Some aspects of barodesy

Lecture in Assisi

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Institut für Infrastruktur
Universität Innsbruck

Constitutive laws . . .

. . . are boring

because their links to reality ("correspondence rules") are usually hidden in equations and thus missed.

Constitutive laws are not a mere tool for numerical calculations . . .

They should rather provide a frame to understand reality, i.e.

a guideline to explore an unkown world

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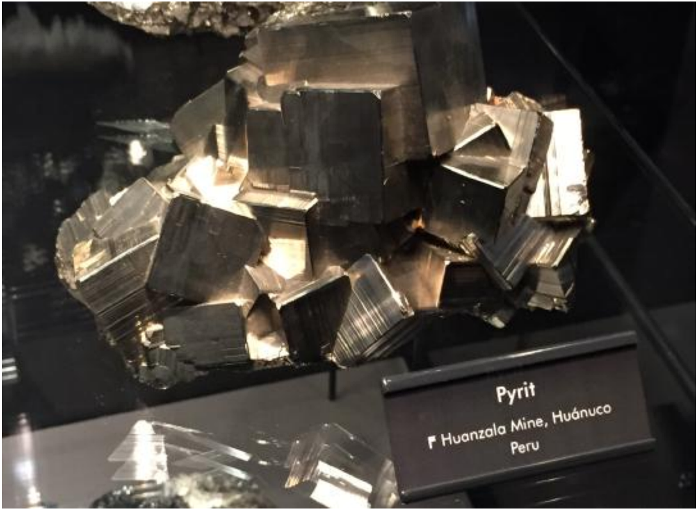
Matter and mathematics are inherently related

Crystal growth



Perfect mathematical structures can be formed in niches
(mathematics develops within niches of peace)

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What about fragmented and weathered matter?

Constitutive equations for soil

Who can answer the following questions:

- Which phenomena should they describe?
- Write down a constitutive equation for soil
- Which are the most widespread equations?
- Which are their advantages and disadvantages?
- How can they be calibrated?
- Why do we need them?
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I know that my car has a motor but I don't know (and I don't care) how it works.

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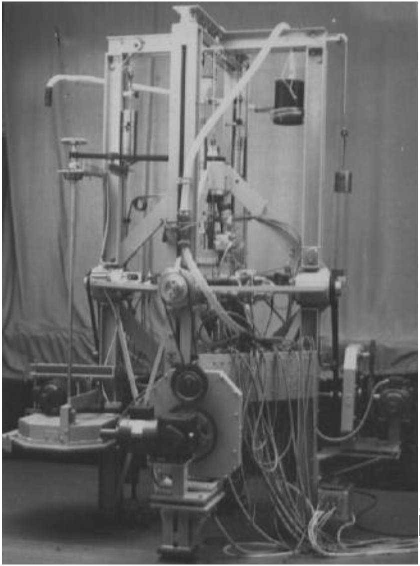
True triaxial apparatus by Goldscheider



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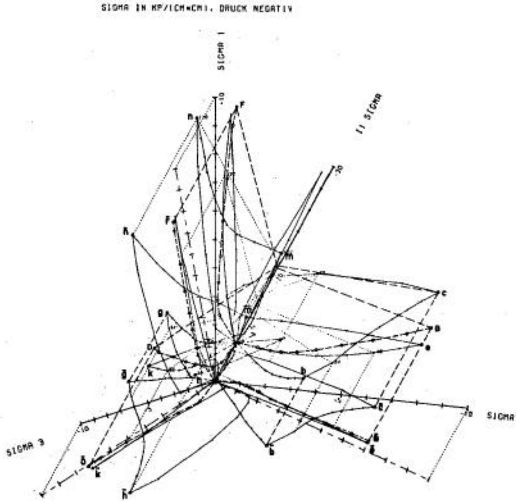
Goldscheider's explorations in stress space (1967)

His results were pretty confusing. . .



Goldscheider's rules

Goldscheider extracted form his results two rules.

From these two rules barodesy can be inferred!

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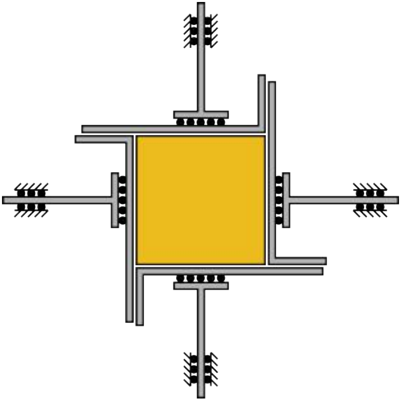
Experiments by Goldscheider1967 (1967)

True triaxial tests with sand, concept by **Hambly**



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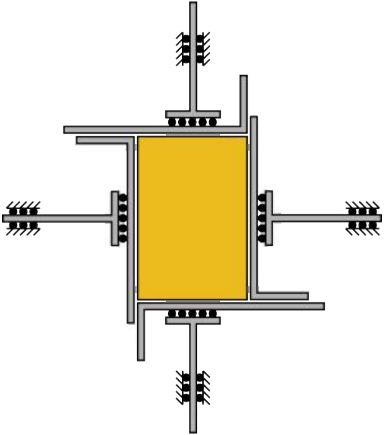

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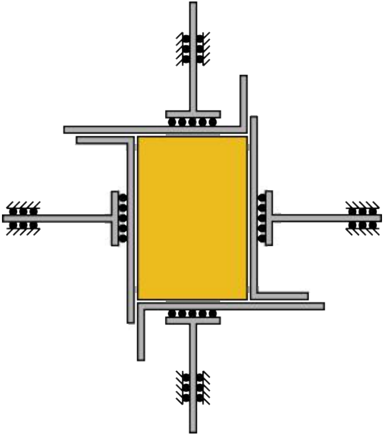

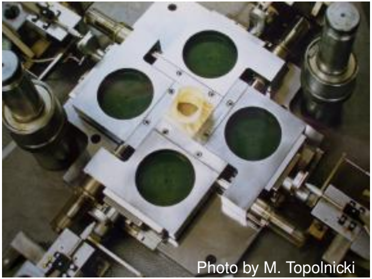

Photo by M. Topolnicki

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1^{st} Goldscheider rule

Prop. strain paths (PεP): $\varepsilon_1 : \varepsilon_2 : \varepsilon_3 = \text{const.}$
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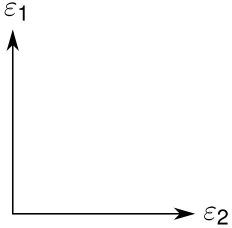

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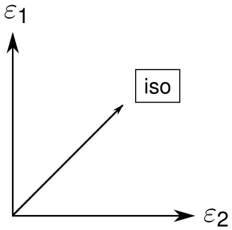

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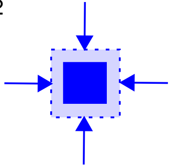

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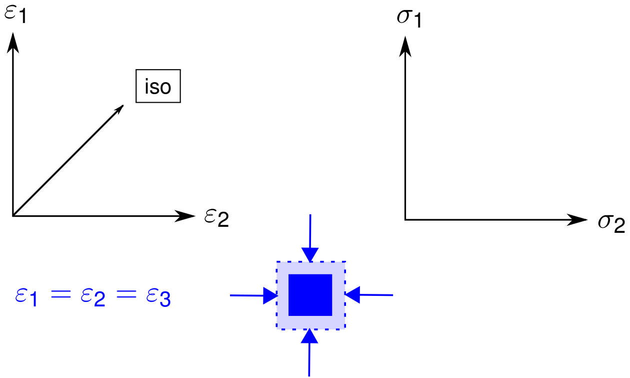

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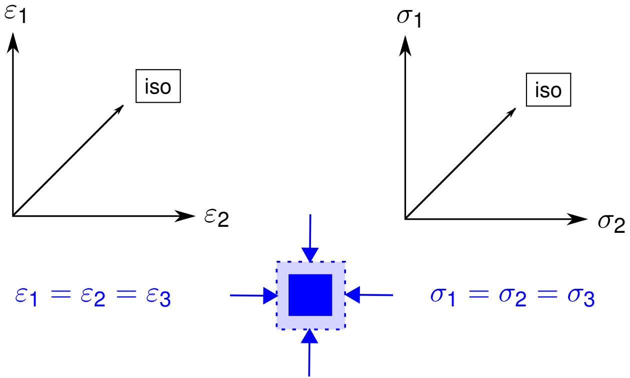

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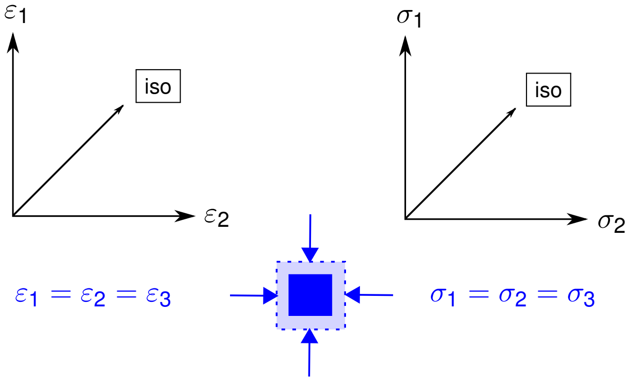

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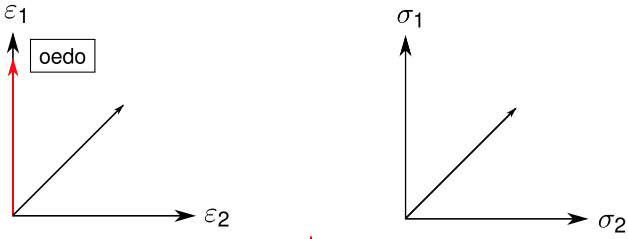

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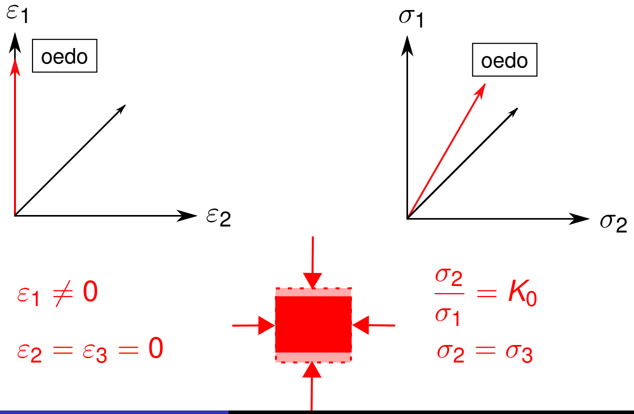

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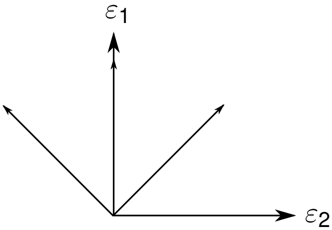

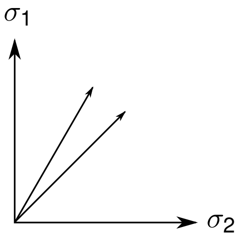

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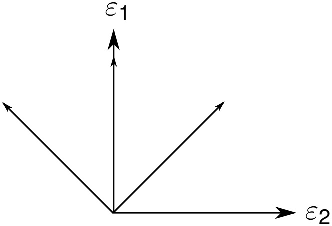

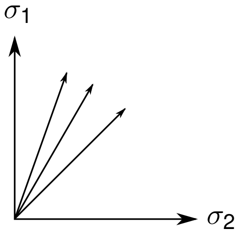

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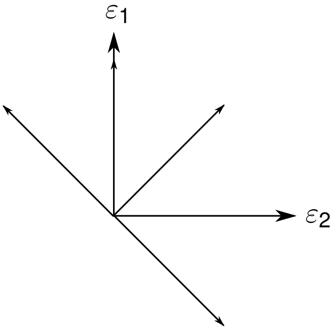

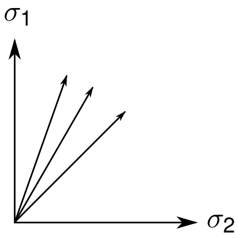

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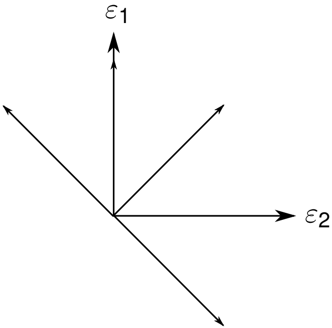

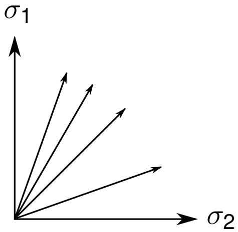

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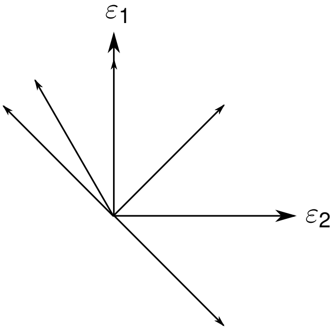

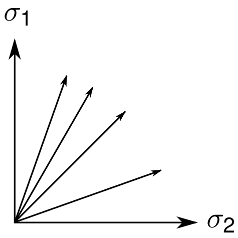

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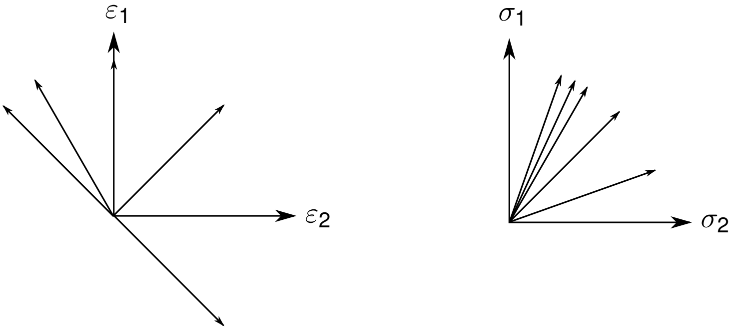

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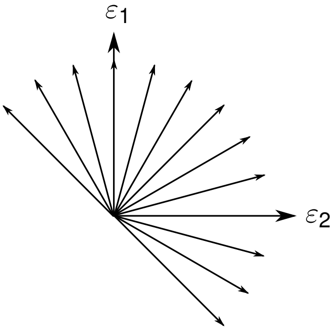

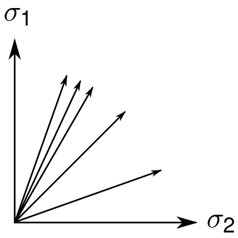

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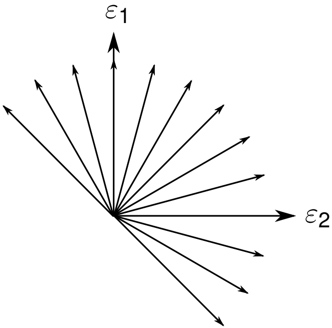

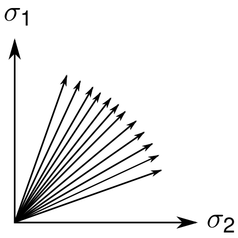

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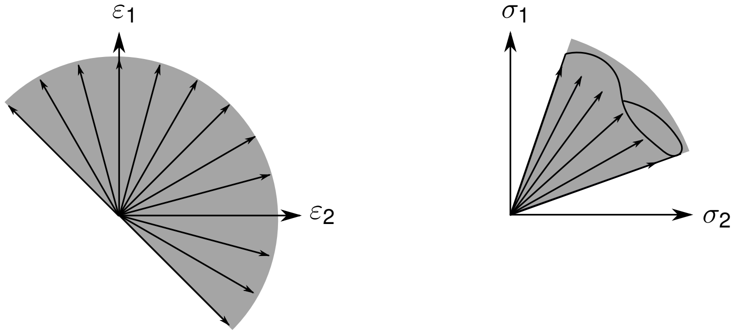

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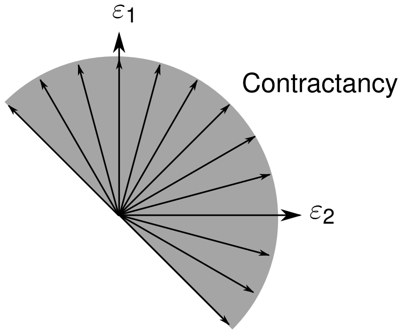
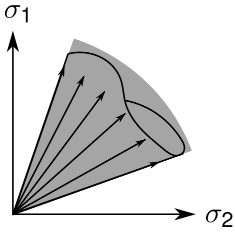
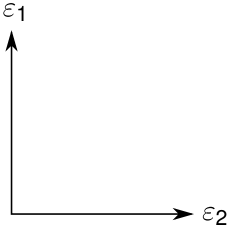
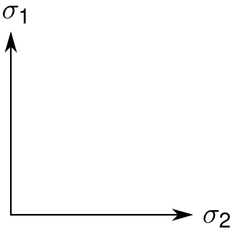




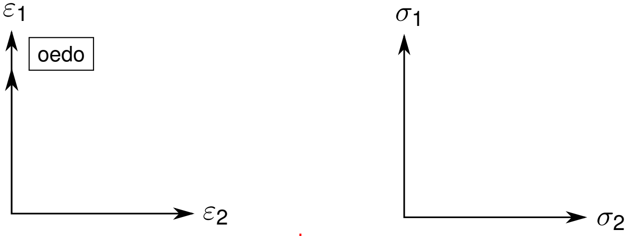
2nd Goldscheider rule





Similar results for clay by **Topolnicki (1987)**

2^{nd} Goldscheider rule



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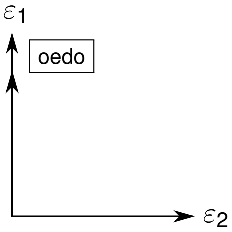

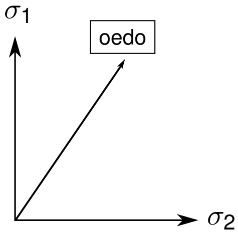

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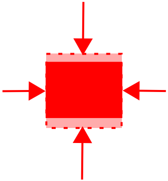

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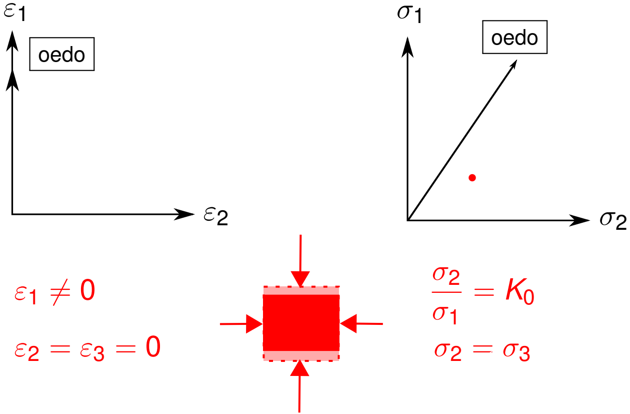

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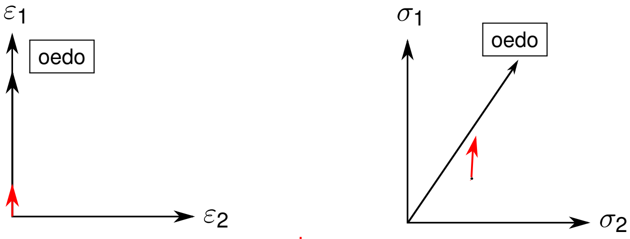

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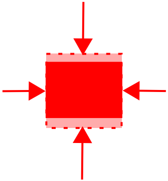

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2^{nd} Goldscheider rule

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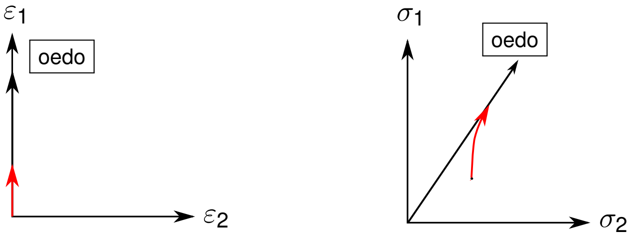

$\varepsilon_1 \neq 0$

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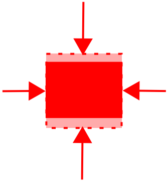

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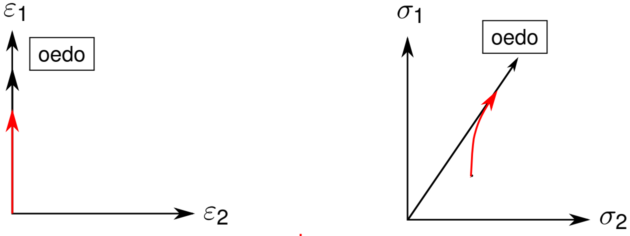

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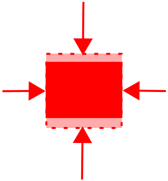

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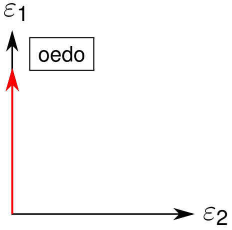

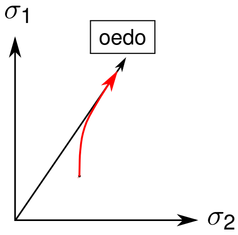

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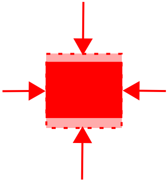

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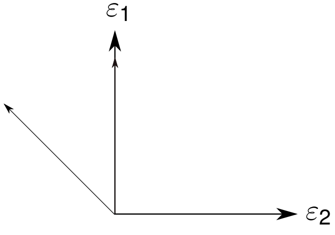

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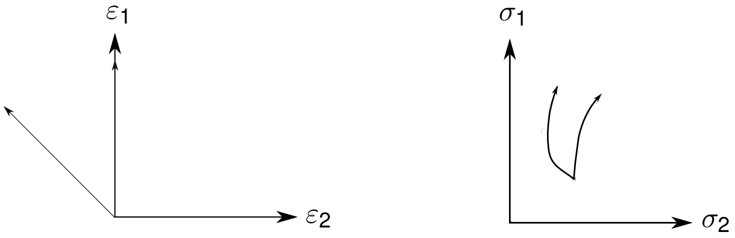

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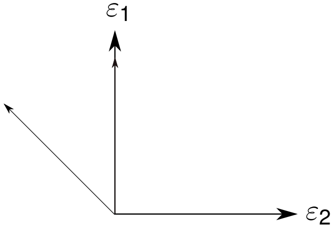

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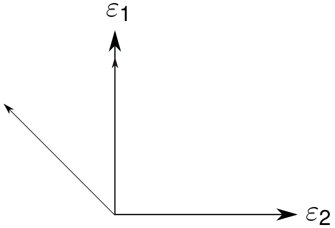

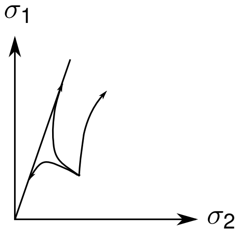

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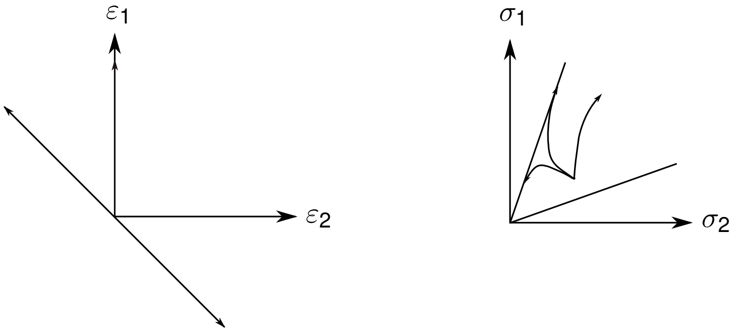

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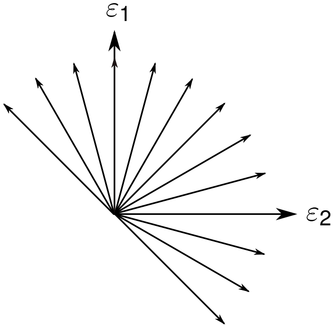

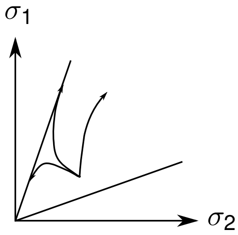

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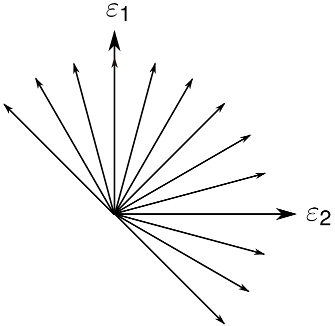

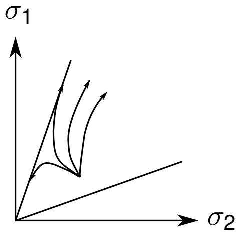

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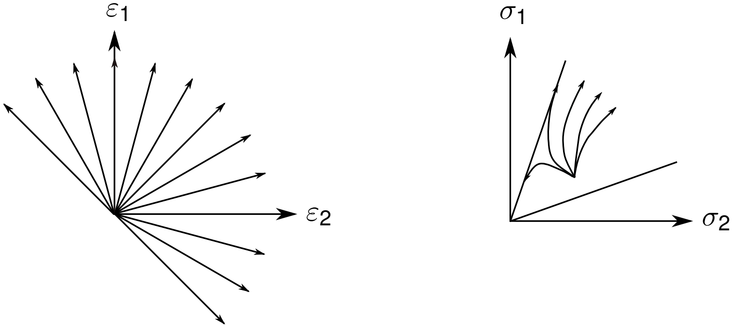

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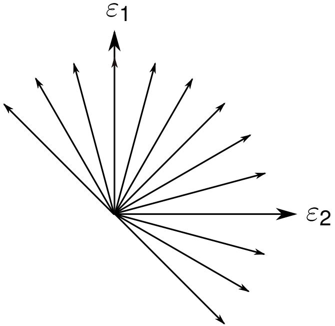

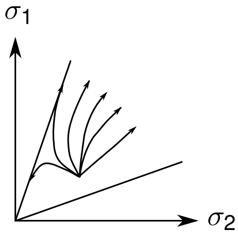

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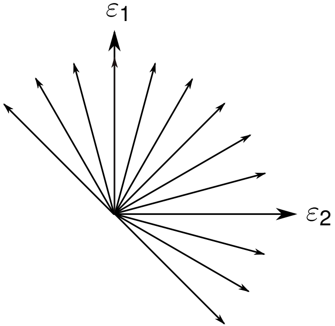

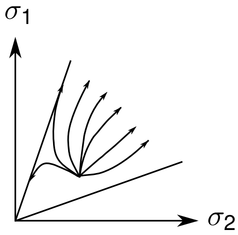

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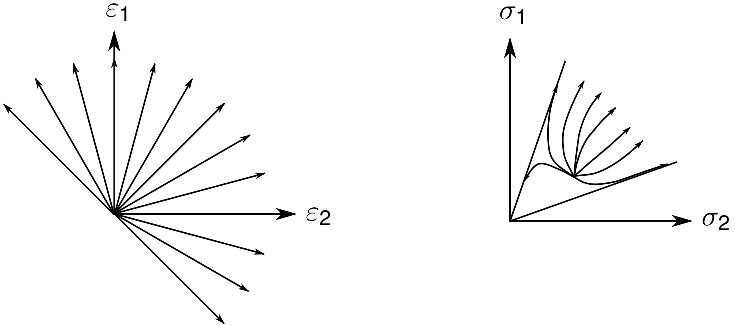

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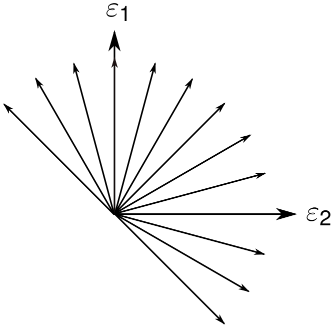

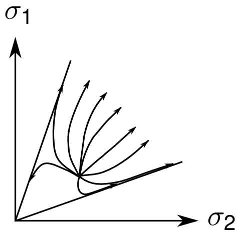

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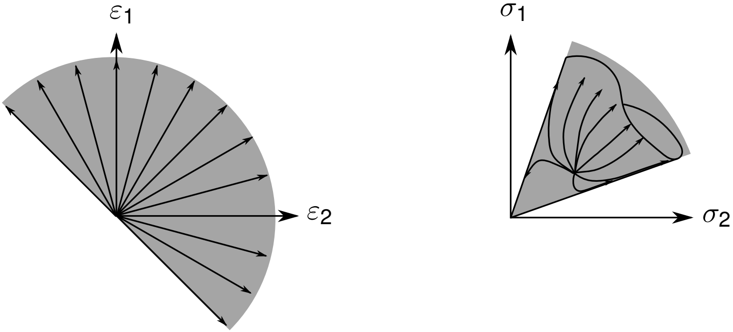

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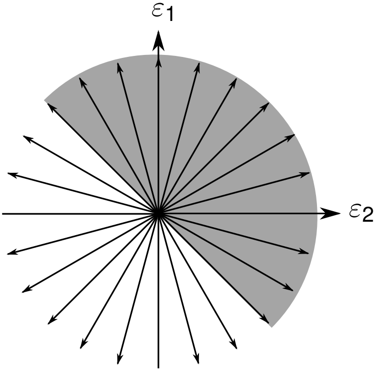

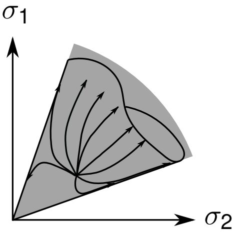

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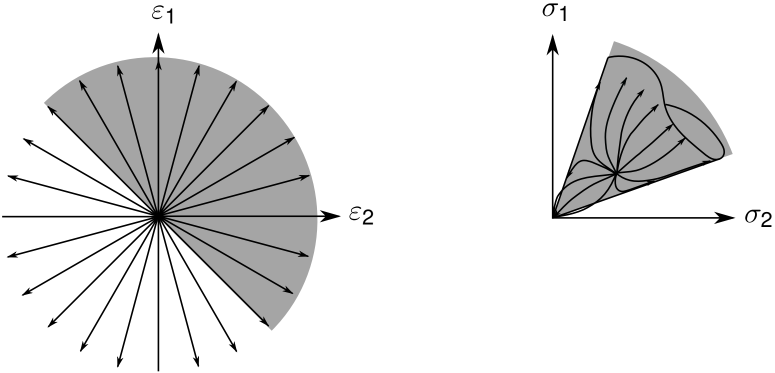

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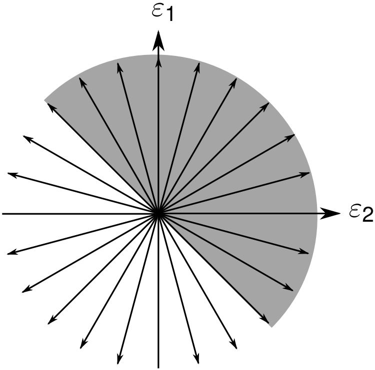

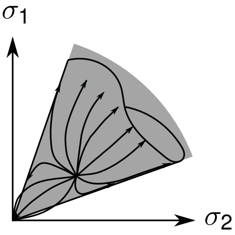

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Goldscheider's first rule:

Proportional paths in strain and stress spaces

Proportional stress paths starting from the stress-free state lead to proportional strain paths

$\mathbf{R}^0$: direction of a proportional stress path
$\mathbf{D}^0$: direction of a proportional strain path

This rule doesn't sound very exciting. However:

$\mathbf{R}^0$ doesn't always coincide with $\mathbf{D}^0$!

How to determine the relation $\mathbf{R}(\mathbf{D})$?

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Volume reducing proportional strain paths ('consolidations') produce stress paths within the compressive octant

This means:

$$\mathrm{tr}\,\mathbf{D}^0 = D_1^0 + D_2^0 + D_3^0 < 0 \rightarrow R_1 R_2 R_3 < 0 \tag{2}$$

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This cone is the critical state surface.

α is related to the critical friction angle φ_c:

With $R_2/R_1 = K_c$ and

$$K_c = \frac{1 - \sin \varphi_c}{1 + \sin \varphi_c} .$$

we obtain

$$\alpha = \sqrt{\frac{2}{3}} \ln K_c \qquad (4)$$

In general, α depends on the dilatancy $\delta := \operatorname{tr} \mathbf{D}^0$.

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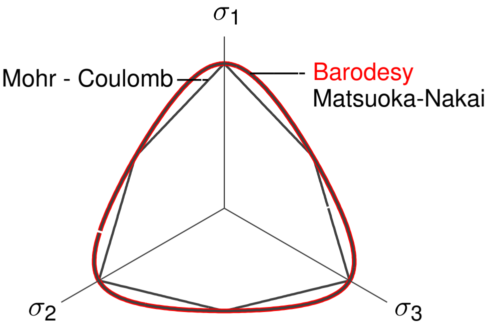

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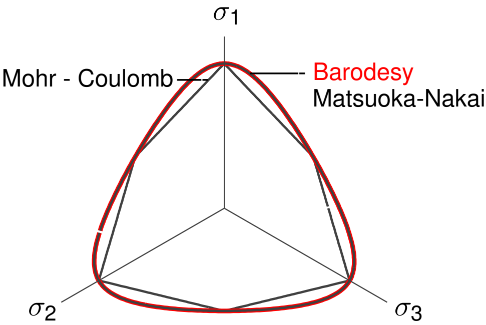

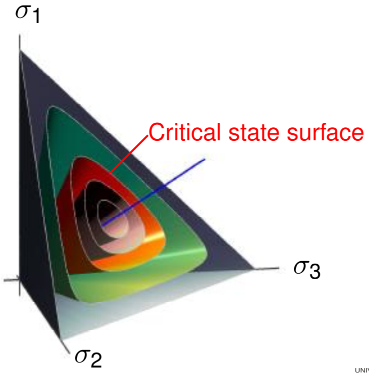

Goldscheider's first rule

We can write a constitutive relation for proportional stress paths:

$$\dot{\mathbf{T}} = h\,\mathbf{R}^0\,\dot{\varepsilon} \tag{5}$$

with $\dot{\varepsilon} := |\mathbf{D}|$. This is a relation of the *rate type*.

h is responsible for the stiffness and depends on $\sigma := |\mathbf{T}|$.

For some reasons, we multiply the right side with the scalar quantity $(f + g)$:

Constitutive relation for proportional stress paths:

$$\dot{\mathbf{T}} = h\,(f + g)\,\mathbf{R}^0\,\dot{\varepsilon} \tag{6}$$

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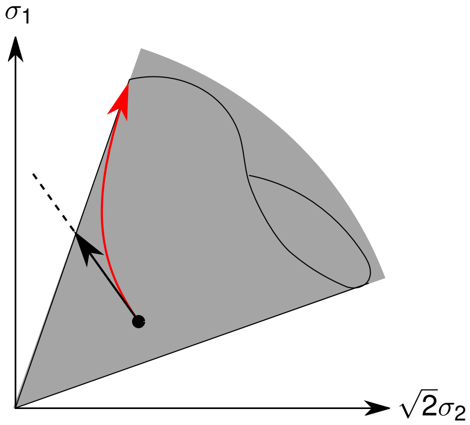

Barodesy

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- includes $\mathbf{T}$
- coincides with it for proportional paths, i.e. for $\mathbf{T}^0 = \mathbf{R}^0$.

Thus, from $\dot{\mathbf{T}} = h\,(f + g)\,\mathbf{R}^0\,\dot{\varepsilon}$ we obtain:

The full constitutive equation of barodesy:

$$\dot{\mathbf{T}} = h(\sigma) \cdot (f\mathbf{R}^0 + g\mathbf{T}^0) \cdot \dot{\varepsilon} \quad . \qquad (7)$$

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Limit states and peaks

Limit states (yield) are characterized by $\dot{\mathbf{T}} = \mathbf{0}$, i.e.

$f\mathbf{R}^0 + g\mathbf{T}^0 = \mathbf{0}$

$\rightsquigarrow$

tensorial equation

$$\mathbf{R}^0 = \mathbf{T}^0 \quad (8)$$

('flow rule' $\rightsquigarrow$ stress-dilatancy relation for peak states)

scalar equation

$$f + g = 0 \quad (9)$$

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Measure of dilatancy:

$$\delta := \frac{\mathrm{tr}\mathbf{D}}{|\mathbf{D}|} = \mathrm{tr}\mathbf{D}^0 \tag{10}$$

hydrostatic compression:	$\delta = -\sqrt{3}$
oedometric compression ($\varphi = 30°$):	$\delta = -\sqrt{2}$
undrained deformation:	$\delta = 0$

$$\delta\,\dot{\varepsilon} = \frac{\mathrm{tr}\mathbf{D}}{\dot{\varepsilon}}\,\dot{\varepsilon} = \mathrm{tr}\mathbf{D} = \frac{\dot{e}}{1+e} \tag{11}$$

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To fulfill equation (9) for limit states, we set

$$f + g = \delta + c_3(e_c - e) \tag{12}$$

e_c: critical void ratio.

Limit states: $f + g = 0$

Critical (residual) limit states: $\delta = 0$ **and** $e = e_c$

Peak limit states: $\delta > 0$ **and** $e < e_c$.

Partitioning:

$$f = \delta + c_3 e_c \tag{13}$$

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$h(\sigma)$ expresses the stress-dependence of stiffness.

Stiffness

- increases (sublinearly) with σ
- should not vanish for $\sigma = 0$
- should not allow compaction below e_{min}

These requirements are fulfilled by:

Stiffness function $h(\sigma)$

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with (12) and (11) $\rightsquigarrow \dot{\sigma} = h\,[\delta + c_3(e_c - e)] \cdot \frac{1}{\delta} \cdot \frac{\dot{e}}{1+e}$

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For $e = e_c$ and $\delta = 0 \rightsquigarrow$ differential equation for *CSL*

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Integration $\rightsquigarrow$ CSL:

$$e_c(\sigma) = \frac{e_{min} + B}{1 - B} \tag{18}$$

with

$$B := \frac{e_{c0} - e_{min}}{e_{c0} + 1}\left(\frac{c_4 + c_5\sigma}{c_4}\right)^{-\frac{1+e_{min}}{c_5}} . \tag{19}$$

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Critical state line $e_c(\sigma)$

Reasoning $\rightsquigarrow$ $\frac{e_c - e}{\delta} = 0$ for CSL.

$\rightsquigarrow$ Differential equation for CSL:

$$\frac{d\sigma}{de_c} = -\frac{c_4 + c_5\sigma}{(1 + e_c)(e_c - e_{min})} \tag{17}$$

Integration $\rightsquigarrow$ CSL:

$$e_c(\sigma) = \frac{e_{min} + B}{1 - B} \tag{18}$$

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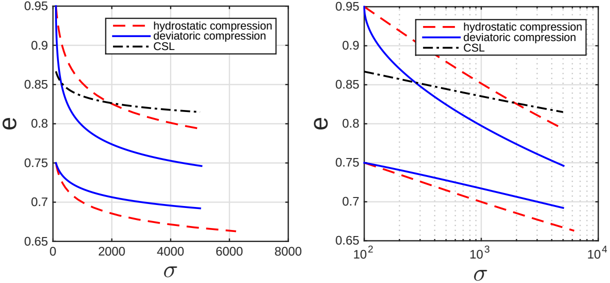
Compression lines

They depend on δ and on initial e.

If the compression includes also deviatoric deformation, then the compressibility

- is larger for $e > e_c$
- is smaller for $e < e_c$



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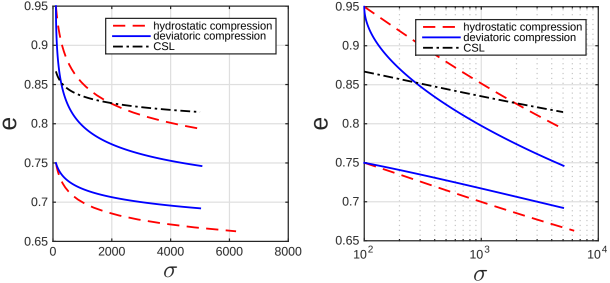

The complete constitutive equation

Barodesy:

$$\dot{\mathbf{T}} = h \cdot \left[(\delta + c_3 e_c)\mathbf{R}^0 - c_3 e \mathbf{T}^0 \right] \cdot \dot{\varepsilon} \tag{20}$$

$$\mathbf{R} = \exp(\alpha \mathbf{D}^0) \tag{21}$$

$$e_c = \frac{e_{min} + B}{1 - B}, \quad B := \frac{e_{c0} - e_{min}}{e_{c0} + 1} \left(\frac{c_4 + c_5 \sigma}{c_4} \right)^{-\frac{1+e_{min}}{c_5}} \tag{22}$$

$$h = -\frac{c_4 + c_5\, \sigma}{e - e_{min}} \tag{23}$$

Current & future research

$$\dot{\mathbf{T}} = h \cdot \left[(\delta + c_3 e_c)\mathbf{R}^0 - c_3 e \mathbf{T}^0 \right] \cdot \dot{\varepsilon} \qquad (24)$$

$$\mathbf{R} = \exp(\alpha \mathbf{D}^0) \qquad (25)$$

- Increased stiffness at cycles
- CSL for calibration
- extension to clay
- anisotropy

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The end

THANK YOU!