POLITECNICO
MILANO 1863

Constitutive modelling of the solid-to-fluid transition in granular matters

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Introduction: very fast landslides

Localized failure

Oppstadhornet mountain, Norway

Diffuse failure: mudflows

Nova Friburgo, Brasil (2009)

Rock avalanche

Val Pola, Sondrio (1987)

Bracigliano-Sarno-Siano-Quindici (1998)

Introduction: from slide to flow

Initiation | **Propagation**

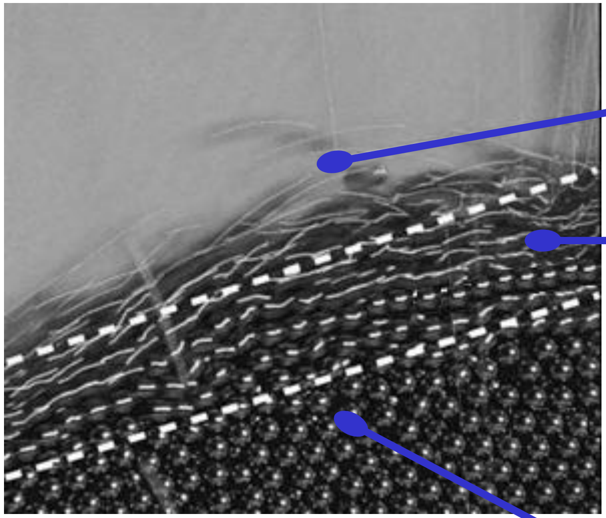
Solid-like material | *Fluid*-like material

Soil mechanics | **Fluid dynamics**

Phase transition

Dry debris flow - Las Colinas, El Salvador (2001)

Introduction: dry granular flows



(Jeager at al. 1996,
Campbell 2006,
Forterre & Pouliquen, 2008)

Granular gas (collisional)

inelastic collisions
grains bounce in all directions
creating a dilute chaotic medium

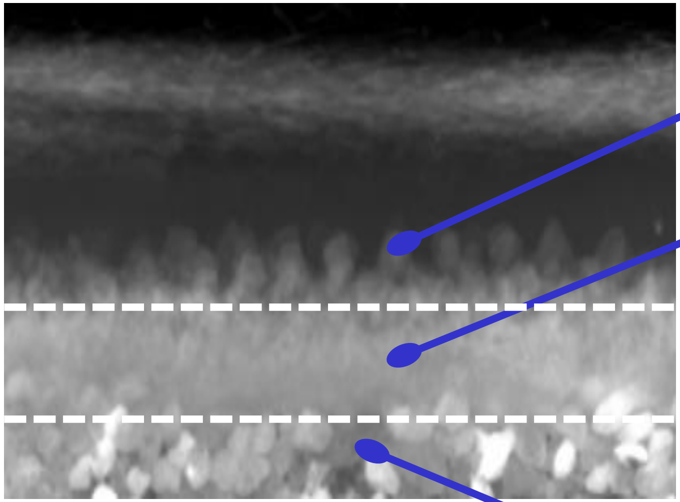
Granular fluid (transition)

inelastic collisions+force chains
both interaction mechanisms cohexist

Granular solid (quasi-static)

force chains
extremely slow deformations
high concentrations

Introduction: saturated granular flows



(Frey 2008)

Turbolent mixture (diffuse collisional)

inelastic collisions + dragging

Particles are dragged by the turbolent fluid and bounce in all directions

Dense mixture (transition)

inelastic collisions+force chains+fluid

turbolence is suppressed by the presence of particles. Relative motion fluid particles.

Saturated solid (quasi-static)

force chains+fluid

extremely slow deformations
high concentrations

State variables

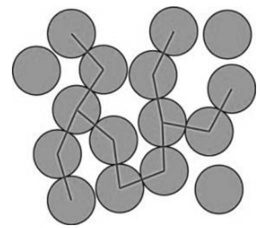
$$e = \frac{V_v}{V_s}$$

Void ratio

+

F

Fabric tensor



Force chains

$$\mathbf{F} = \frac{15}{2}\left(\boldsymbol{\varphi} - \frac{1}{3}\boldsymbol{\delta}\right)$$: Fabric tensor

$$\phi_{IJ} = \frac{1}{C}\sum_{c \in C} n_i^c n_j^c$$

$\mathbf{n}^c$: unit normal contact vector

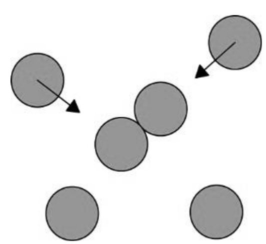
C : total number of contacts

Oda, Satake (1982),
Zhao and Guo (2013)

+

T

Granular temperature



Collisions

$$T = \frac{\left\langle\left\langle \left| \mathbf{u} - \left\langle\left\langle \mathbf{u} \right\rangle\right\rangle \right|^2 \right\rangle\right\rangle}{3}$$

u single particle velocity

$\langle\langle \bullet \rangle\rangle$ average using the single particle velocity distribution function

Campbell (1990), Goldhirsch (2003)

Introduction: numerical approaches

Discontinuum mechanisc

INCEPTION
Solid-like material

Discrete approaches

DEM

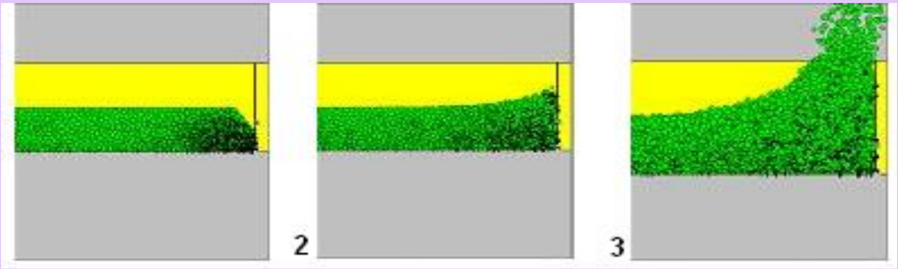
Utili (2006)

FROM MICRO TO MACRO

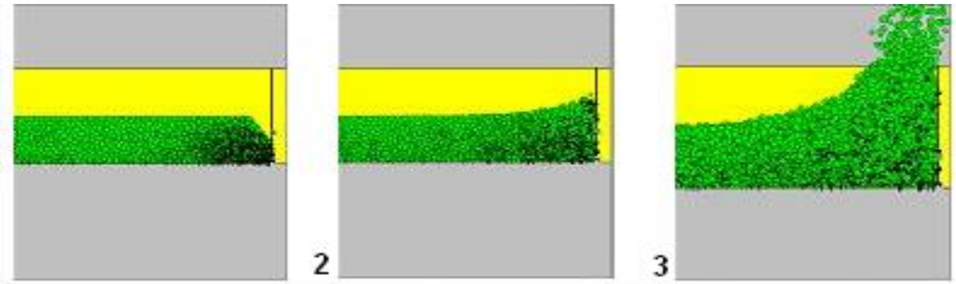
PROPAGATION
Fluid-like material

Discrete approaches DEM

Calvetti et al. (2000, 2014) Salciarini at al. (2010)



Introduction: the impacting force



The double nature of dry granular media

$$MIF = S_P + \Delta_{MIF}$$

$$F_r = \frac{u_0}{\sqrt{gh}} \qquad F_{rM} = \frac{u_M}{\sqrt{gh}}$$

HS contribution ⇐ $S_p = \frac{1}{2}\gamma b h^2 k_P$

HD contribution ⇑

$$\Delta_{MIF} = f_s \Delta_{MIF*} \qquad \Delta_{MIF*} = a_1 F_{rM} F_r + a_2 F_r^{\ 2}$$

$$f_s = \frac{1}{2}\gamma_s h^2 b$$

⇓ **Elastic body contribution**

Continuum mechanics

INCEPTION
Solid-like material

Geomechanical approaches

LIMIT EQUILIBRIUM Utili (2005)

FEM Cascini (2003, 2005, 2008)

Fluid dynamics approaches

PFEM Cremonesi et al. (2011)

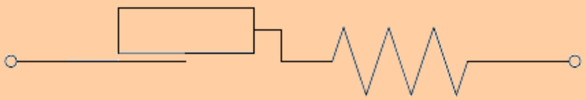
SPH Pastor (2009),Cascini (2014)

MPM Andersen (2010)

FEMLIP Prime et al. (2014)

Solid-like material

Time-independent incremental constitutive relationships
(Elasto-plasticity)

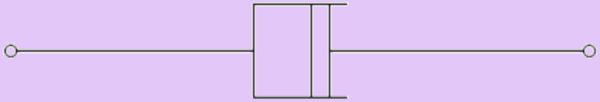


$$\dot{\boldsymbol{\sigma}} = \mathbf{D}\dot{\boldsymbol{\varepsilon}}$$

$$\dot{\boldsymbol{\varepsilon}} = \mathbf{C}\dot{\boldsymbol{\sigma}}$$

Fluid-like material

Time-dependent constitutive relationships
(Newtonian, non-Newtonian fluids)



$$\boldsymbol{\sigma} = \boldsymbol{\Phi}^{v}\dot{\boldsymbol{\varepsilon}}$$

$$\dot{\boldsymbol{\varepsilon}} = \left(\boldsymbol{\Phi}^{v}\right)^{-1}\boldsymbol{\sigma}$$

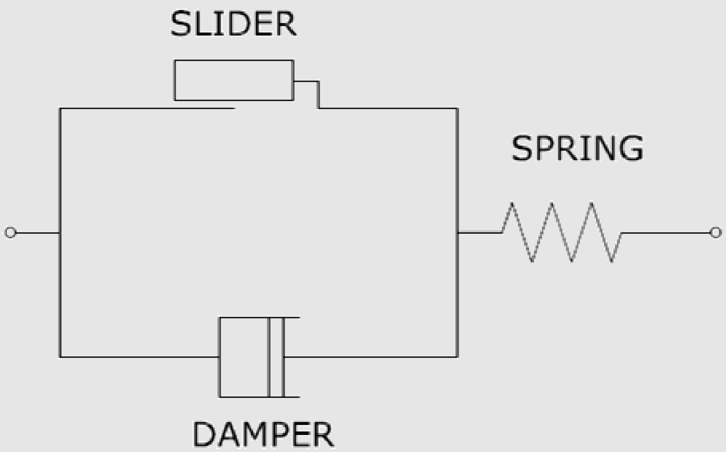
In both cases the material is usually assumed to be uncompressible and characterized by a constant concentration

but e is not constant !!!

Transition: models in series

Elasto-visco-plasticity

Perzyna (1966)



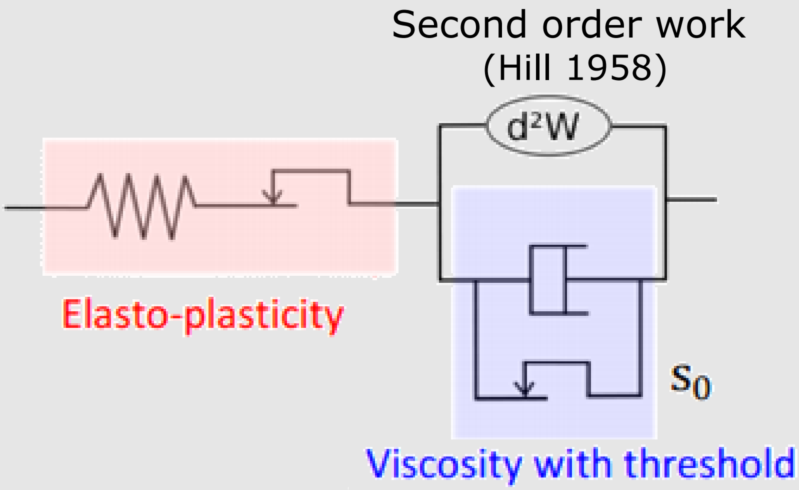
$$\dot{\boldsymbol{\varepsilon}} = \mathbf{C}^e \dot{\boldsymbol{\sigma}} + \dot{\boldsymbol{\varepsilon}}^{vp}$$

$$= \mathbf{C}^e \dot{\boldsymbol{\sigma}} + \Phi(\boldsymbol{\sigma}')\,\mathbf{m}$$

$$\dot{\boldsymbol{\varepsilon}} = \mathbf{C}^e \dot{\boldsymbol{\sigma}} + (\boldsymbol{\Phi}^v)^{-1} \boldsymbol{\sigma}$$ → visco-elasticity

Elasto-visco-plasticity

Prime, Dufour and Darve (2014)



Presentation outline

- **Theoretical approachl:**

•THE PARALLEL SYSTEM

•STEADY CONDITIONS

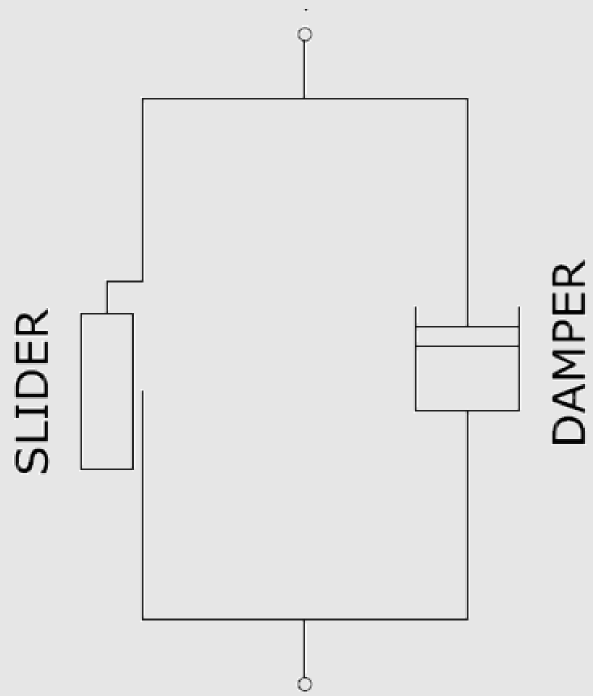
•THE CONSTITUTIVE RELATIONSHIP

- **Numerical results:**

Conclusions

Transition: models in parallel

Johnson & Jackson (1987, 1990)
Savage (1998)
Louge (XXX)
Lee & Huang (2010)
Vescovi et al.(2013)



Theoretical approach: the parallel scheme

Static	Kinematic	State variables
$\boldsymbol{\sigma}$	$\boldsymbol{\varepsilon}$	e T

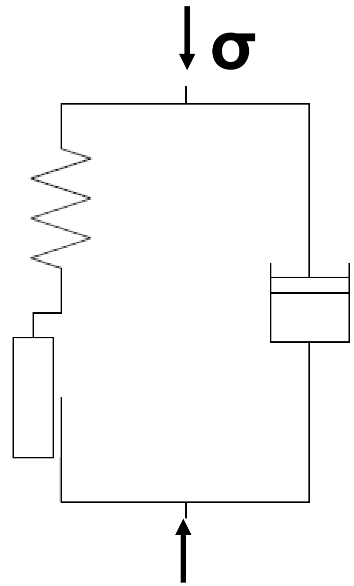
Energy balance for the REV

Produced energy		Kinetic fluctuating transported energy		Variation in elastically stored energy		Dissipated energy
$\dot{W}$	$=$	$\dot{E}_{k,f}$	$+$	$\dot{E}_{el}$	$+$	Γ

$$\boldsymbol{\sigma} = \boldsymbol{\sigma}_q + \boldsymbol{\sigma}_c$$

$$\dot{\boldsymbol{\varepsilon}} = \dot{\boldsymbol{\varepsilon}}^e + \dot{\boldsymbol{\varepsilon}}^p$$

Strain additivity
(hp of small transformation)



$$\Gamma = \Gamma_q + \Gamma_c$$

Theoretical approach: the parallel scheme

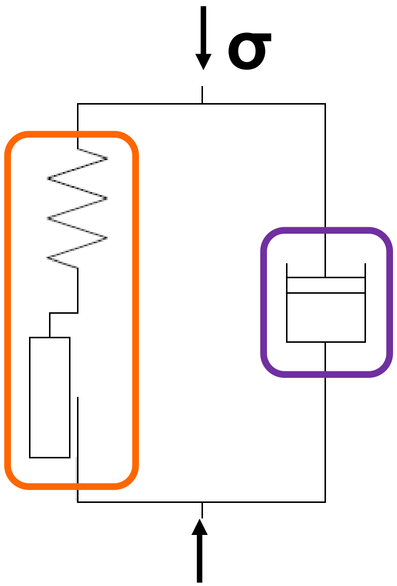
Static	Kinematic	State variables
$\boldsymbol{\sigma}$	$\boldsymbol{\varepsilon}$	e T

Energy balance for the REV

Produced energy		Kinetic fluctuating transported energy		Variation in elastically stored energy		Dissipated energy
$\dot{W}$	$=$	$\dot{E}_{k,f}$	$+$	$\dot{E}_{el}$	$+$	Γ

Quasi-Static Contribution

Elasto-plastic theories based on the critical state concept



Collisional contribution

Kinetic theory for granular gases

Theoretical approach: balance of energy

Static

$\boldsymbol{\sigma}$

Kinematic

$\boldsymbol{\varepsilon}$

State variables

e

T

Energy balance for the REV

Produced energy		Kinetic fluctuating transported energy		Variation in elastically stored energy		Dissipated energy
$\dot{W}$	$=$	$\dot{E}_{k,f}$	$+$	$\dot{E}_{el}$	$+$	Γ

$$(\boldsymbol{\sigma}_q + \boldsymbol{\sigma}_c) : \dot{\boldsymbol{\varepsilon}} = \frac{3}{2}\rho_p \frac{1}{1+e}\dot{T} + \boldsymbol{\sigma}_q : \dot{\boldsymbol{\varepsilon}}^e + \boldsymbol{\sigma}_q : \dot{\boldsymbol{\varepsilon}}^p + \Gamma_c$$

Balance of the kinetic fluctuating energy (UNIFORM CONDITIONS)

$$\boldsymbol{\sigma}_c \dot{\boldsymbol{\varepsilon}} = \frac{3}{2}\rho_p \frac{1}{1+e}\dot{T} + \Gamma_c$$

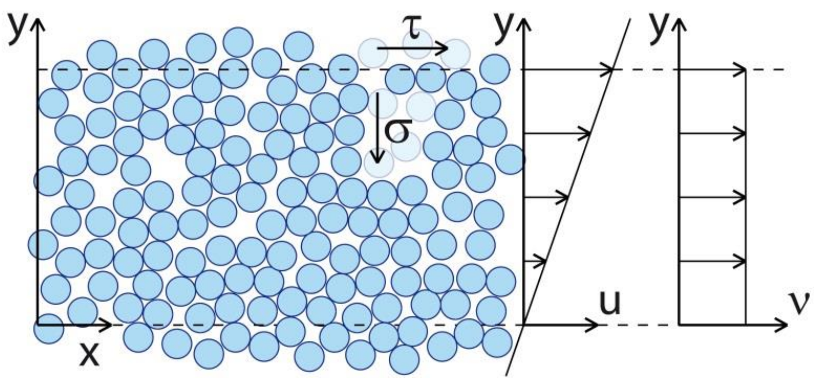
Research goals

Formulation of a constitutive model capable of capturing the response of **dry** granular flows from quasi-static to dynamic conditions and based on an ***extended critical state theory***

Critical state: a **steady state** for which $T \rightarrow 0 \;\; \Rightarrow \;\; \dot{\gamma} \rightarrow 0$

The case of simple-shear conditions

2



<u>Hypotesis</u>

- Identical soft spheres of diameter d and density ρ_p
- Simple shear conditions
- Dry granular materials

$$\underline{u} = (u, 0); \quad \partial/\partial x = 0$$

$$\boldsymbol{\sigma} = \begin{bmatrix} \sigma \\ \tau \end{bmatrix}$$

$$\boldsymbol{\varepsilon} = \begin{bmatrix} v \\ \gamma \end{bmatrix}$$

$$e$$

$$T$$

$$\boldsymbol{\sigma} = \begin{bmatrix} \sigma_q + \sigma_c \\ \tau_q + \tau_c \end{bmatrix}$$

Energy balance for the REV

Produced energy | Kinetic fluctuating transported energy | Variation in elastically stored energy

$$\dot{W} = \dot{E}_{k,f} + \dot{E}_{el} + \Gamma$$

Dissipated energy

$$\dot{W} = \Gamma$$

$$\tau\dot{\gamma} = \Gamma$$

Dissipated energy $\Gamma = \Gamma_q + \Gamma_c$

$$\tau_q\dot{\gamma} + \tau_c\dot{\gamma} = \Gamma_q + \Gamma_c$$

$$\boxed{\tau_c\dot{\gamma} = \Gamma_c}$$

Critical state locus (Simple shear DEM simulations: Chialvo et al 2012)

$$G_1 = \sigma_q - f_0 \frac{K_P}{d} = 0$$

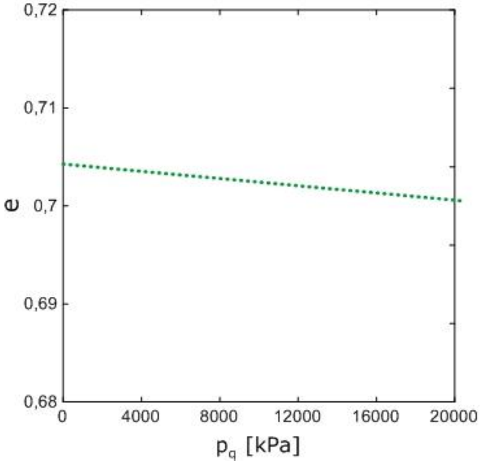
$$f = \tau_q - \sigma_q \tan\phi_{ss}' = 0$$

$$f_0(e) = \begin{cases} a\left(\dfrac{e_c - e}{(1+e)(1+e_c)}\right)^{\frac{3}{2}} \neq 0 & e < e_c \\ 0 & e \geq e_c \end{cases}$$

e_c: critical void ratio

a: material coefficient

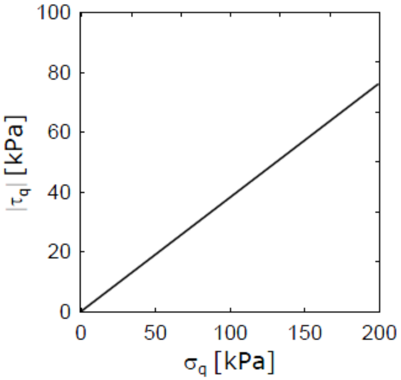
Sun & Sundaresan (2011)



$$\tau_q = \sigma_q \tan\phi_{ss}'$$

$$\sigma_q = \frac{K_P}{d} f_0(e)$$

p_q



ϕ_{ss}' = internal friction angle at the critical state

$K_p = \pi dE/8$ particle stiffness for linear elastic contacts

Modified kinetic theory for granular gases

$$\sigma_c = \rho_p f_1 f_4 T$$

$$\tau_c = \rho_p d f_2 f_4 T^{1/2} \dot{\gamma}$$

$$\Gamma_c = \rho_p \frac{f_3}{L} f_4 T^{3/2}$$

f_1, f_2, f_3 = functions of e and the coefficient of restitution (Garzò & Dufty 1999)

L = correlation lenght (Jenkins 2006, 2007) accounting for the correlated motion of particles

$$f_4 = \left[1 + 2\frac{d}{s}\left(\rho_p \frac{T}{E_p}\right)^{1/2}\right]^{-1}$$

influence of the particle stiffness on the collisions

s mean separation distance (Hwang & Hutter, 1995)

$$\sigma = \frac{K_P}{d} f_0 + \rho_P d^2 f_1 f_4 f_5 \dot{\gamma}^2$$

$$\tau = \frac{K_P}{d} f_0 \tan\phi_{ss}' + \rho_P d^2 f_2 f_4 f_5^{1/2} \dot{\gamma}^2$$

quasi-static collisional

Energy balance: $T = d^2 f_5 \dot{\gamma}^2$

$f_5 = f_5(e,L)$

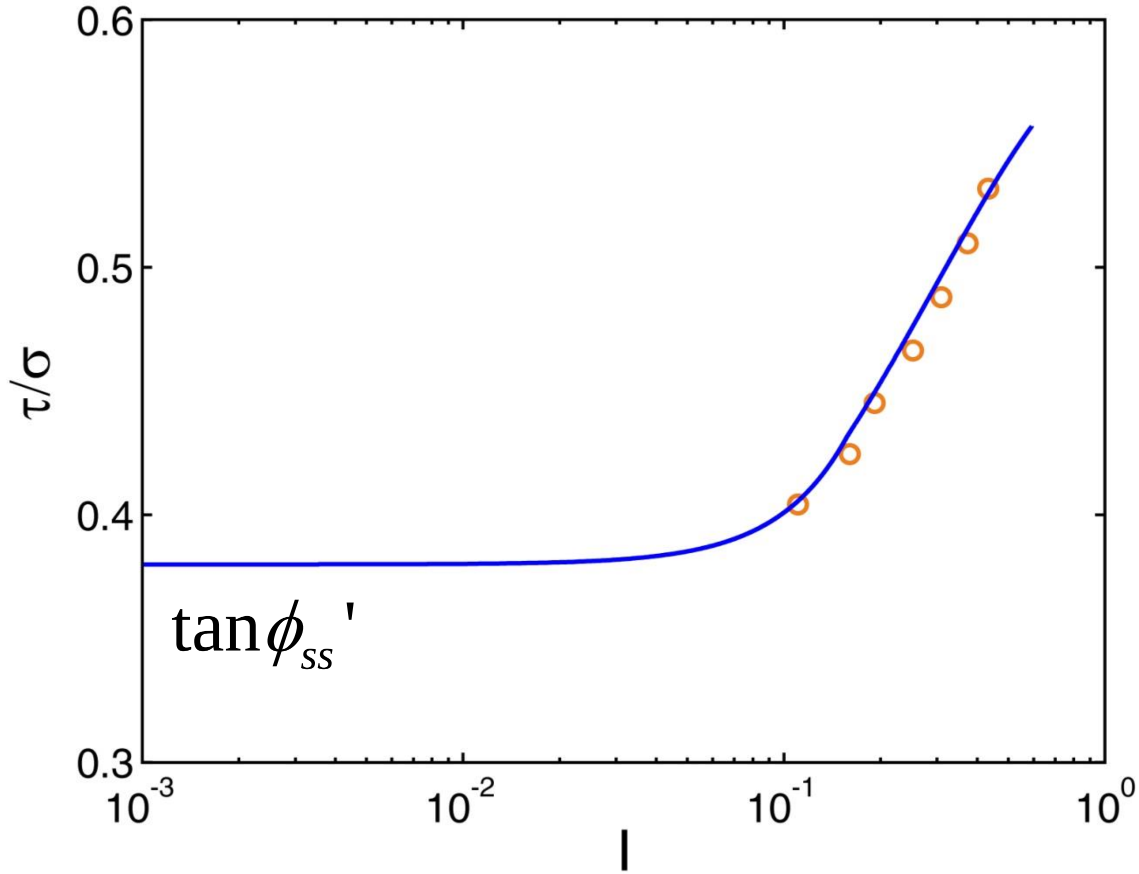
Critical state: particular **steady state** for which $T \to 0 \Rightarrow \dot{\gamma} \to 0$

Collisional regime: $e > e_c$

- D. Berzi, C. di Prisco, D. Vescovi. Constitutive relations for steady, dense granular flows. ***Physical Review E***, **84**, 031301 (2011).
- D. Vescovi, C. di Prisco, D. Berzi. From solid to granular gases: the steady state for granular materials. ***Int. J. Numer. Anal. Meth. Geomech.***, **37**, 2937 (2013).

Steady, simple shear flows: SOME RESULTS

Comparisons with the experiments on steady, fully developed flows of glass spheres on inclined planes performed by Pouliquen (1999)



$$I = d\,\dot{\gamma}\sqrt{\frac{\rho_p \nu}{\sigma}}$$

$$\nu = \frac{1}{1+e}$$

State variables at critical state: 3D conditions

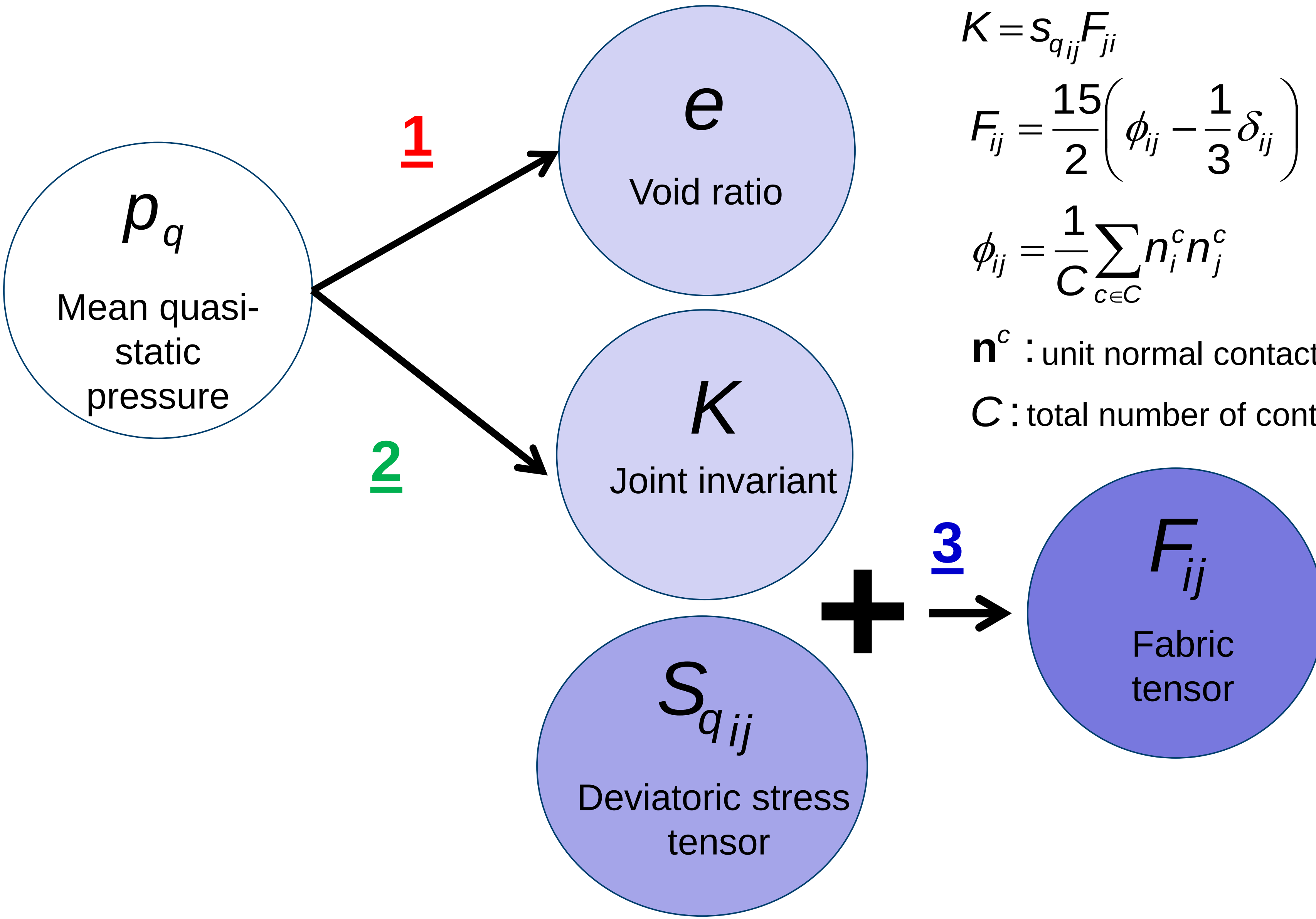
$$K = s_{q_{ij}} F_{ji}$$

$$F_{ij} = \frac{15}{2}\left(\phi_{ij} - \frac{1}{3}\delta_{ij}\right)$$

$$\phi_{ij} = \frac{1}{C}\sum_{c \in C} n_i^c n_j^c$$

$\mathbf{n}^c$: unit normal contact vector

C : total number of contacts



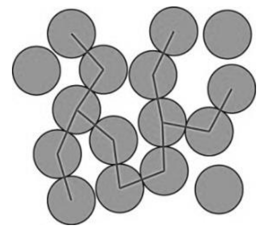
State variables

$$e = \frac{V_v}{V_s}$$

Void ratio

+

~~**F** Fabric tensor~~



Force chains

$$\mathbf{F} = \frac{15}{2}\left(\boldsymbol{\varphi} - \frac{1}{3}\boldsymbol{\delta}\right) \text{: Fabric tensor}$$

$$\phi_{IJ} = \frac{1}{C}\sum_{c \in C} n_i^c n_j^c$$

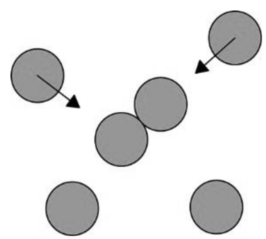
$\mathbf{n}^c$: unit normal contact vector

C : total number of contacts

Oda, Satake (1982),
Zhao and Guo (2013)

+

T Granular temperature



Collisions

$$T = \frac{\left\langle\left\langle \left|\mathbf{u} - \left\langle\left\langle \mathbf{u} \right\rangle\right\rangle\right|^2 \right\rangle\right\rangle}{3}$$

$\mathbf{u}$ single particle velocity

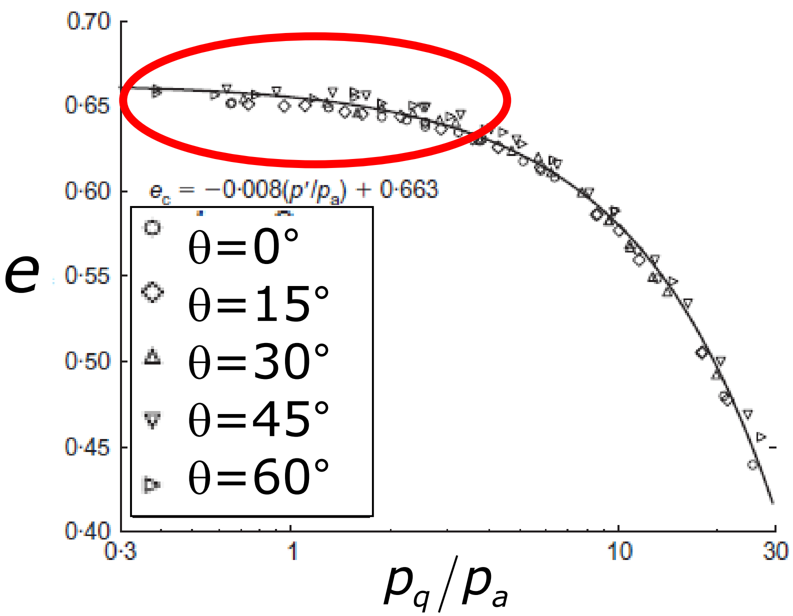
$\langle\langle \bullet \rangle\rangle$ average using the single particle velocity distribution function

Campbell (1990), Goldhirsch (2003)

Uniqueness of the critical state locus

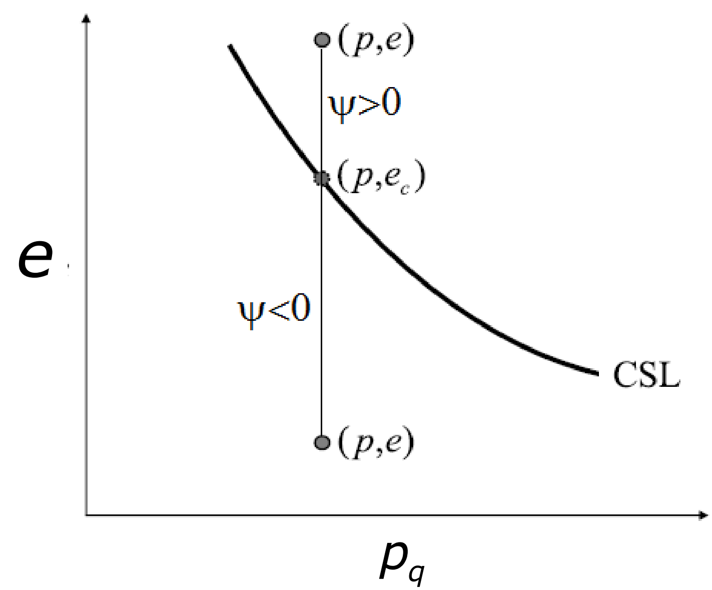
The critical state locus is not influenced by the initial state.
Although soil is highly anisotropic at critical state, the e-p_q critical state line is independent on the loading path.

DEM triaxial tests
Zhao and Guo (2013)



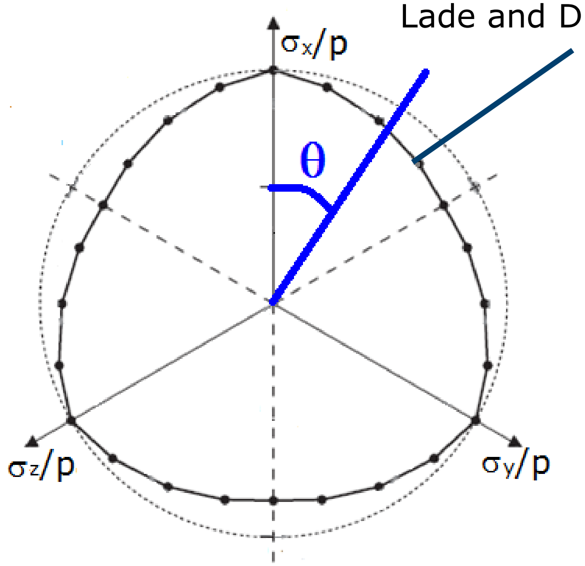
$$p = \frac{\mathrm{Tr}(\boldsymbol{\sigma})}{3}$$

Thermodynamic proof
Li and Dafalias (2012)

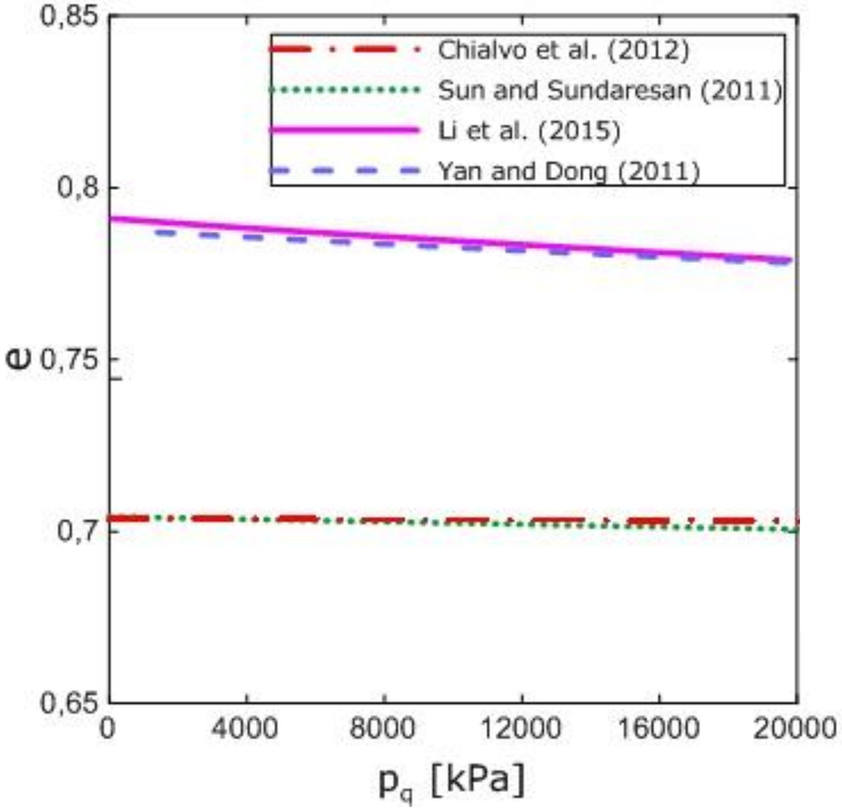


Shape of the critical state locus

DEM triaxial tests



Thornton (2000)
Ng (2004)
Thornton and Zhang (2010)
Zhao and Guo (2013)
Barreto and O'Sullivan (2013)
Huang et al. (2014)



Theoretical approach: quasi-static contribution

Critical state locus definition

$$G(p_q, q_q, e) = \begin{cases} G_1 = \dfrac{p_q}{K_L} - f_0(e) = 0 \\[2ex] G_2 = \dfrac{q_q}{K_L} - \dfrac{p_q}{K_L} M_c(\beta, \tan\phi_{ss})\,\rho_q(\beta, \vartheta) = 0 \end{cases}$$

$q = \sqrt{\frac{3}{2}\mathbf{s}:\mathbf{s}}$: equivalent deviatoric stress

$\mathbf{s}$: deviatoric stress tensor

$K_L = \dfrac{4E_p}{6(1-\nu_p^2)}$: particle stiffness (Linear contact model)

E_p, υ_p : particle Young modulus and Poisson ratio

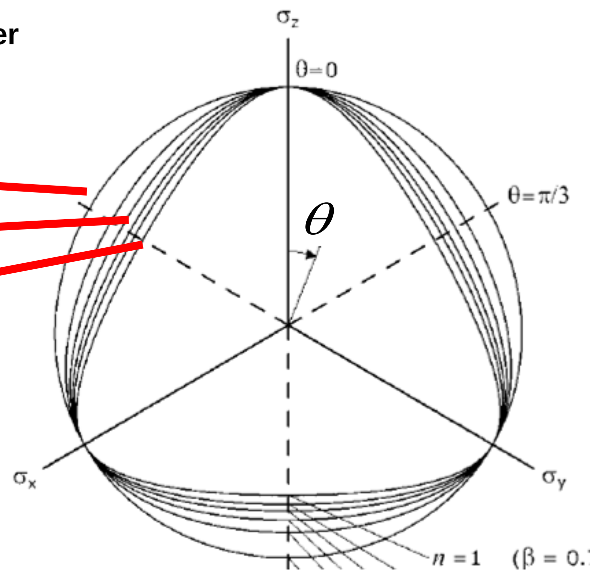
β = constitutive parameter

β Shape parameter

Drucker pragher $\beta \to \infty$

Lade Dunkan $\beta = 0{,}779$

Matsuoka Nakai $\beta = 0{,}714$

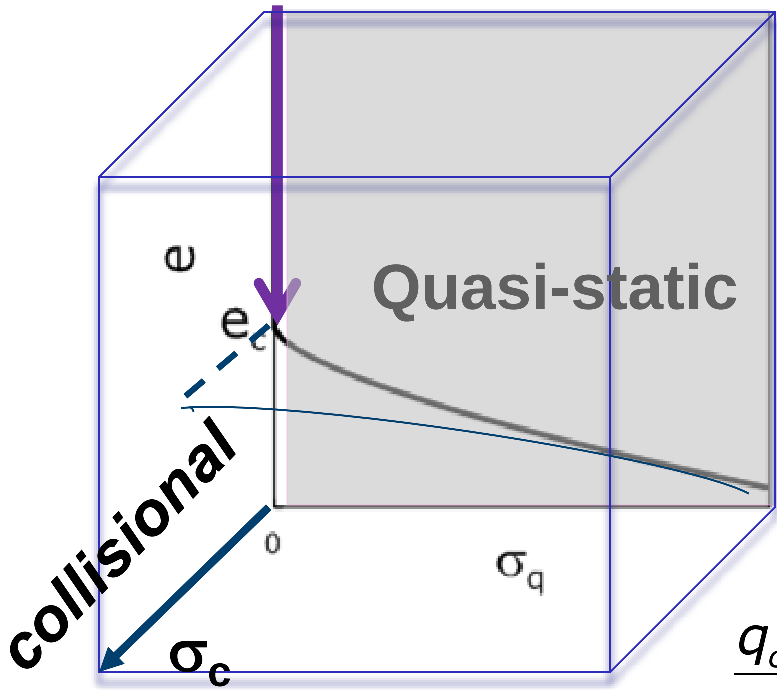


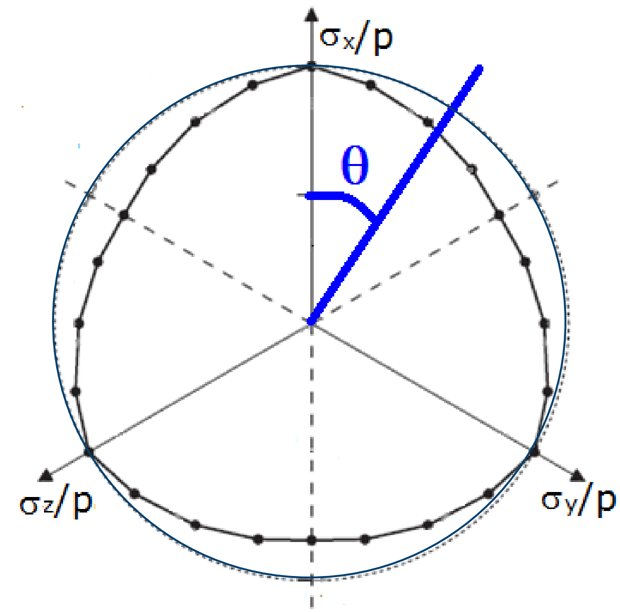
Mortara (2011)

$M_c = M_c(e_c, \varepsilon_r)$: stress ratio in triaxial compression

ε_r: effective coefficient of restitution, Jenkins & Zhang (2002)

Theoretical approach: steady conditions





$$\left.\frac{q_c}{p_c}\right|_{e=e_c} = \frac{q_q}{p_q}$$

$$\left.\frac{q_c}{p_c} = f_1\left(\frac{1}{f_2 f_3}\frac{L}{d}\right)^{1/2}\right|_{e=e_c} = \frac{q_q}{p_q} = M_c \rho_q(\theta)$$

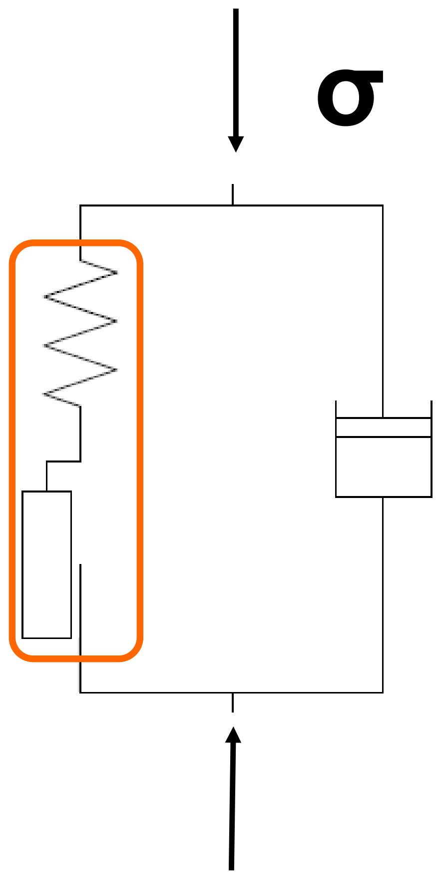
New expression for L

$$L = \max\left\{d, \frac{d^2 f_e(e)\rho_c(\theta)\dot{\varepsilon}_d}{\sqrt{T}}\right\}$$

Quasi-Static contribution

Elasto-plastic theories based on the critical state concept

Been & Jefferies (1985)
Wood (1990, 1994)
Manzari & Dafalias (1997)
Li et al. (1999)
Li & Dafalias (2000)
Li (2002)
Dafalias & Manzari (2004)



Theoretical approach: quasi-static contribution

incremental formulation $\dot{\boldsymbol{\sigma}}_q = \mathbf{D}^q \dot{\boldsymbol{\varepsilon}}$

Elastic response

$\mathbf{D}^e(\boldsymbol{\sigma}_q)$

$f < 0$

$f = 0 \cap \dot{f} < 0$

Standard perfect elasto-plastic response

$\mathbf{D}^{ep}(\boldsymbol{\sigma}_q, e)$

$f = \dot{f} = 0 \cap G_1 \neq 0$

Critical response

$\mathbf{D}^{cr}(\boldsymbol{\sigma}_q, e)$

$f = \dot{f} = G_1 = \dot{G}_1 = 0$

$\cap$

$e < e_c$

Collisional

$\mathbf{0}$

$f = \dot{f} = G_1 = 0$

$\cap$

$e \geq e_c$

f: yield locus

Theoretical approach: quasi-static contribution

Elastic response $\dot{\boldsymbol{\sigma}}_q = \mathbf{D}^q \dot{\boldsymbol{\varepsilon}}$

$$f < 0$$

$$f = 0 \cap \dot{f} < 0$$

$$\mathbf{D}^e(\boldsymbol{\sigma}_q) = \left(\frac{\partial^2 E_{el}}{\partial \boldsymbol{\sigma}_q \otimes \partial \boldsymbol{\sigma}_q} \right)^{-1}$$

Hyperelastic stiffness matrix

Houlsby (1985), Loret (1985),
Lade et al. (1987), Borja et al. (1997),
Sulem et al. (1999), Einav et al. (2004),
Houlsby et al. (2005)

$$E_{el} = E_{el}(\boldsymbol{\sigma}_q) = \frac{p_0^{2-n}}{K_L^{1-n}\bar{k}(1-n)(2-n)} - \frac{\sigma_{q,kk}}{\bar{k}(1-n)}$$

$$p_0^2 = \frac{\sigma_{q,mm}\sigma_{q,mm}}{9} + \frac{\bar{k}(1-n)s_{q,mn}s_{q,mn}}{\bar{g}}$$

Gibbs energy

Houlsby et al. (2005)

$\bar{k}$: material parameters associated with the volumetric stiffness

$\bar{g}$: material parameters associated with the shear stiffness

n : non-dimensional constant

Theoretical approach: quasi-static contribution

Standard perfect elasto-plastic response

$$\dot{\boldsymbol{\sigma}}_q = \mathbf{D}^q \dot{\boldsymbol{\varepsilon}} \qquad f = \dot{f} = 0 \cap G_1 \neq 0$$

$$\mathbf{D}^{ep}(\boldsymbol{\sigma}_q, e) = \mathbf{D}^e - \frac{\mathbf{D}^e : \dfrac{\partial g}{\partial \boldsymbol{\sigma}_q} \otimes \dfrac{\partial f}{\partial \boldsymbol{\sigma}_q} : \mathbf{D}^e}{\dfrac{\partial f}{\partial \boldsymbol{\sigma}_q} : \mathbf{D}^e : \dfrac{\partial g}{\partial \boldsymbol{\sigma}_q}}$$

$$f = f(p_q, q_q) = KG_2(p_q, q_q)$$

Yield locus

$$g(p_q, q_q, e) = q_q + \delta p_q \left(\frac{p_q}{2K_L} - f_0 \right)$$

Plastic potential

$$D(p_q, e) = \frac{\dot{\varepsilon}_v^p}{\dot{\varepsilon}_d^p} = \frac{\partial g / p_q}{\partial g / q_q} = \delta G_1(p_q, e)$$

State-dependent dilatancy

Been & Jefferies (1985)
Li & Dafalias (2000)

$\varepsilon_v = \mathrm{tr}(\boldsymbol{\varepsilon})$: volumetric strain

$\varepsilon_d = \sqrt{\frac{2}{3} \mathbf{e} : \mathbf{e}}$: equivalent deviatoric strain

$\mathbf{e}$: strain deviator tensor

Critical response

Redaelli et al. (2015)

$$\dot{\boldsymbol{\sigma}}_q = \mathbf{D}^q \dot{\boldsymbol{\varepsilon}} \qquad f = \dot{f} = G_1 = \dot{G}_1 = 0 \cap e < e_c$$

$$\dot{f} = \frac{\partial f}{\partial \mathbf{s}_q} : \dot{\mathbf{s}}_q + \frac{\partial f}{\partial p_q} \dot{p}_q = 0$$

Consistency rule

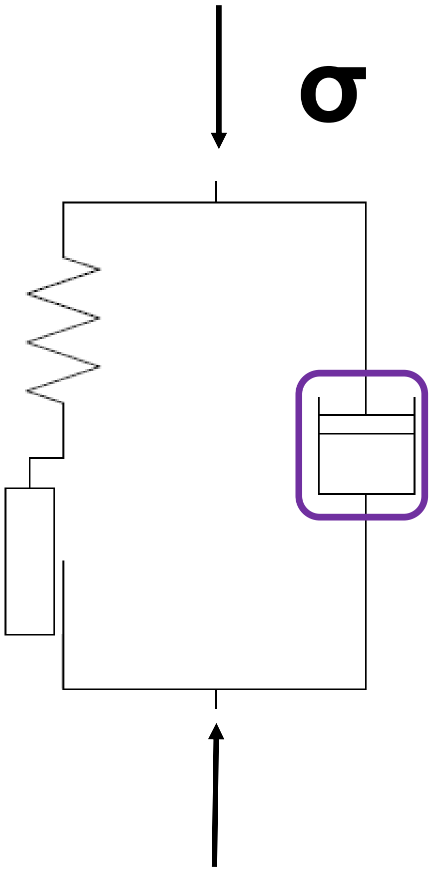
$$\dot{G}_1 = \frac{\partial G_1}{\partial p_q} \dot{p}_q + \frac{\partial G_1}{\partial e} \dot{e} = 0$$

Extended consistency rule

$$\left. \begin{aligned} \dot{\varepsilon}_v^p &= \dot{\varepsilon}_v - \dot{\varepsilon}_v^e \\ \dot{e}_{ij}^p &= \dot{\lambda} \frac{\partial g}{\partial s_{ij}} \end{aligned} \right\}$$

Plastic strains

Theoretical approach: collisional contribution



Collisional contribution

Kinetic theories for granular gases

Garzo & Dufty (1999), Jenkins & Zhang (2002), Jenkins (2006, 2007), Berzi (2014), Vescovi et al. (2014)

Theoretical approach: collisional contribution

$$\boldsymbol{\sigma}_c = \boldsymbol{\Phi}^v \dot{\boldsymbol{\varepsilon}} + \mathbf{h}^v$$

$$\Phi^v_{ijkl}(e,T) = \begin{cases} \rho_p d\left(\frac{4}{3} f_2 + f_3\right) f_4 T^{1/2} & \text{if } i=j=k=l \\ \rho_p d\left(\frac{-2}{3} f_2 + f_3\right) f_4 T^{1/2} & \text{if } i=j \cap k=l \cap i \neq k \\ \rho_p d^2 f_2 f_4 T^{1/2} & \text{if } i=k \cap j=l \cap i \neq j \\ \rho_p d^2 f_2 f_4 T^{1/2} & \text{if } i=l \cap j=k \cap i \neq j \\ 0 & \text{otherwise} \end{cases}$$

$$h^v_{mn}(e,T) = \begin{cases} \rho_p f_1 f_4 T & \text{if } m=n \\ 0 & \text{otherwise} \end{cases}$$

$f_i(e,\varepsilon_r,e_m)$, $f_r(e,T,E_p,\nu_p)$ Garzò and Dufty (1999), Vescovi et al. (2014)

ε_r: effective coefficient of restitution, Jenkins and Zhang (2002)

μ_P = interparticle friction coefficient

Theoretical approach: constitutive relationship

$$\boldsymbol{\sigma}^{(i)} = \boldsymbol{\sigma}_q^{(i)} + \boldsymbol{\sigma}_c^{(i)}$$

$$\boldsymbol{\sigma}^{(i)} = \boxed{\mathbf{D}^q : \dot{\boldsymbol{\varepsilon}}^{(i)} \Delta t + \boldsymbol{\sigma}_q^{(i-1)}} + \boxed{\boldsymbol{\Phi}^v : \dot{\boldsymbol{\varepsilon}}^{(i)} + \mathbf{h}}$$

$$\boldsymbol{\sigma} = \mathbf{D}^{vep} : \dot{\boldsymbol{\varepsilon}} + \mathbf{c}$$

$$\mathbf{D}^{vep}(\boldsymbol{\sigma}_q, e, T) = \mathbf{D}^q(\boldsymbol{\sigma}_q, \mathbf{r}, e)\Delta t + \boldsymbol{\Phi}^{\text{v}}(e, T)$$

$$\mathbf{c}(\boldsymbol{\sigma}_q^{i-1}, e, T) = \boldsymbol{\sigma}_q^{i-1} + \mathbf{h}^{v}(e, T)$$

Theoretical approach: state variables evolution

Evolution of void ratio

$$\dot{e} = -\dot{\varepsilon}_v (1 + e)$$

Mass balance under uniform conditions

Evolution of granular temperature

$$\dot{T} = \frac{2}{3}\frac{(1+e)}{\rho_p}\left(\boldsymbol{\sigma}_c : \dot{\boldsymbol{\varepsilon}} + \Gamma_c\right)$$

Balance of kinetic fluctuating energy under uniform conditions

$$\Gamma_c = \rho_p \frac{f_3}{L} f_4 T^{3/2}$$

Energy dissipated by collisions

For a general boundary value problem

Balance of the kinetic fluctuating energy

$$\boldsymbol{\sigma}_c : \dot{\boldsymbol{\varepsilon}} = \frac{3}{2}\rho_p \frac{1}{1+e}\dot{T} + \nabla \cdot \mathbf{q} + \Gamma_c$$

Divergence of the energy flux

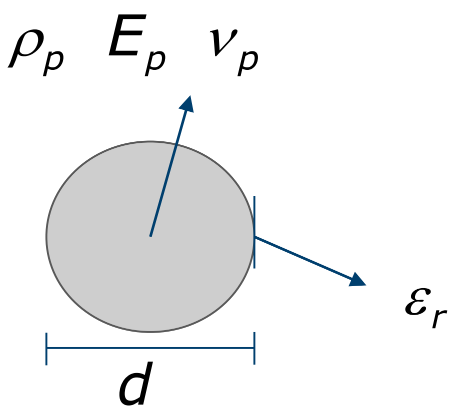
$$\mathbf{q} = -\kappa \nabla T - \mu \nabla \rho$$

ρ: material density
κ: thermal conductivity
μ: coefficient of density gradient

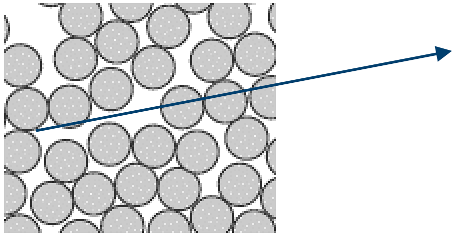
Non-local effect at the origin of the spatial propagation of the phase transition phenomenon within large masses of soil

Theoretical approach: parameters

Micro-mechanical paremeters
(referred to the single particle)



Macro-mechanical paremeters
(referred to the material)



$\bar{k}$ $\bar{g}$ n — elastic energy

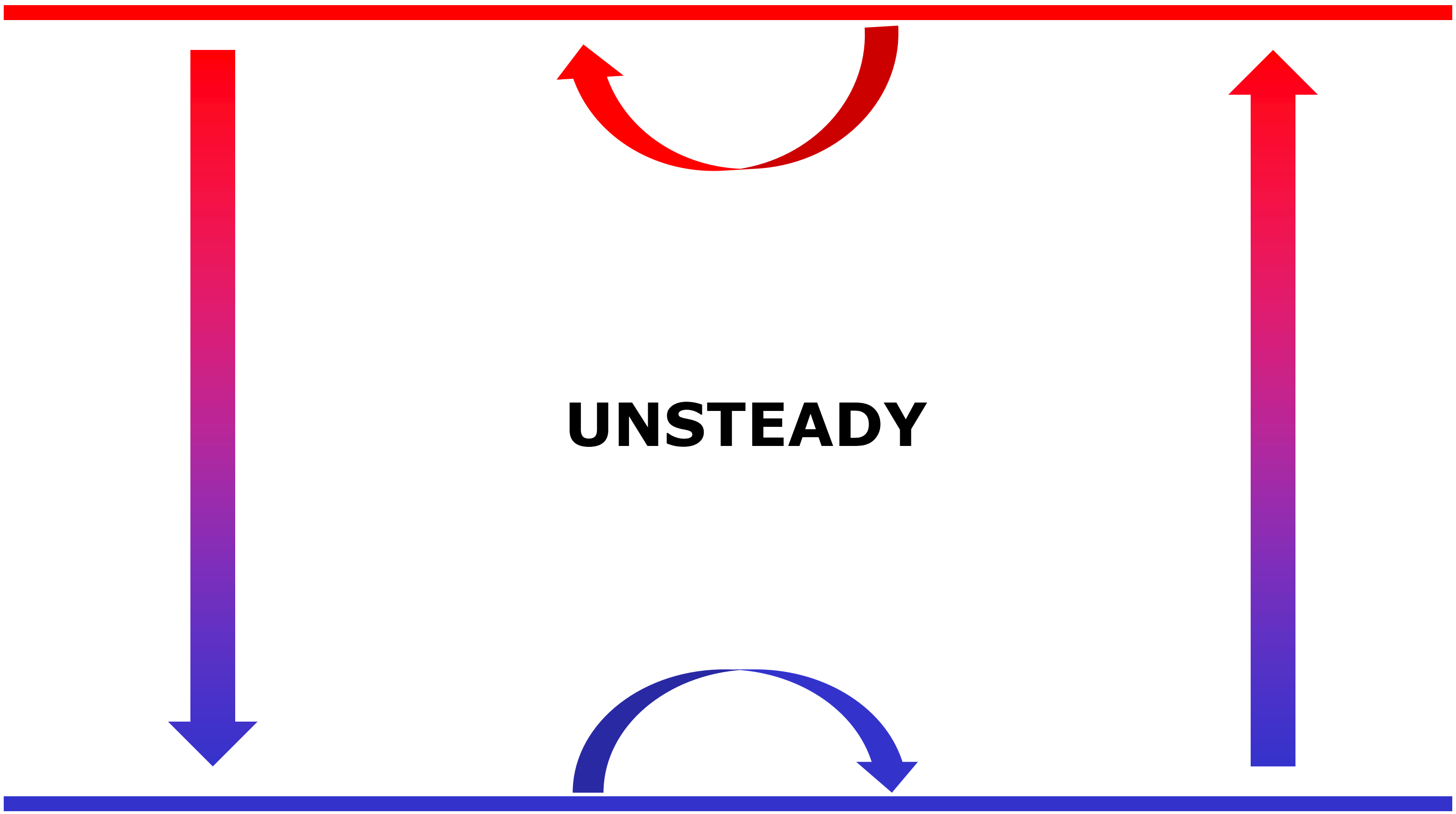
a e_c — critical state locus

δ — plastic potential

e_m — kinetic functions

STEADY

$$f = G_1 = \dot{T} = \dot{\varepsilon}_v = \ddot{\varepsilon}_d = 0$$



$$T_0 = \dot{\boldsymbol{\varepsilon}}_0 = 0$$

STATIC

Theoretical approach: unsteady conditions

STEADY

$$f = G_1 = \dot{T} = \dot{\varepsilon}_v = \ddot{\varepsilon}_d = 0$$

Collisional

$$f = \dot{f} = G_1 = 0$$
$$e \geq e_c$$
$$\boldsymbol{\sigma} = \boldsymbol{\Phi}^v : \dot{\boldsymbol{\varepsilon}} + \mathbf{h}$$

Critical

$$f = \dot{f} = G_1 = \dot{G}_1 = 0$$
$$e < e_c$$
$$\boldsymbol{\sigma} = (\mathbf{D}^{cr}\Delta t + \boldsymbol{\Phi}^v) : \dot{\boldsymbol{\varepsilon}} + \mathbf{c}$$

Visco-elastic

$$f < 0 \cup f = 0 \cap \dot{f} < 0$$
$$G_1 \neq 0$$
$$\boldsymbol{\sigma} = (\mathbf{D}^{e}\Delta t + \boldsymbol{\Phi}^v) : \dot{\boldsymbol{\varepsilon}} + \mathbf{c}$$

Visco-elasto-plastic

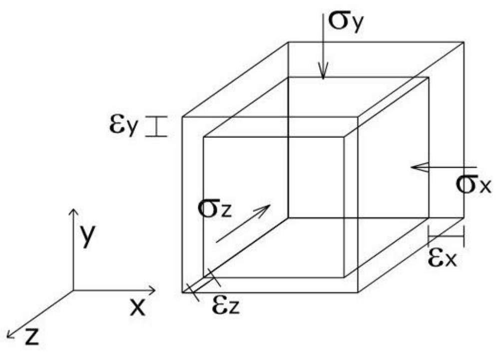
$$f = \dot{f} = 0$$
$$G_1 \neq 0$$
$$\boldsymbol{\sigma} = (\mathbf{D}^{ep}\Delta t + \boldsymbol{\Phi}^v) : \dot{\boldsymbol{\varepsilon}} + \mathbf{c}$$

$$T_0 = \dot{\boldsymbol{\varepsilon}}_0 = 0$$

STATIC

→ loading
← – – unloading

True triaxial: constant pressure tests

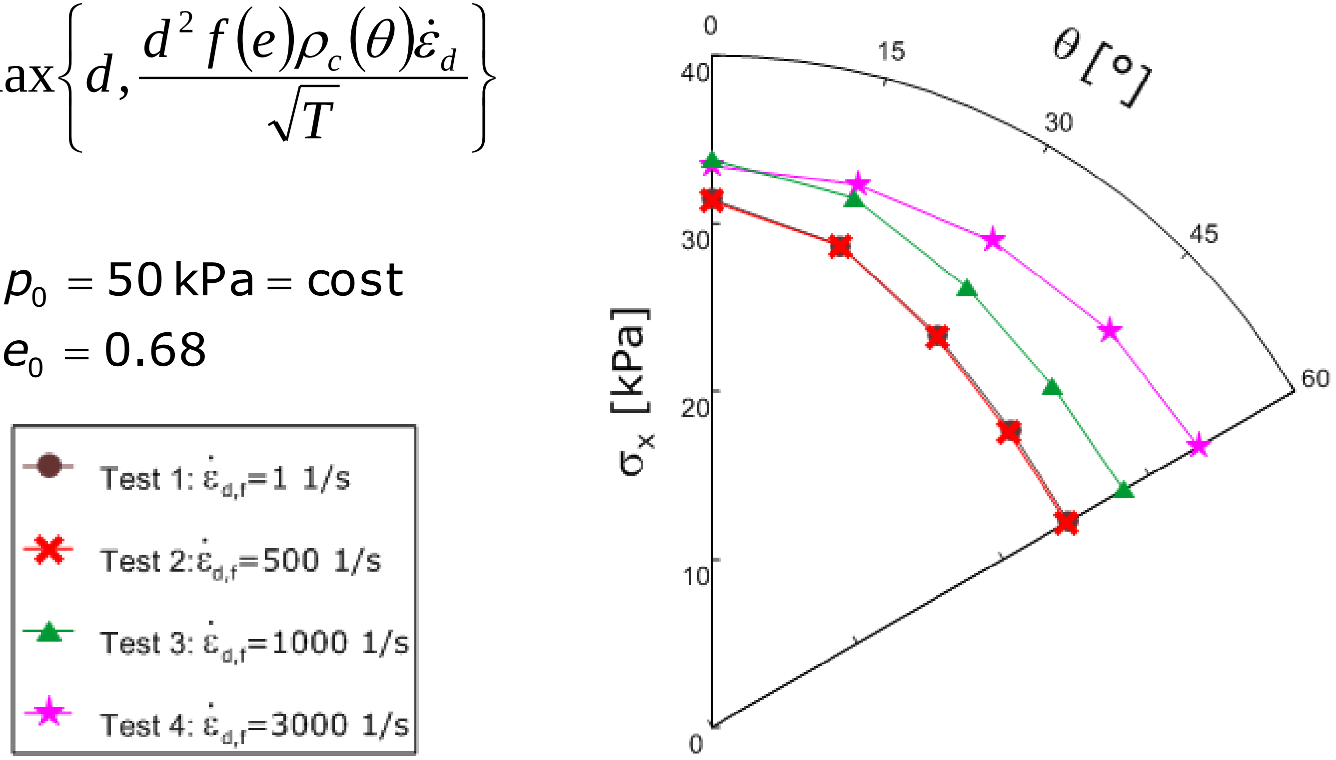


$$\dot{\varepsilon}_{d,f} \uparrow \longrightarrow T \uparrow$$

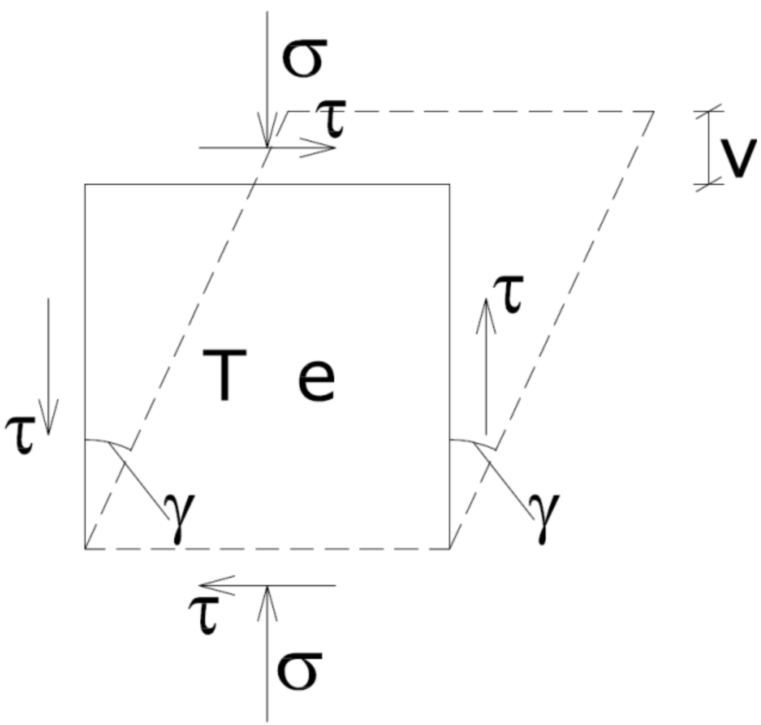
$$L = \max\left\{ d, \frac{d^2 f(e) \rho_c(\theta) \dot{\varepsilon}_d}{\sqrt{T}} \right\}$$

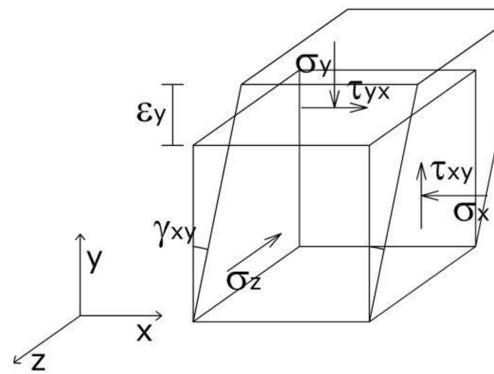
$p_0 = 50\,\text{kPa} = \text{cost}$

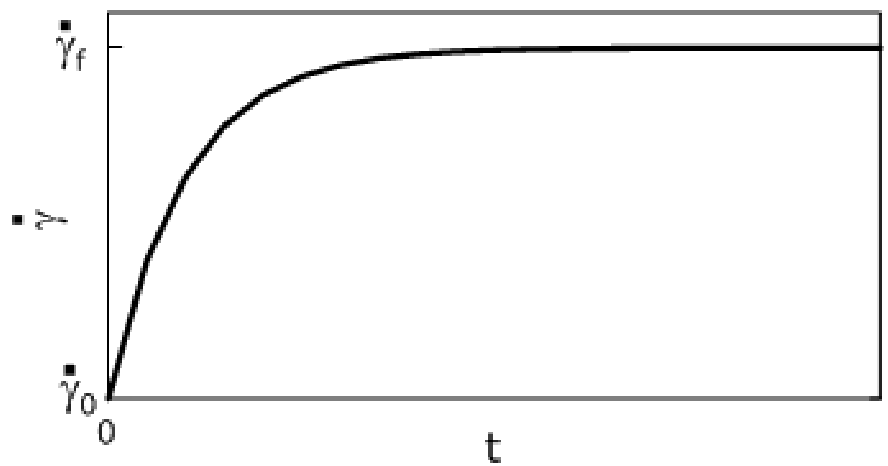
$e_0 = 0.68$



Simple shear: τ-σ formulation

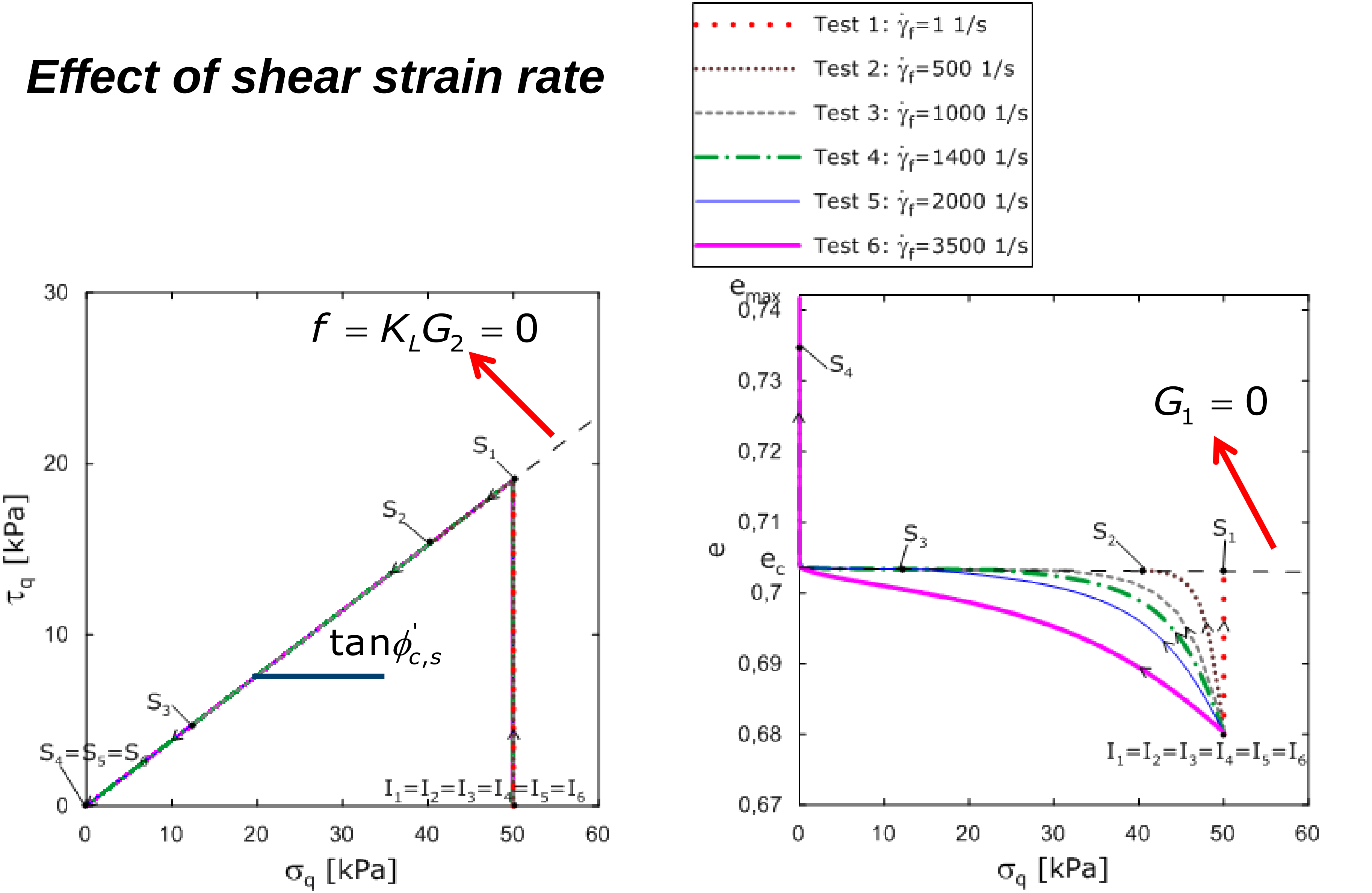






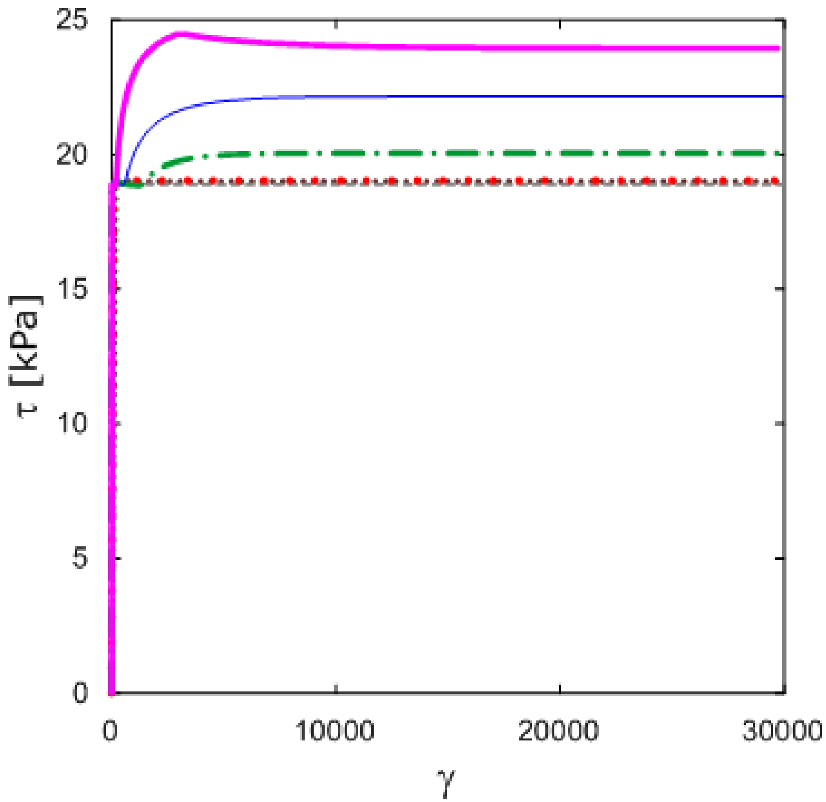
Simple shear: constant pressure tests

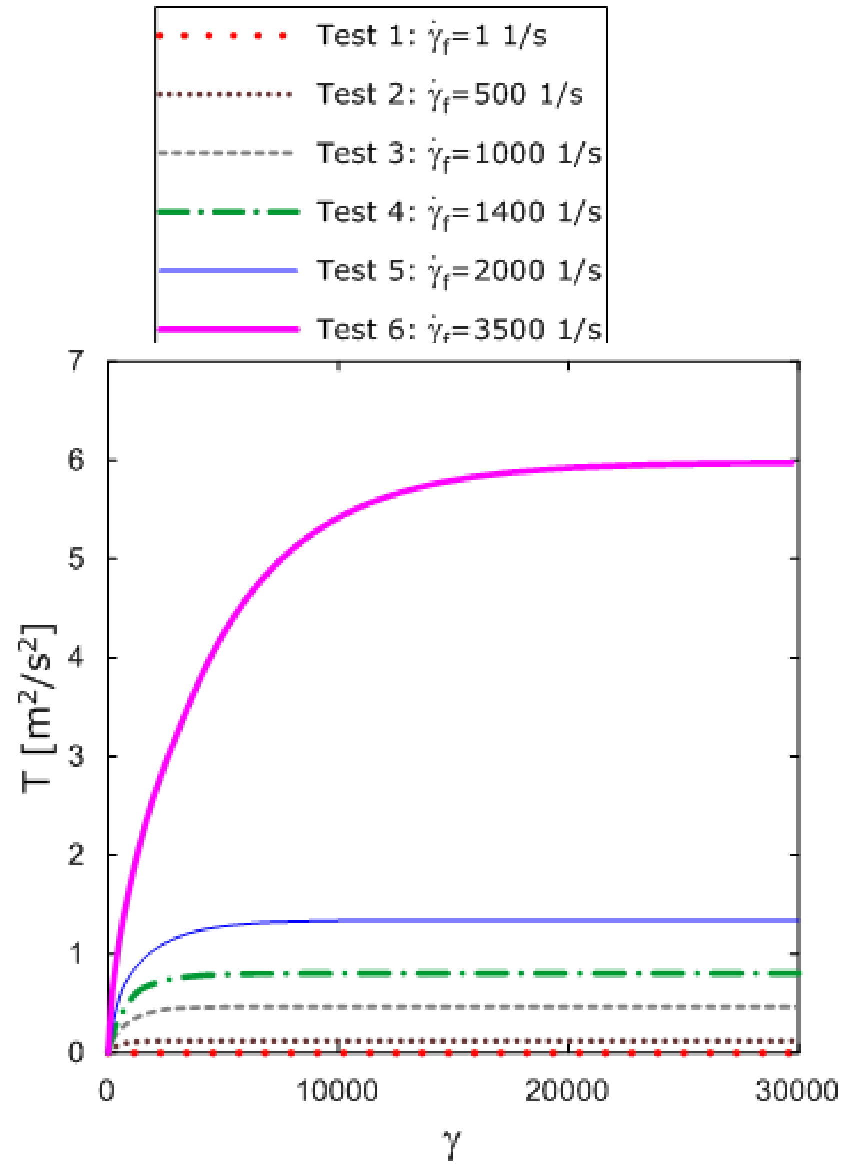
Effect of shear strain rate



Simple shear: constant pressure tests

Effect of shear strain rate

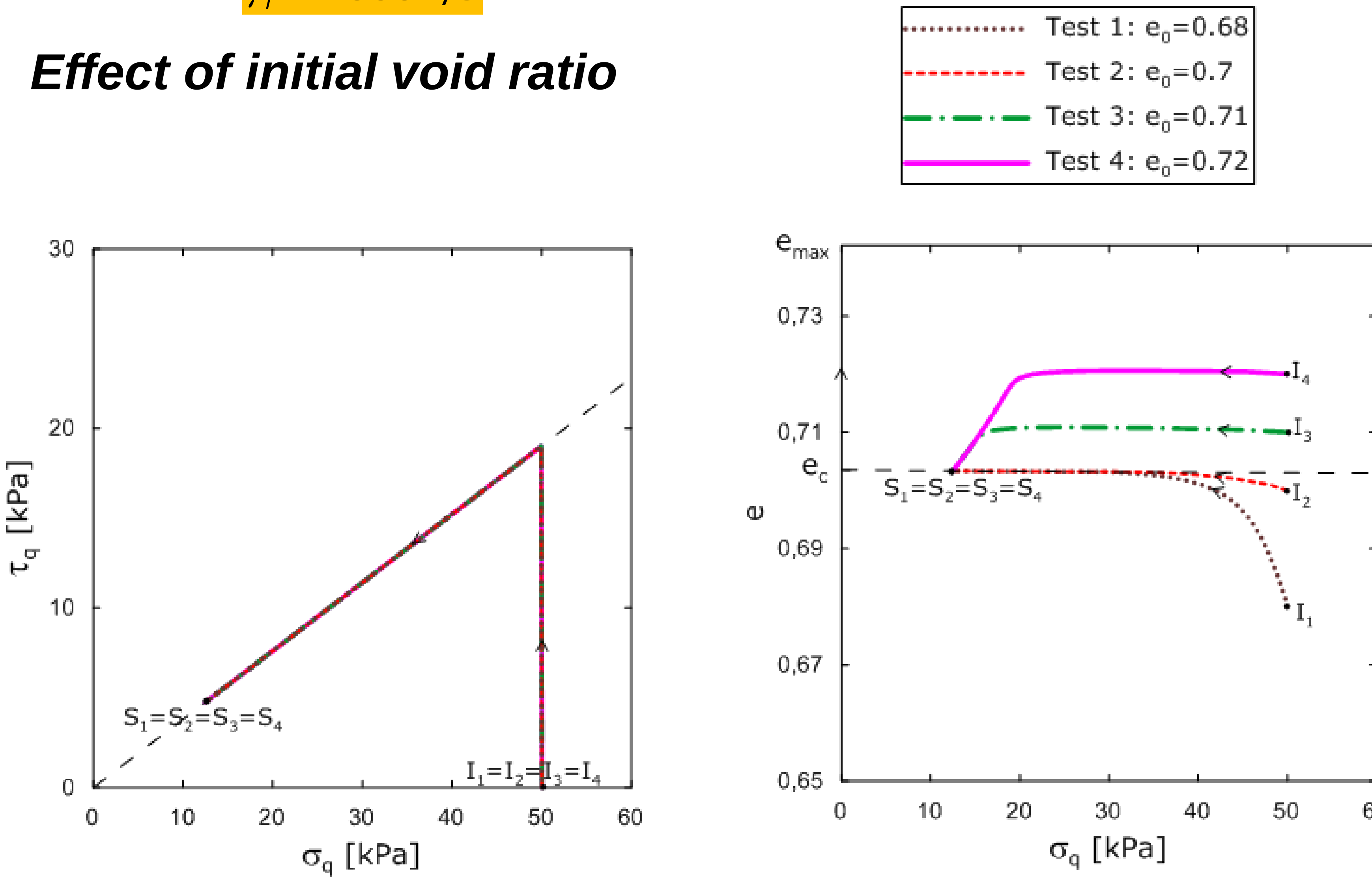




Simple shear: constant pressure tests

$\dot{\gamma}_f = 1000\,1/s$

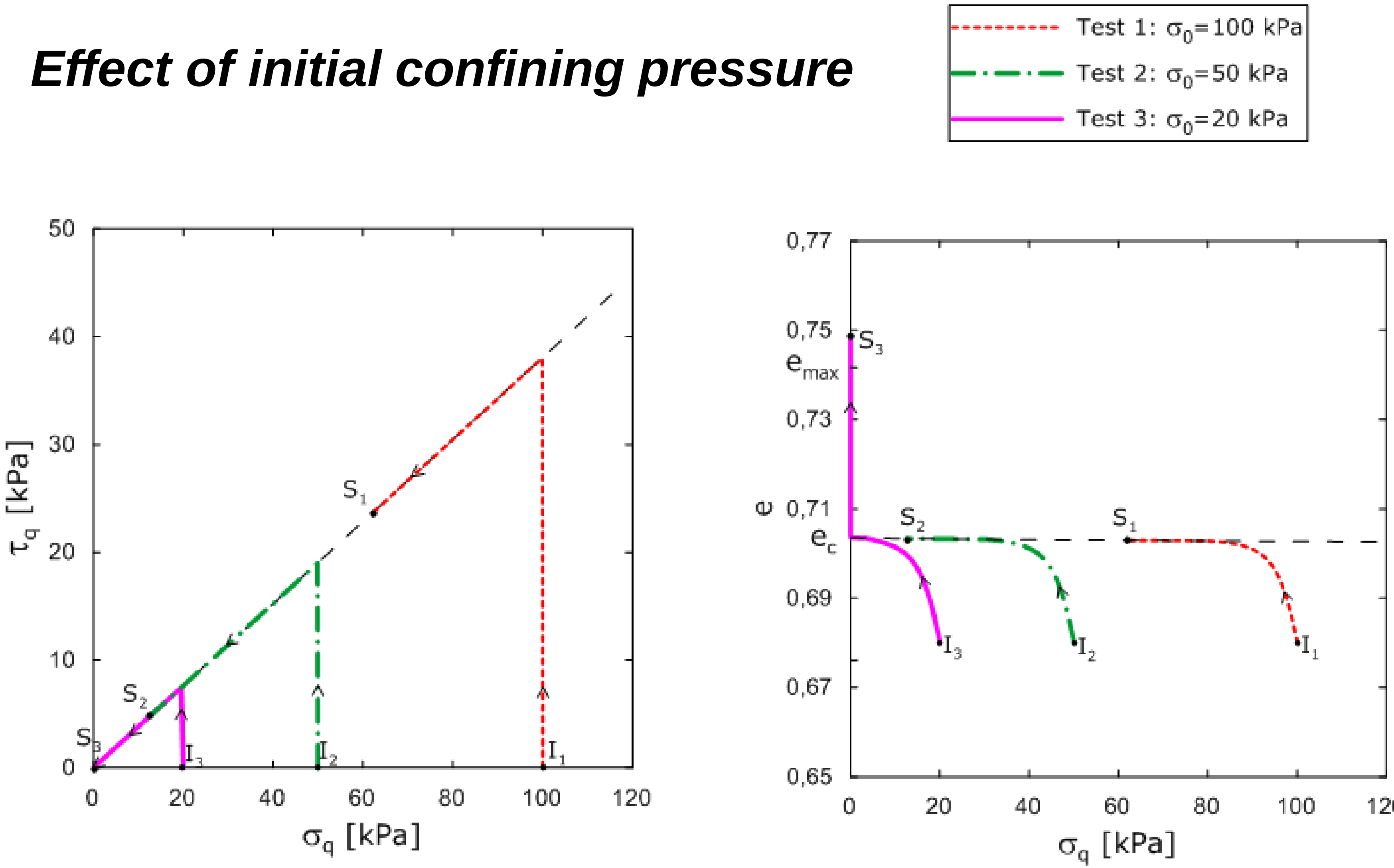
Effect of initial void ratio



Simple shear: constant pressure tests

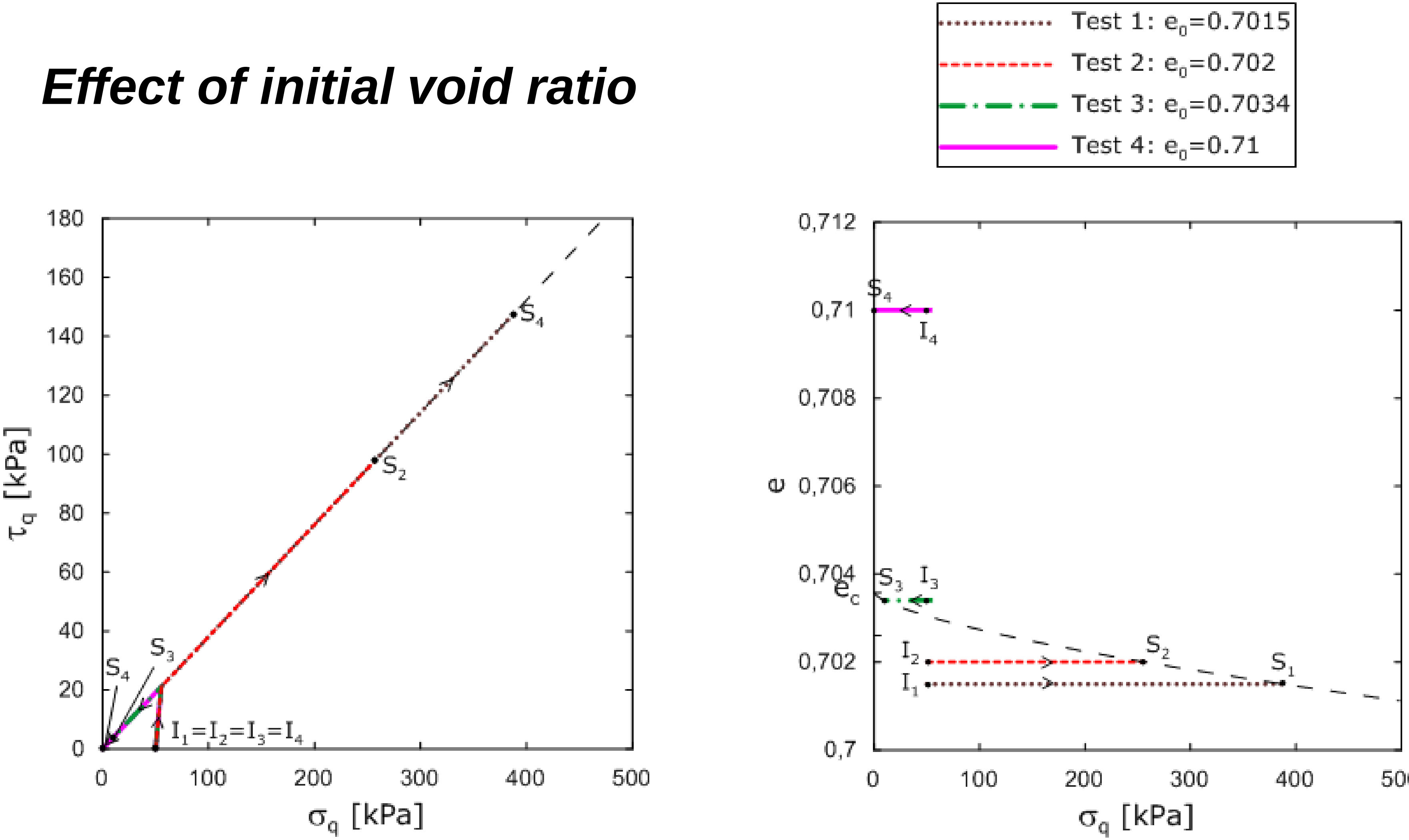
$\dot{\gamma}_f = 1000\,1/s$

Effect of initial confining pressure



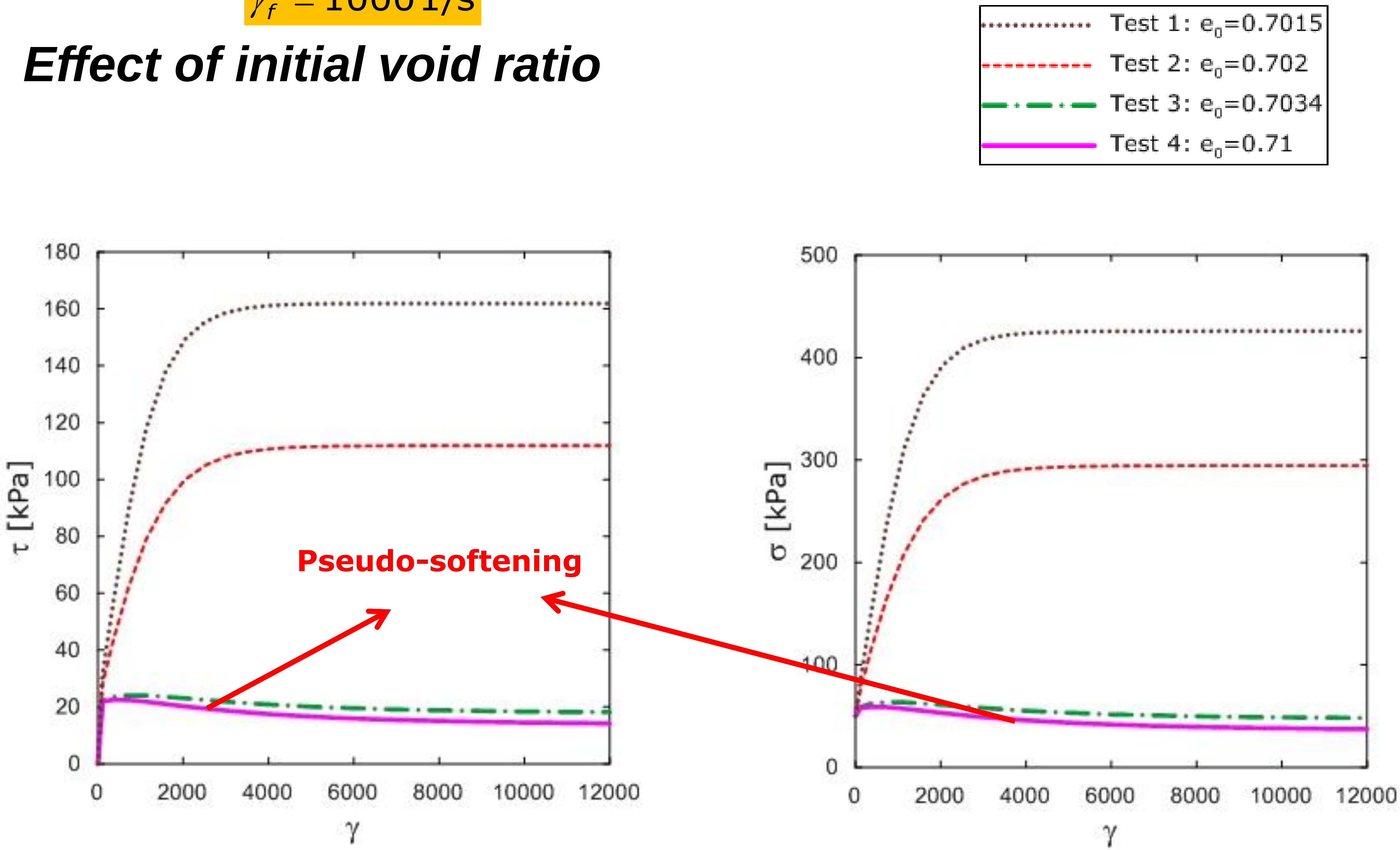
$\dot{\gamma}_f = 1000\,1/s$

Effect of initial void ratio



$\dot{\gamma}_f = 1000\,1/s$

Effect of initial void ratio



Conclusions

• The constitutive model is capable of simulating both the **inception and the post-collapse** behaviour of the granular material.

•The constitutive model is based on the definition of **four distinct unsteady regimes**: visco-elastic, visco-elasto-plastic, critical and collisional.

•Under the **critical regime**, the void ratio evolution is governed by the mean quasi-static pressure via the critical state locus definition.

•The numerical results testify the capability of the model to take into account the **dependence** of the mechanical behaviour of granular matters **on: the initial void ratio, the imposed pressure and the imposed deviatoric strain rate**

•The **critical state is interpreted as a peculiar steady state** taking place for a nullifying granular temperature (quasi-static conditions)

•There is a **continuous transition from the collisional to the quasi-static regime**: this has been obtained by re-defining the correlation length L