Recent advances on hypoplasticity with explicit asymptotic states

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Outline

1. Explicit incorporation of asymptotic states into hypoplasticity

2. Recent developments and applications of the framework
   - Incorporation of stiffness anisotropy
   - Variable ASBS shape in spudcan simulations
   - Coupled Thermo-Hydro-Mechanical double structure model for partially saturated expansive clays
   - ASBS anisotropy – soft clay modelling
   - Modelling of peat

Asymptotic states

- Asymptotic behaviour addressed in the lecture by *Prof. Kolymbas*: each *strain increment direction* is associated with the *asymptotic stress increment direction* and with the void ratio vs. mean stress dependency (*normal compression line*).

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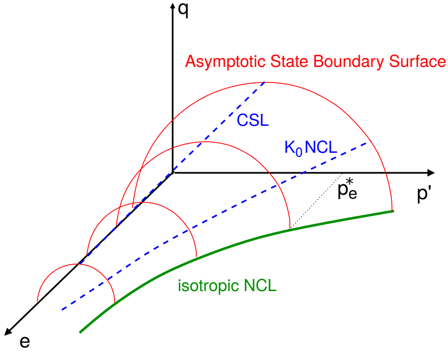

Explicit asymptotic states in hypoplasticity

General hypoplastic equation:

$$\mathring{\boldsymbol{\sigma}} = f_s(\boldsymbol{\mathcal{L}} : \mathbf{D} + f_d \mathbf{N} \|\mathbf{D}\|)$$

Principle of derivation: normalised stress $\boldsymbol{\sigma}_n = \boldsymbol{\sigma}/p_e^*$ does not change in asymptotic loading.

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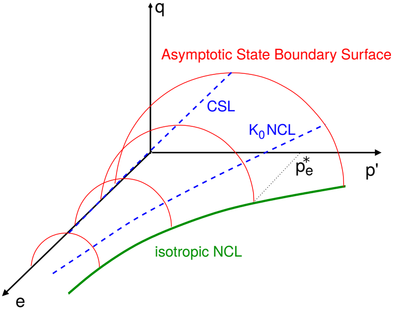

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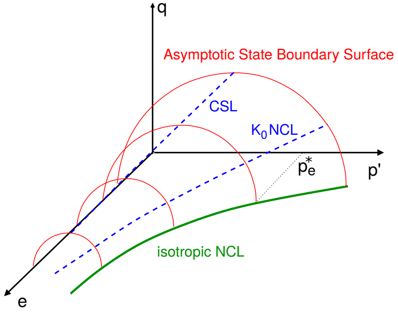

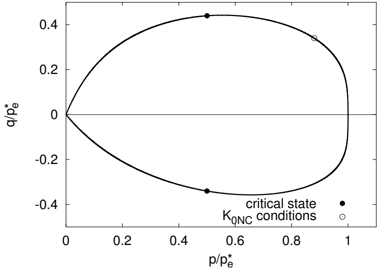

Explicit asymptotic states in hypoplasticity

The rate of normalised stress follows from its definition $\boldsymbol{\sigma}_n = \boldsymbol{\sigma}/p_e^*$

$$\dot{\boldsymbol{\sigma}}_n = \frac{\dot{\boldsymbol{\sigma}}}{p_e^*} - \frac{\boldsymbol{\sigma}}{p_e^{*2}} \dot{p}_e^*$$

Let's now select an expression for the isotropic normal compression line. We choose the Butterfield (1979) law

$$\ln(1+e) = N - \lambda^* \ln(p_e^*/p_r)$$

from which follows

$$\dot{p}_e^* = -\frac{p_e^*}{\lambda^*} \operatorname{tr} \mathbf{D}$$

and thus

$$\dot{\boldsymbol{\sigma}}_n = \frac{f_s}{p_e^*}\left(\mathcal{L} : \mathbf{D} + f_d \mathbf{N} \|\mathbf{D}\|\right) + \frac{\boldsymbol{\sigma}}{p_e^* \lambda^*} \operatorname{tr} \mathbf{D}$$

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Explicit asymptotic states in hypoplasticity

As already indicated, $\boldsymbol{\sigma}_n$ is constant during proportional loading. Thus, $\dot{\boldsymbol{\sigma}}_n = \mathbf{0}$, and

$$-\frac{\boldsymbol{\sigma}}{\lambda^*}\operatorname{tr}\mathbf{D}^A = f_s\left(\mathcal{L} : \mathbf{D}^A + f_d^A\mathbf{N}\|\mathbf{D}^A\|\right)$$

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So, we now have

$$-\mathcal{A} : \boldsymbol{d} = f_s f_d^A \mathbf{N}$$

which also means

$$\mathbf{N} = -\frac{\mathcal{A} : \boldsymbol{d}}{f_s f_d^A}$$

This expression for $\mathbf{N}$ can be substituted back into the original hypoplastic equation

Which can be manipulated to

$$\dot{\sigma} = f_s \mathcal{L} : \mathbf{D} - f_d \left(\frac{\mathcal{A} : \boldsymbol{d}}{f_d^A} \right) \|\mathbf{D}\|$$

This equation *inherently* implies normal compression behaviour according to the Butterfield (1979) law. f_d^A controls the shape of the state boundary surface, and $\boldsymbol{d}$ is the asymptotic strain rate direction. Any shape of the ASBS can be included, *as in elasto-plasticity*.

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Clay hypoplastic model with explicit ASBS

- Clay hypoplastic model from [Mašín, D. (2013). Clay hypoplasticity with explicitly defined asymptotic states. **Acta Geotechnica** 8, No. 5, 481-496.].
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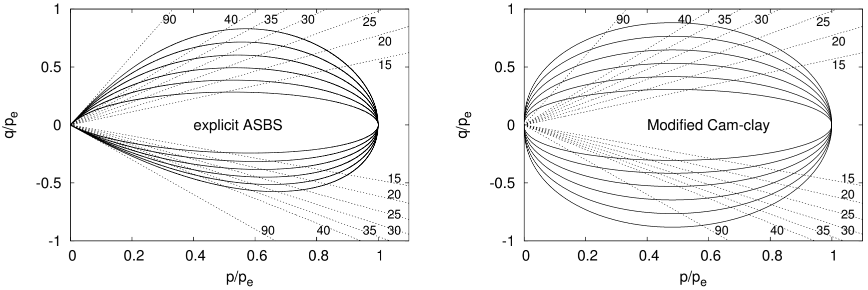
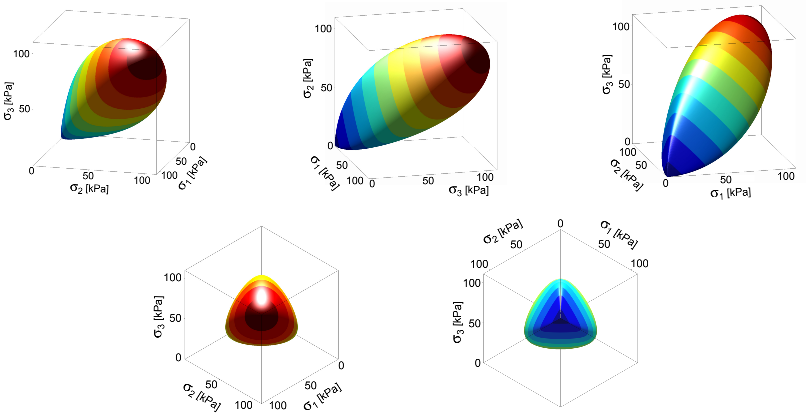


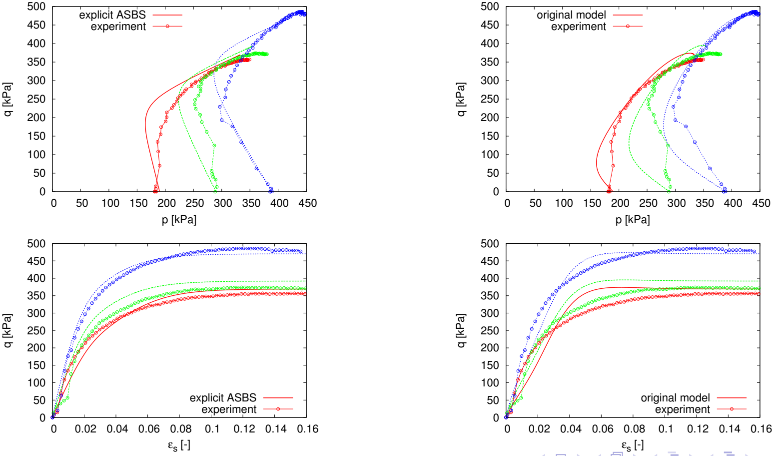
Clay hypoplastic model with explicit ASBS

- A shape of the asymptotic state boundary surface, which has Matsuoka-Nakai (1974) deviatoric cross-sections



Clay hypoplastic model with explicit ASBS

- The new model improves predictions when compared with the original 2005 model.



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Incorporation of stiffness anisotropy

- In the new formulation, ASBS is *independent* of the (hypo)elastic tensor $\mathcal{L}$:

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- It is thus (rather) straightforward to replace the isotropic $\mathcal{L}$ of the original model by *transversely isotropic* $\mathcal{L}$.

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with anisotropic elasticity by

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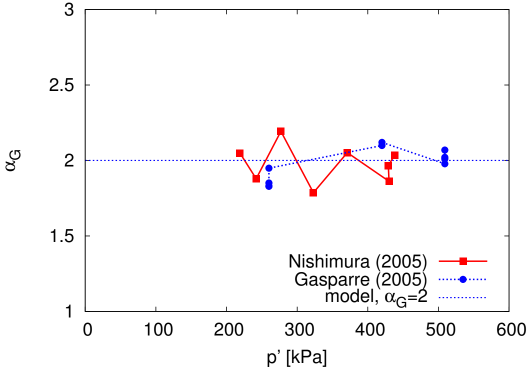
Predictions of model with anisotropic stiffness

- Stiffness anisotropy controlled a parameter α_G. Calibrated using tests on London clay with *horizontally* and *vertically* mounted *bender elements* (Gasparre 2005, Nishimura 2005)



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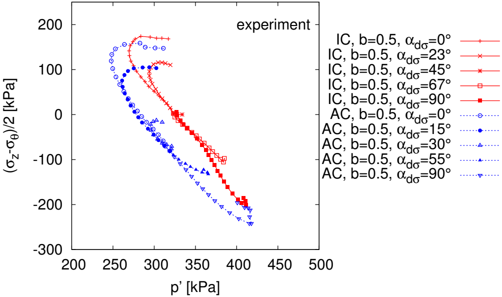

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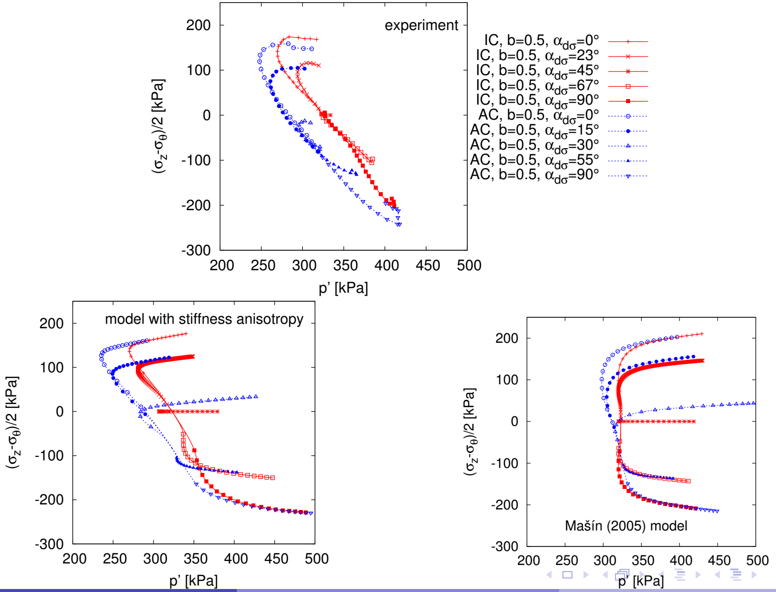

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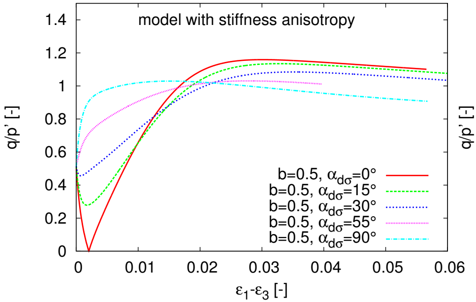
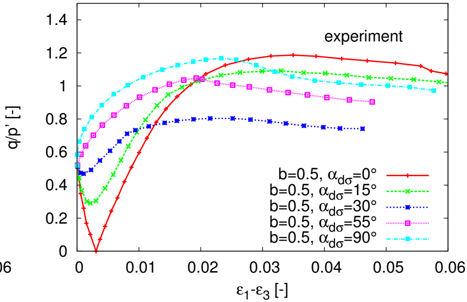
- Model evaluated using *hollow cylinder tests* on London clay by Nishimura et al. (2007)





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- Model implemented into *finite element codes* (Abaqus, Plaxis...). Available for free at *soilmodels.info*.
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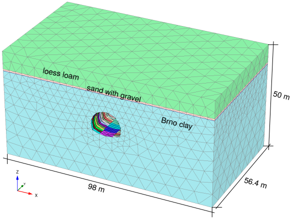

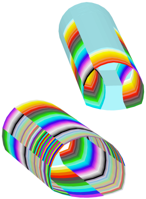

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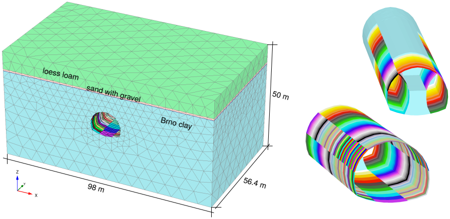
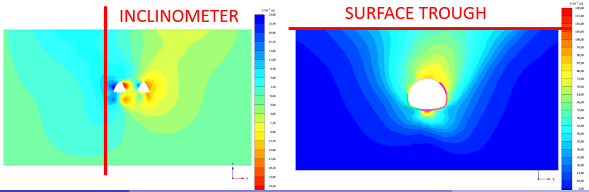
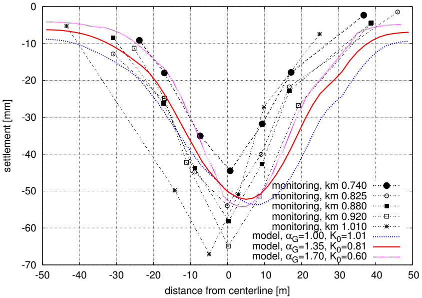


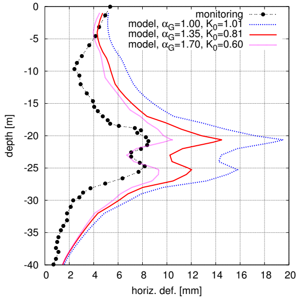
- Monitoring available: *surface settlement trough* and *inclinometer data.*



Predictions of model with anisotropic stiffness

- The model predicted remarkably well both the *surface settlement trough* and the *inclinometer measurements*





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Variable ASBS shape in spudcan simulations

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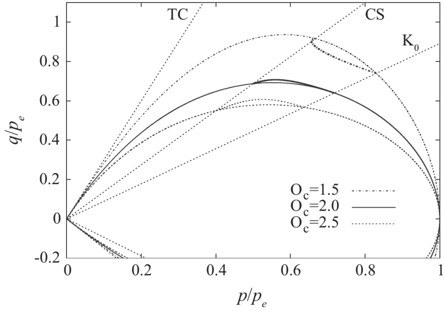

- Adopted in the spudcan foundation simulation by [Ragni, R., Wang, D., Mašín, D., Bienen, B., Cassidy, M. J. and Stanier, A. S. (2016). Numerical modelling of the effects of consolidation on jack-up spudcan penetration. **Computers and Geotechnics** 78, 25-37.]..

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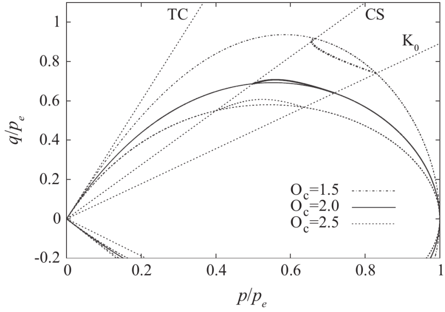

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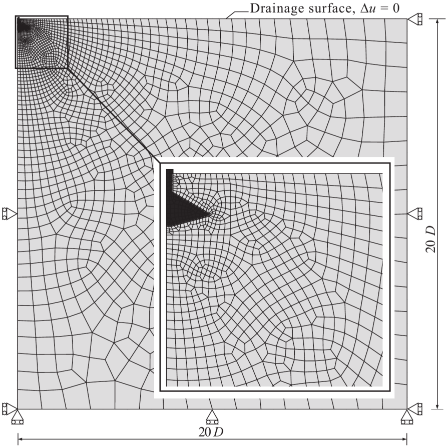
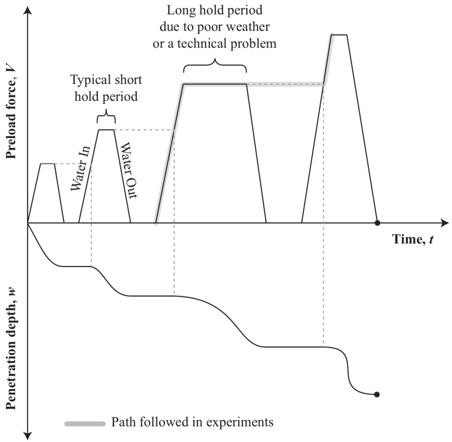
Variable ASBS shape in spudcan simulations

- "*Spudcan*" is a circular footing used as a foundation of *offshore platforms*.



Variable ASBS shape in spudcan simulations

- Occasional *delay* of the spudcan penetration (bad weather, technical problem) causes the soil beneah to *consolidate* - thus increase the bearing capacity and the danger of punch-through.



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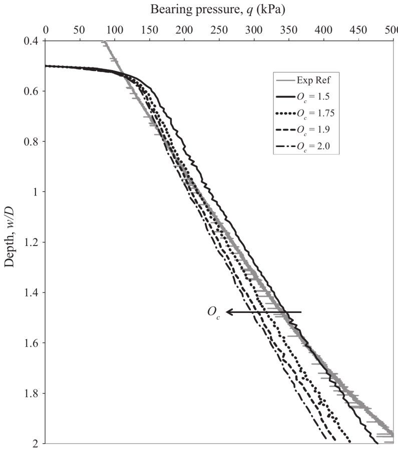

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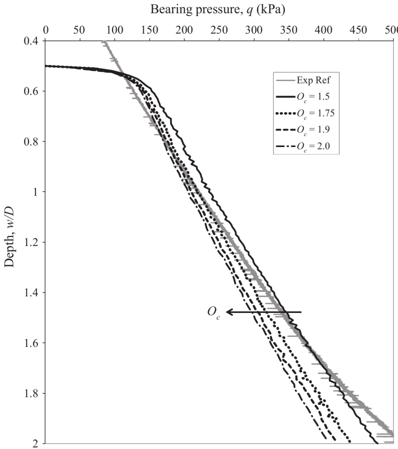
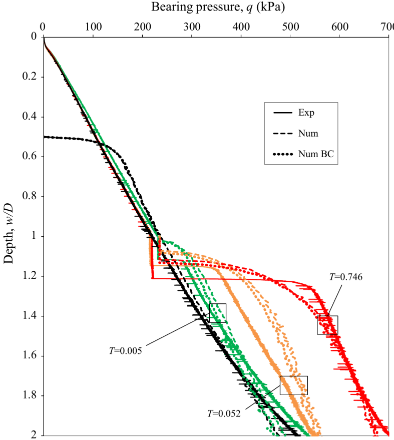
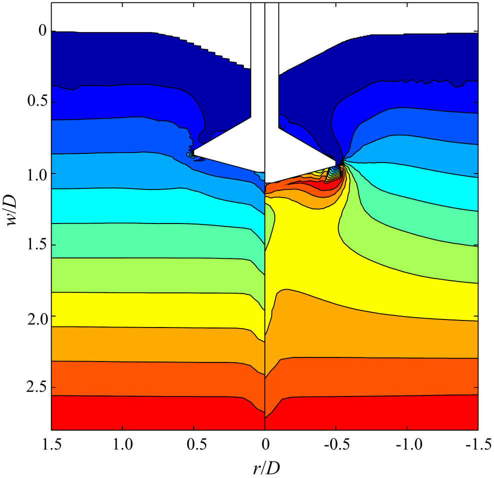




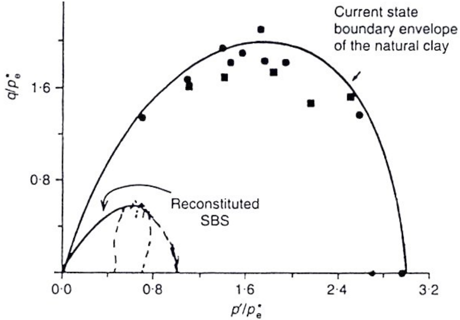
Variable ASBS shape in spudcan simulations

- *Undrained shear strength* before (left) and after (right) period of consolidation:



Variable ASBS shape in spudcan simulations

- Soil used in the experiments is *sensitive* (it has *meta-stable structure*).



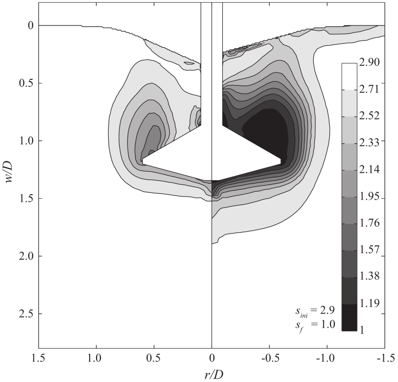
Cotecchia and Chandler, 2000

Rate of structure degradation controlled by a parameter k in the model

$$\dot{s} = -\frac{k}{\lambda^*}(s - s_f)\dot{\epsilon}^d$$

Variable ASBS shape in spudcan simulations

- Numerical simulation can be used to visualise *structure degradation* around the spudcan. Plots are showing *current sensitivity*



Left: *lower rate* of structure degradation k=0.05

Right: *higher rate* of structure degradation k=0.2

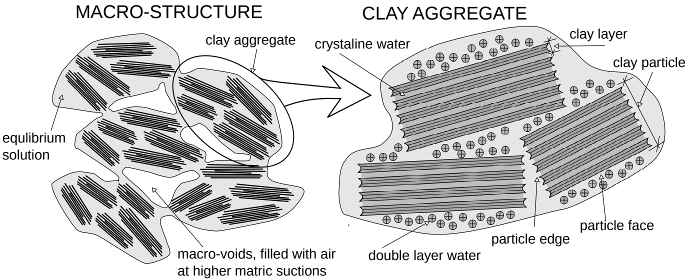
Outline

1 Explicit incorporation of asymptotic states into hypoplasticity

2 Recent developments and applications of the framework
- Incorporation of stiffness anisotropy
- Variable ASBS shape in spudcan simulations
- Coupled Thermo-Hydro-Mechanical double structure model for partially saturated expansive clays
- ASBS anisotropy – soft clay modelling
- Modelling of peat

Double structure model for expansive soils

- Expansive clay soils have double-porous structure. The behaviour of *aggregates* differ from the behaviour of *macrostructure*, which forms a material with apparently larger particles (*double structure* modelling concept by *Alonso et al., 1999*).



Double structure hydro-mechanical coupling

- Two *mechanical models* (macrostructure $\mathbf{G}^M$ and microstructure $\mathbf{G}^m$), defined using *effective stresses* $\boldsymbol{\sigma}^M$ and $\boldsymbol{\sigma}^m$.

$$\dot{\boldsymbol{\sigma}}^M = \mathbf{G}^M(\boldsymbol{\sigma}^M, \mathbf{q}^M, \dot{\boldsymbol{\epsilon}}^M) \quad \text{with} \quad \boldsymbol{\sigma}^M = \boldsymbol{\sigma}^{net} - \mathbf{1}s\chi^M$$
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- *Hydro-mechanical coupling*: Hydraulic strain measure S_r^M depends on mechanical quantity $\dot{\epsilon}^M$. Mechanical response influenced by χ^M, which may depend on a hydraulic quantity S_r^M.

Double structure hydro-mechanical coupling

- Coupling between the two structural levels:

$$\dot{\epsilon} = \dot{\epsilon}^M + f_m \dot{\epsilon}^m$$

where $0 \leq f_m \leq 1$ specifies occlusion of macropores by microstructure. f_m is a function of *relative density*.

⇒ $f_m = 0$ . . . aggregates can freely swell into macrovoids without imposing global deformation - typical behaviour of *open structure*.

⇒ $f_m = 1$ . . . aggregates cannot ocupy the macrovoids, global strain is equal to aggregate strain - typical behaviour of *dense structure*.

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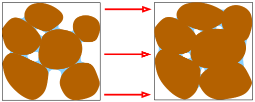

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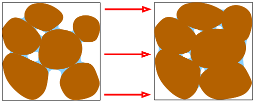

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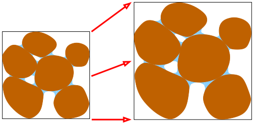
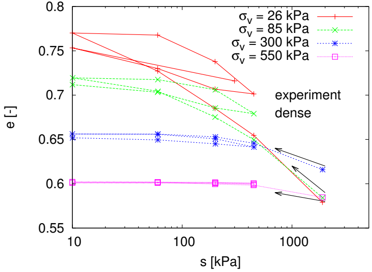


Double structure model for expansive soils

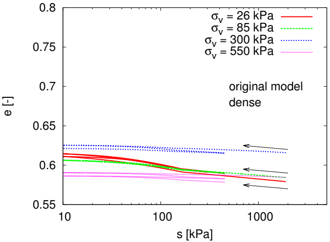
Oedometric free-swell test

High density fabric

Experimental data (Romero 1999)



Basic model



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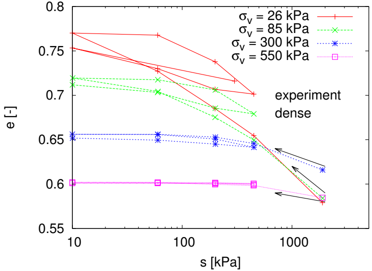

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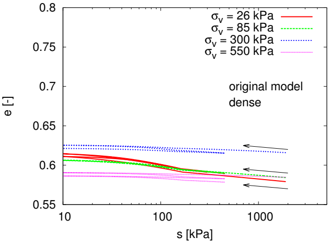
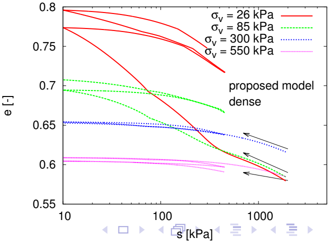


Double structure



Thermo-hydro-mechanical model

Thermal effects have been incorporated recently: [Mašín, D. (2016). Coupled thermo-hydro-mechanical double structure model for expansive soils, **Journal of Engineering Mechanics ASCE** (under review)].

General rate expression of the model reads:

$$\mathring{\sigma}^M = f_s \left[\mathcal{L} : (\mathbf{D} - f_m \mathbf{D}^m) + f_d \mathbf{N} \|\mathbf{D} - f_m \mathbf{D}^m\| \right] + f_u \left(\mathbf{H}_s + \mathbf{H}_T \right)$$

⇒ For prescribed $\mathbf{D}$, $\dot{s}$ and $\dot{T}$ the formulation is implicit!

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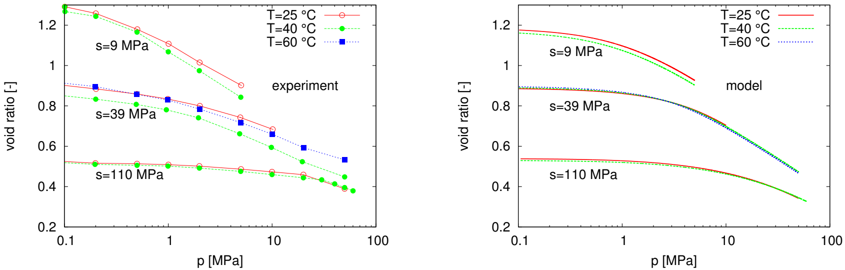
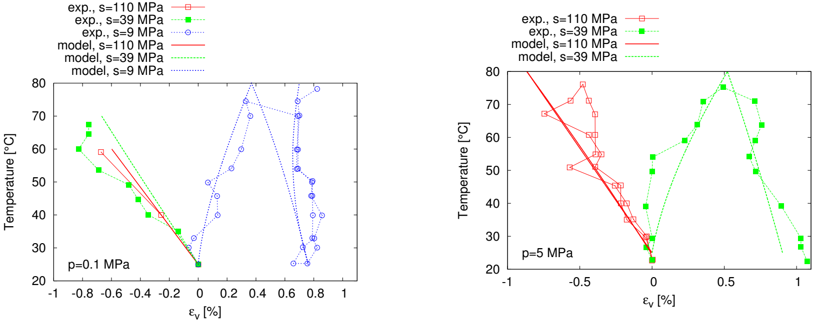
Thermo-hydro-mechanical model

- Normal compression lines, MX80 bentonite, data by Tang et al. (2008).



Predictions of heating-cooling tests



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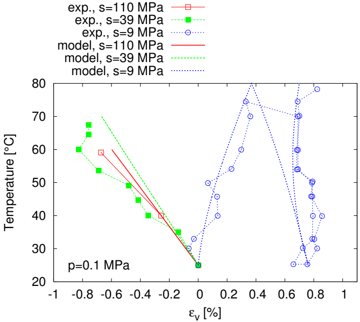
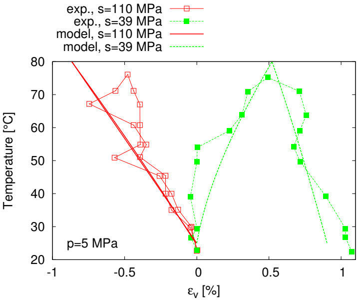
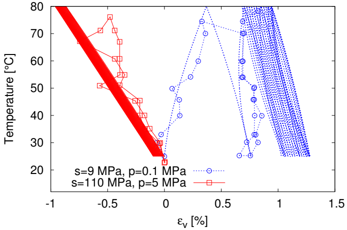


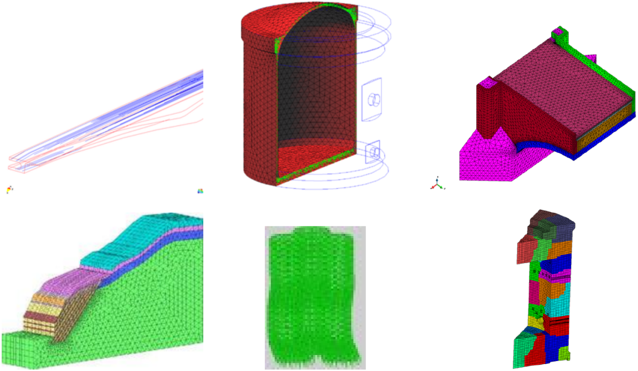




Cyclic heating-cooling test: no ratchetting problem as in standard hypoplasticity.

Coupled THM finite element simulations

- The model has been implemented into coupled THM 3D finite element code *SIFEL* of *Czech Technical University* (Prof. Kruis).



- Now planned to be used in simulations of *Mock-up experiments* of *nuclear waste repositories* (project underway).

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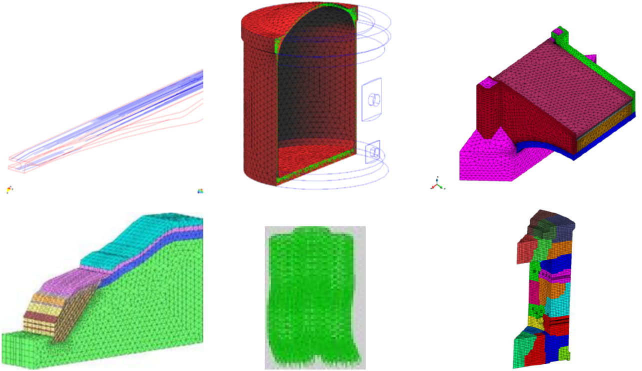

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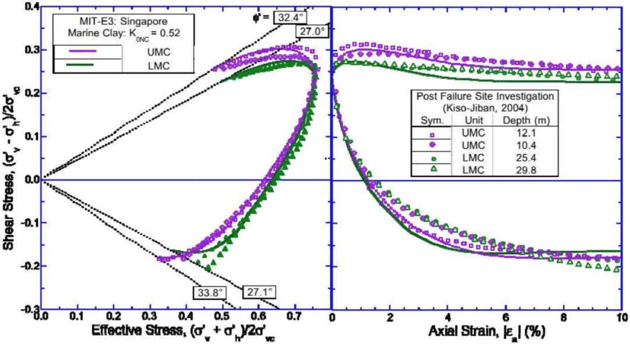
Outline

ASBS anisotropy – soft clay modelling

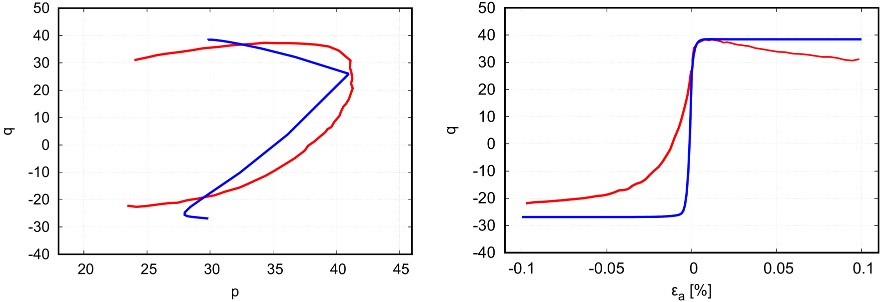
- Normally consolidated soft clays show a pronounced *strength anisotropy*.



Data on Singapore marine clay by Corral and Whittle, 2010

ASBS anisotropy – soft clay modelling

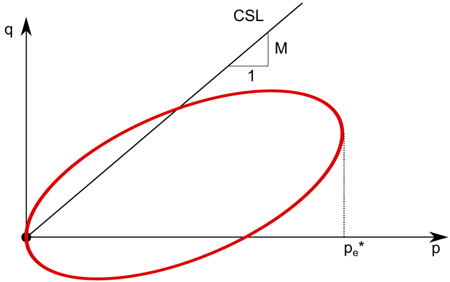
- The *strength anisotropy* cannot be modelled with an isotropic shape of the state boundary surface.



Predictions of the soft clay behaviour with the basic clay model, Jerman 2016

ASBS anisotropy – soft clay modelling

- In *elasto-plastic models* that this behaviour is modelled by *rotation of the state boundary surface*.



- In hypoplasticity, *rotation of the ASBS* was not possible without the explicit formulation.

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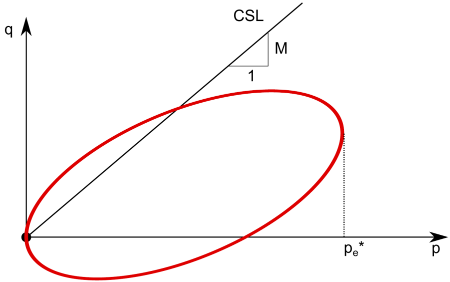

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ASBS anisotropy – soft clay modelling

- *Hypoplastic model with anisotropic SBS* – PhD research by Jan Jerman, see also *poster session*
- Approach by Gajo and Wood (2001), in which "*skewed*" stress coordinates are adopted for ASBS calculation (equations are unmodified), *flow rule* is calculated using the original stress coordinates.

$$\sigma_{skew} = \sigma + \frac{1}{3}\mathrm{tr}(\sigma)\beta$$

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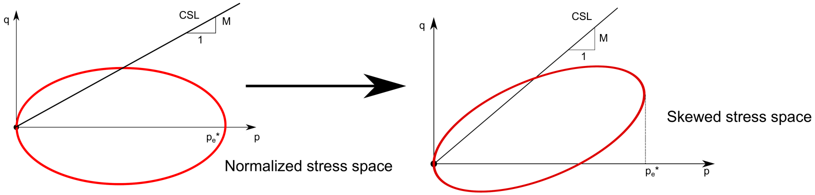
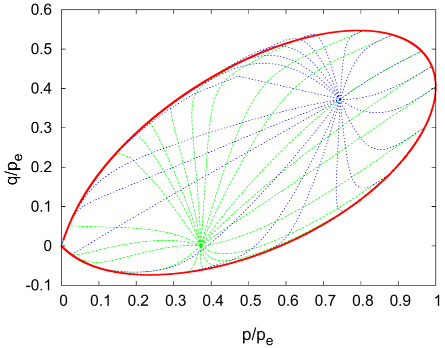


ASBS anisotropy – soft clay modelling

- Modified model with *rotated ASBS*.
- Improves predictions of *strength*, still incorrect *stiffness* – project underway



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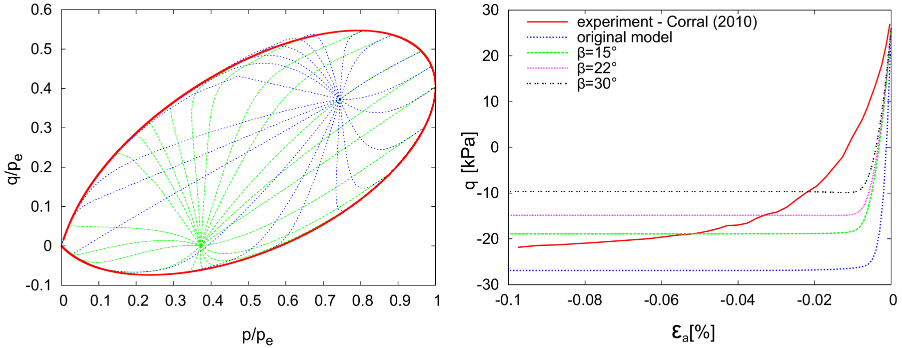
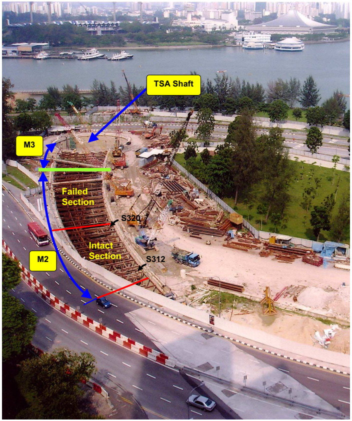
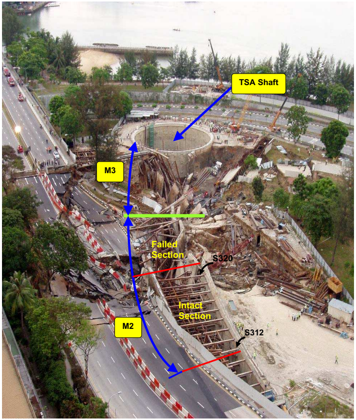


ASBS anisotropy – soft clay modelling

- Final goal: application in simulation of boundary value problems



Overview of Affected Project Site Prior to Accident



Overview of Affected Project Site After the Accident

Outline

1. Explicit incorporation of asymptotic states into hypoplasticity

2. Recent developments and applications of the framework
   - Incorporation of stiffness anisotropy
   - Variable ASBS shape in spudcan simulations
   - Coupled Thermo-Hydro-Mechanical double structure model for partially saturated expansive clays
   - ASBS anisotropy – soft clay modelling
   - Modelling of peat

Modelling of peat

- *Modelling of peat* – PhD research by Stefano Muraro, TU Delft, under supervision of Cristina Jommi.

Modelling of peat

- *Peat* is an unusual material, extremely high friction angle $\varphi_c = 64°$, high void ratios $e \approx 8$, low stresses relevant ($\approx$ 10 kPa).
- It comes as no surprise that models developed for *clays* ($\varphi_c = 22°$, high void ratios $e \approx 0.8$, low stresses relevant ($\approx$ 100 kPa) are not perfectly fitting experiments.
- Explicit formulation of hypoplasticity allows us to tweak the model

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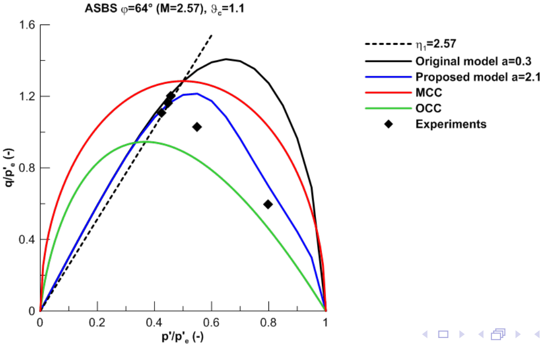
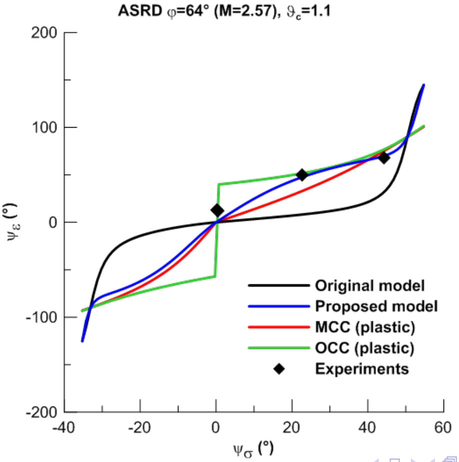


Modelling of peat

- *Directions of asymptotic strain increment* for the given *stress path direction*



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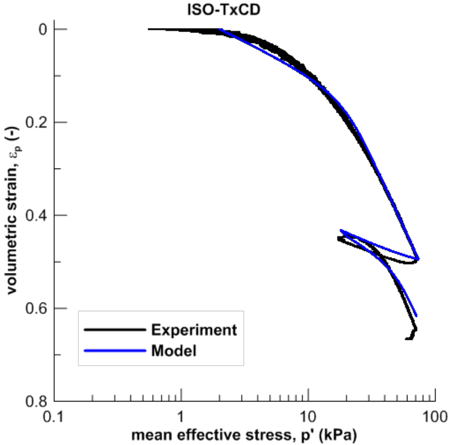
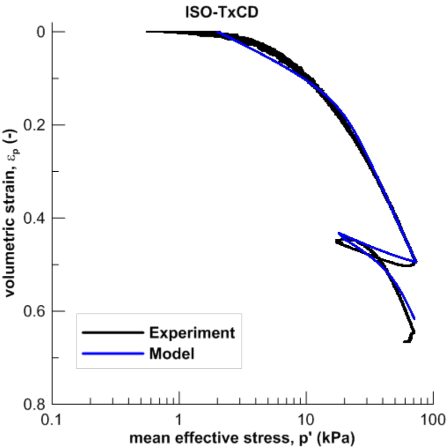


Isotropic compression

Modelling of peat



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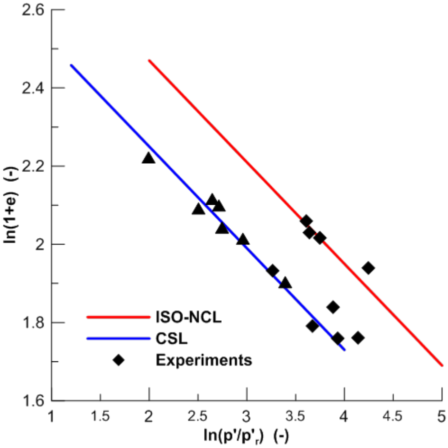
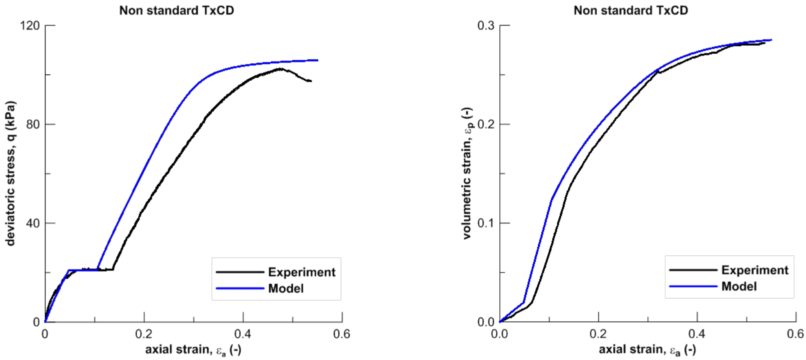


Critical states and isotropic NCL

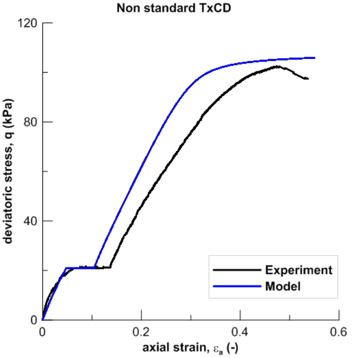
Modelling of peat

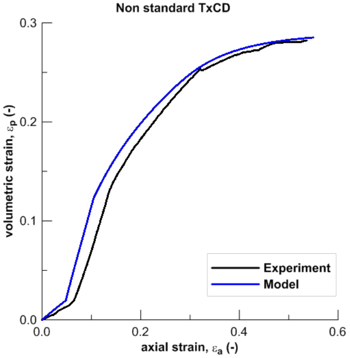


p-constant – q-constant – p-constant test

- Modelling of peats: *rate effects* to be included: project underway

Modelling of peat





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Summary

- A method presented to incorporate *arbitrary pre-defined* asymptotic state boundary into *hypoplasticity*.
- The method enables further *development of hypoplastic models*, with *various engineering applications*.