passing Interface

Element loop workflow

Make a choice : Concentrate on the lower level « atomic » task



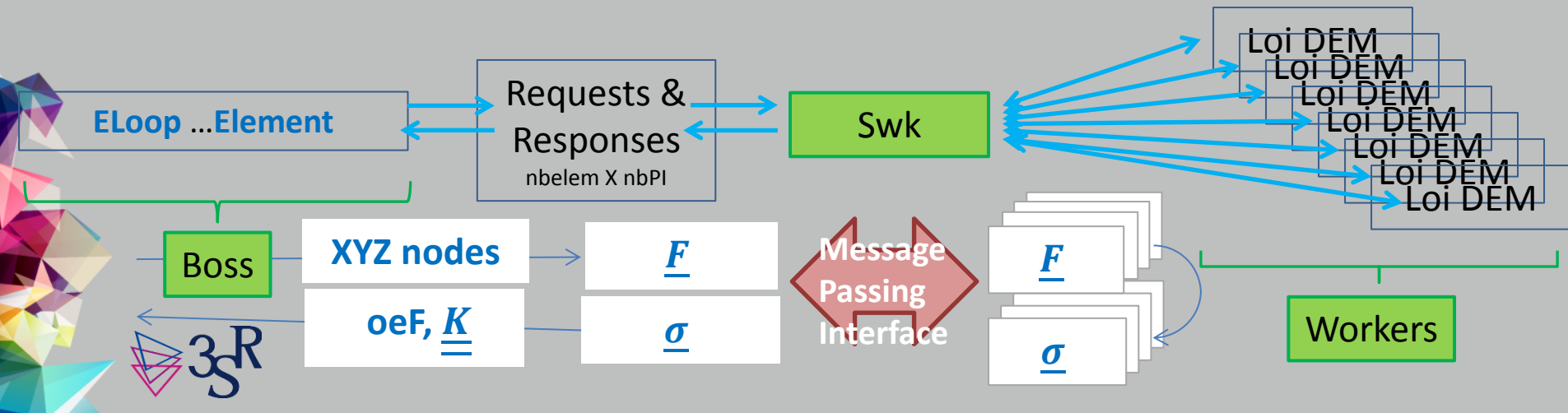
Element loop organisation



- Sequential :



- Parallel MPI with 3 levels: boss, supervisor, wk



Performances

example : Biaxial Test 128 elements with 2nd gradient

▶ **Sequential REV 400** : step 15 reached in 174 minutes

▶ **Parallel MPI REV 400**

Nb wks	time	acceler. factor	scalability
1	172'	NA	NA
10	21'	8,2	82%
50	8'	21	43%

▶ **Sequential REV 1600** : step 15 reached in 3401 minutes

▶ **Parallel MPI REV 1600** : 77 minutes

Nb wks	time	acceler. factor	scalability
48	77'	37	75%

scalability increases with increasing REV size, (for a given number of workers)

- Scalability improves with relative « heaviness » of the micro problem
 - Still some acceleration possible with more refined intermediate data recording strategies (experimental, spring 2016, quasi 100% efficiency but increased implementation complexity)
- ... and a priori no limitation on the number of computers & nodes



Conclusions & Perspectives

CONCLUSIONS (final)

- We have presented an **integrated Two-scale numerical approach** for granular materials: combining FEM (at macro scale) and DEM (at micro scale).
 - Illustration by two examples of BVP :
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PERSPECTIVES

- 3D approach (OK spring 2016, still experimental)
- hydromechanical coupling (idem, ongoing work)



references

Grenoble –Alpes 3SR-Lab team

- ▶ [1] Nitka M., Combe G., Dascalu C., Desrues J. Two-scale modeling of granular materials: a DEM-FEM approach, *Granular Matter* vol.13 No 3, pp. 277-281, (2011)
- ▶ [2] Nguyen T.K., Combe G., Caillerie D., Desrues J. FEM x DEM modelling of cohesive granular materials: numerical homogenisation and multi-scale simulation, *Acta Geophysica* vol.62 No 5, pp. 1109-1126, (2014)

... more refs to appear shortly

Hong-Kong Univ. team

- ▶ [3] Guo Ning and Zhao Jidong. A coupled FEM/DEM approach for hierarchical multiscale modelling of granular media. *International Journal for Numerical Methods in Engineering* 99.11, 789-818 (2014)

... more refs recently appeared

