Micromechanical Aspects of Natural Formation Process of Geological Grains

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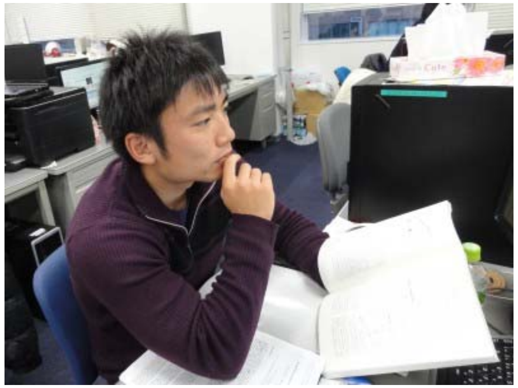


IV International workshop
Modern Trends in Geomechanics
Assisi, May 16 -18 2016

The 4th International Workshop on Modern Trends in Geomechanics,
Assisi, May 16, 2016

Introduction

Geomaterials are natural materials formed by long-time geological process.

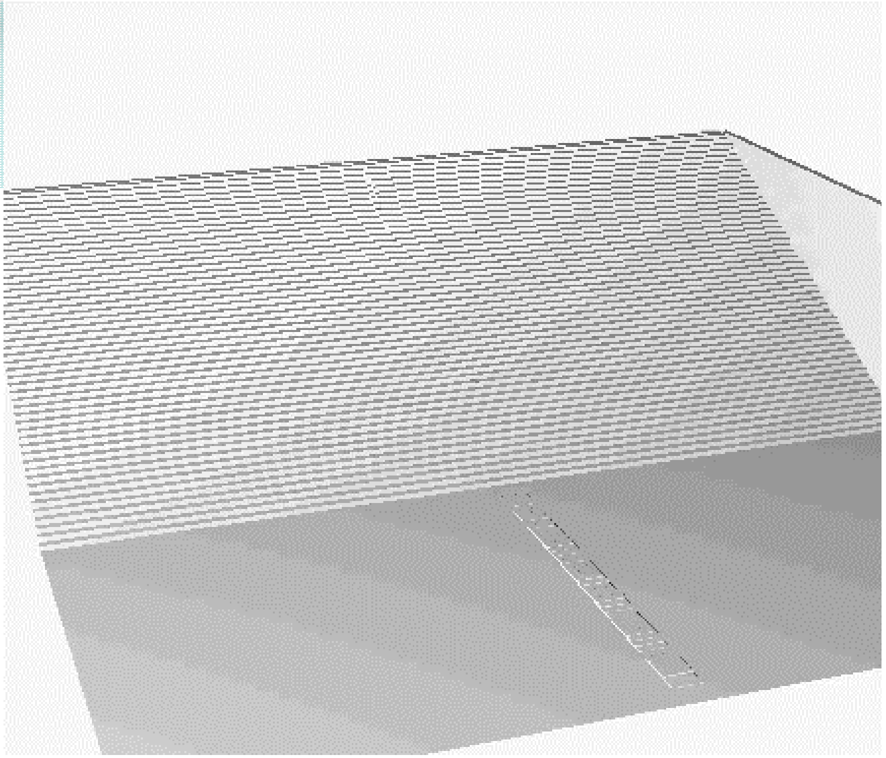
Their physical and mechanical properties differ in different places and change in time as well.

A big challenge in geomechanics is to develop a **simulation tool** for quantitatively evaluating such properties.

Erosion and Sedimentation

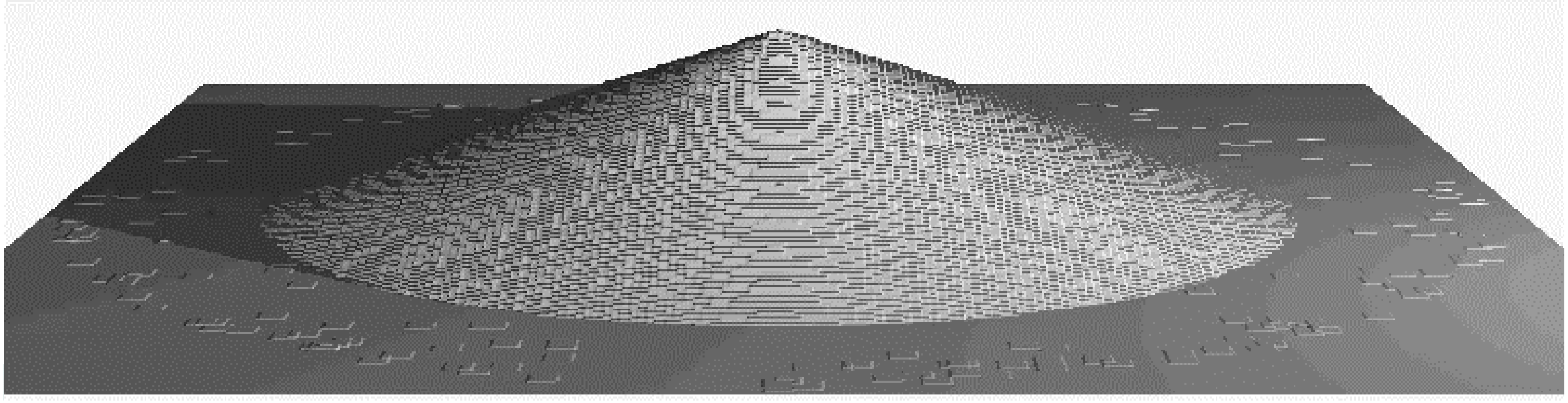
Geological process simulation
by depth-integrated particle method
considering grain segregation
(*Matsushima, JGS annual meeting, 2014,2015*)

The erosion/sedimentation model is based on the experimental observation, but evolution of GSD is not considered.

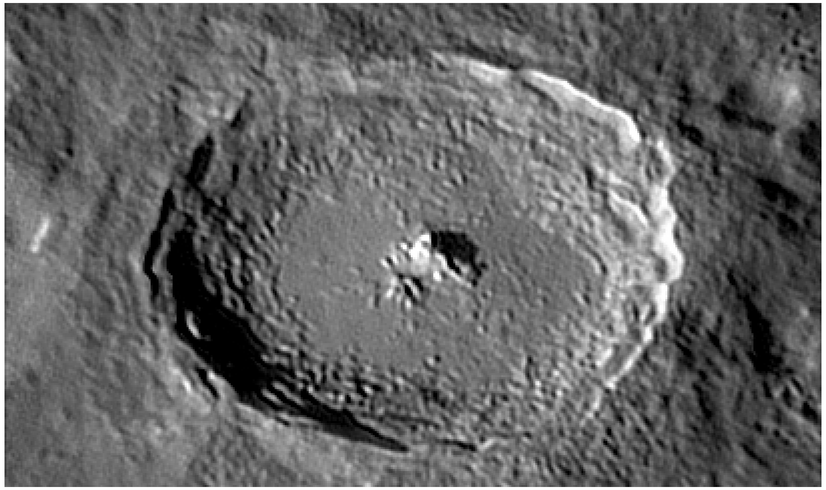


↑ Formation of alluvial fan

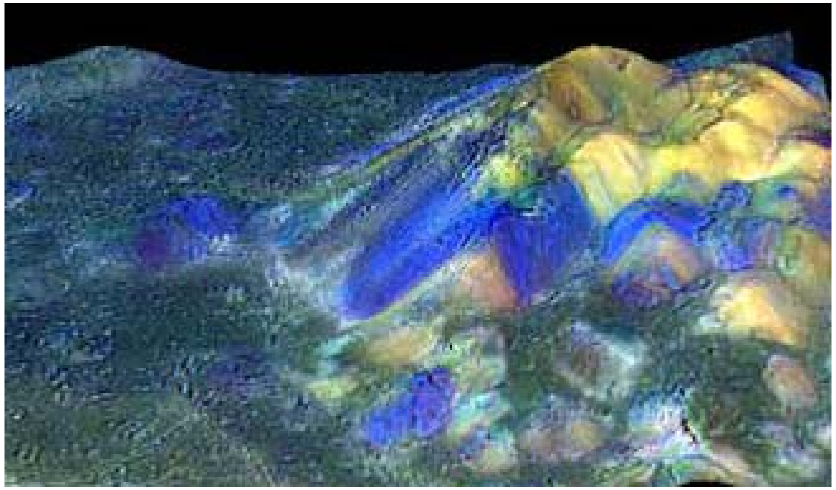
↓ Formation of erosion valley



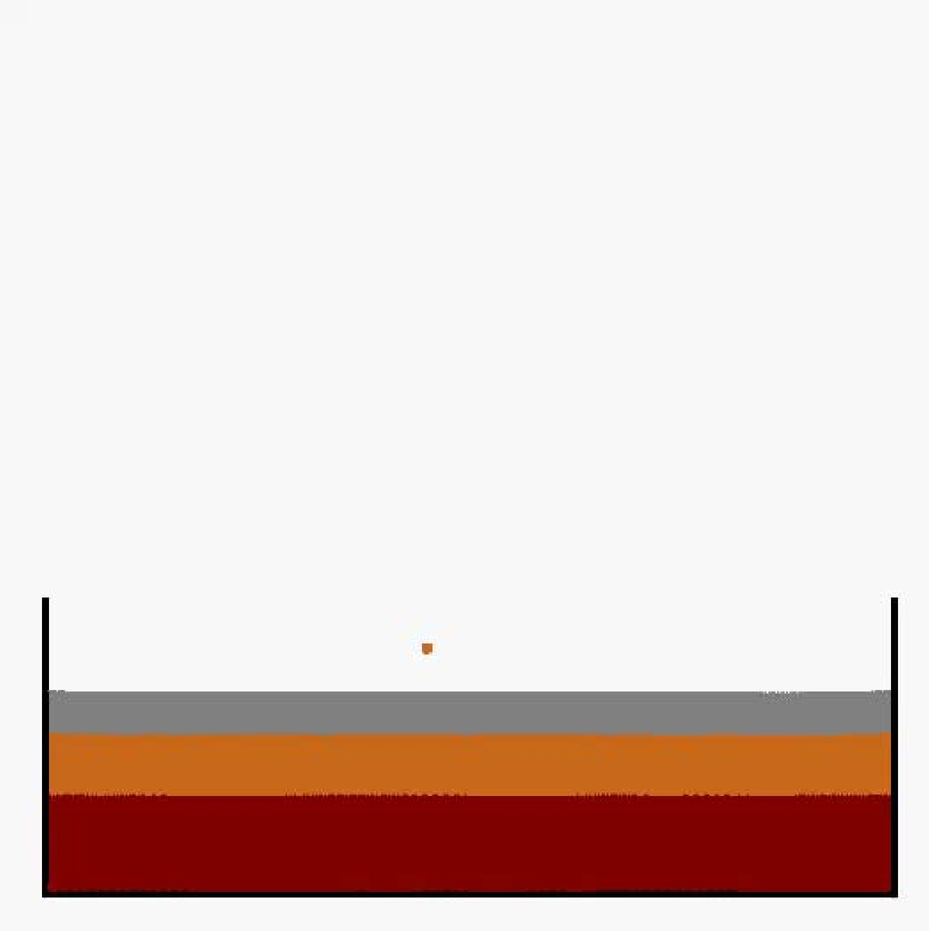
Impact cratering



Tycho Crater (D=85km)



Ohtake et al.: Nature, 461,10,2009



Formation of central peak by SPH (*unpublished*)

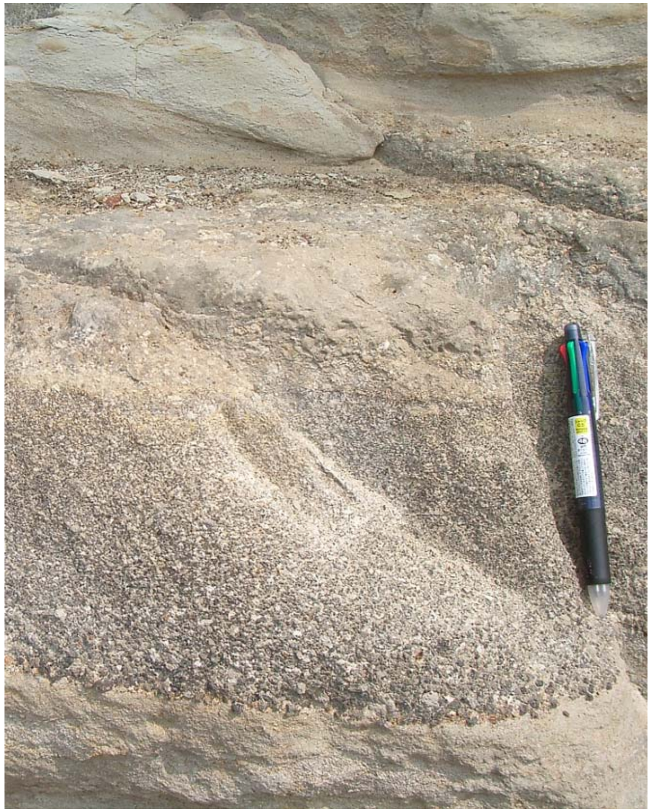
CE: Drucker-Prager EP model

Importance of grain scale processes

On solid planetary surfaces, geomaterials are composed of a lot of solid geological particles of various sizes and shapes.

Such particles were formed and evolved either by crushing , agglomeration or solidification process under a certain natural environment.

These processes greatly affect the bulk mechanical properties of the layers



grain segregation
in geological layers

Grant-in-Aid for Scientific Research (B) 2014-16

“Micromechanics of compression, shear and solidification of geo-materials under high pressure”

Takashi Matsushima (PI)
Takahiro Hatano (University of Tokyo)
Keiko Watanabe (University of Ritsumeikan)
Masuhiro Beppu (National Defense Academy)
Hiroko Kitajima (Texas A&M University)

Various experiments with common geo-materials

Materials used

Gifu sand

mountain sand

90.2% SiO_2

D_{mean}=2.38mm

angular

Kashima sand

river sand

96.3% SiO_2

D_{mean}=2.08mm

round

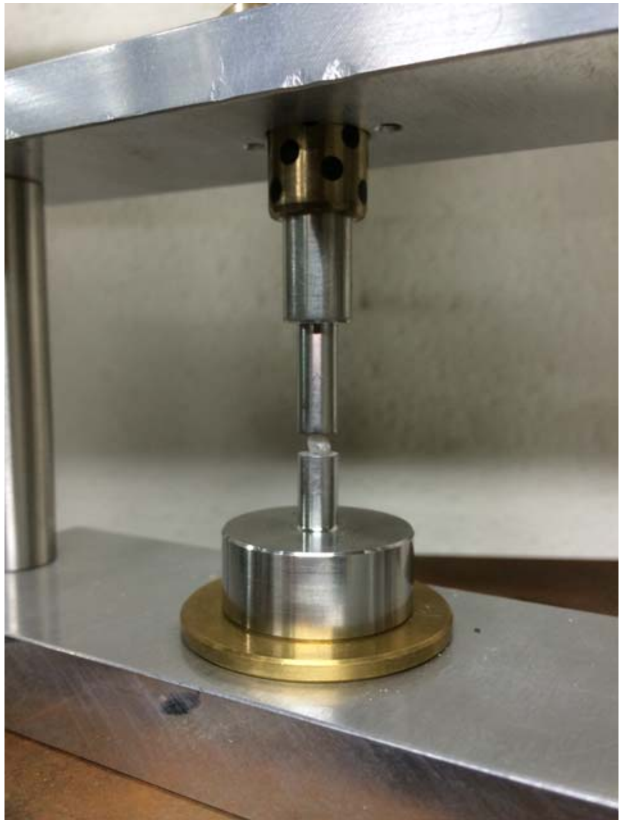
Experimental program

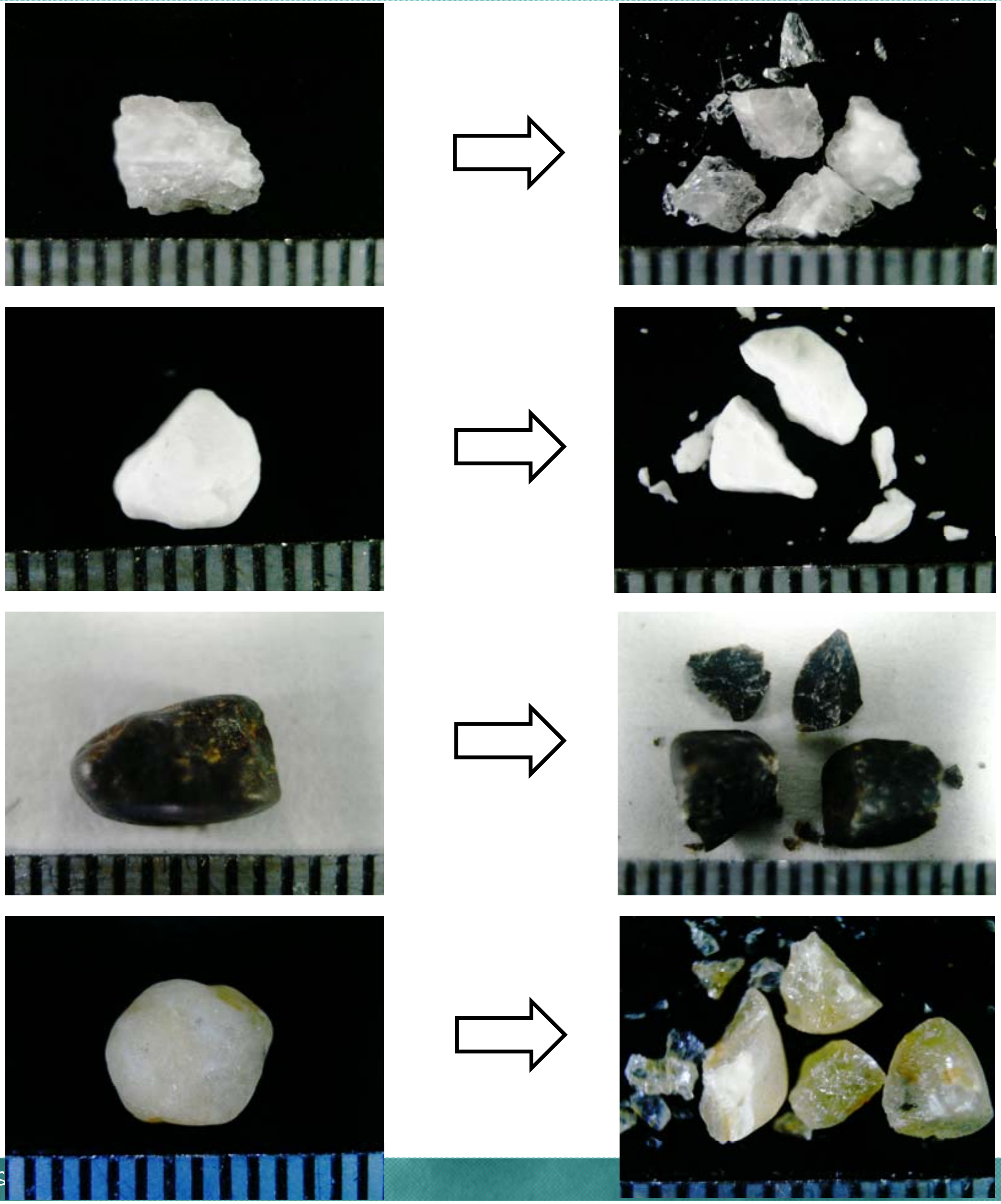
today's talk

[1] Single grain crushing test	Sato
[2] One-dimensional compression (ODC) test	Sato
[3] Rotary shear (RS) test	Kitajima, Sato
[4] High-speed projectile impact test	Watanabe
[5] Explosion test	Beppu

[1] Single grain crushing test

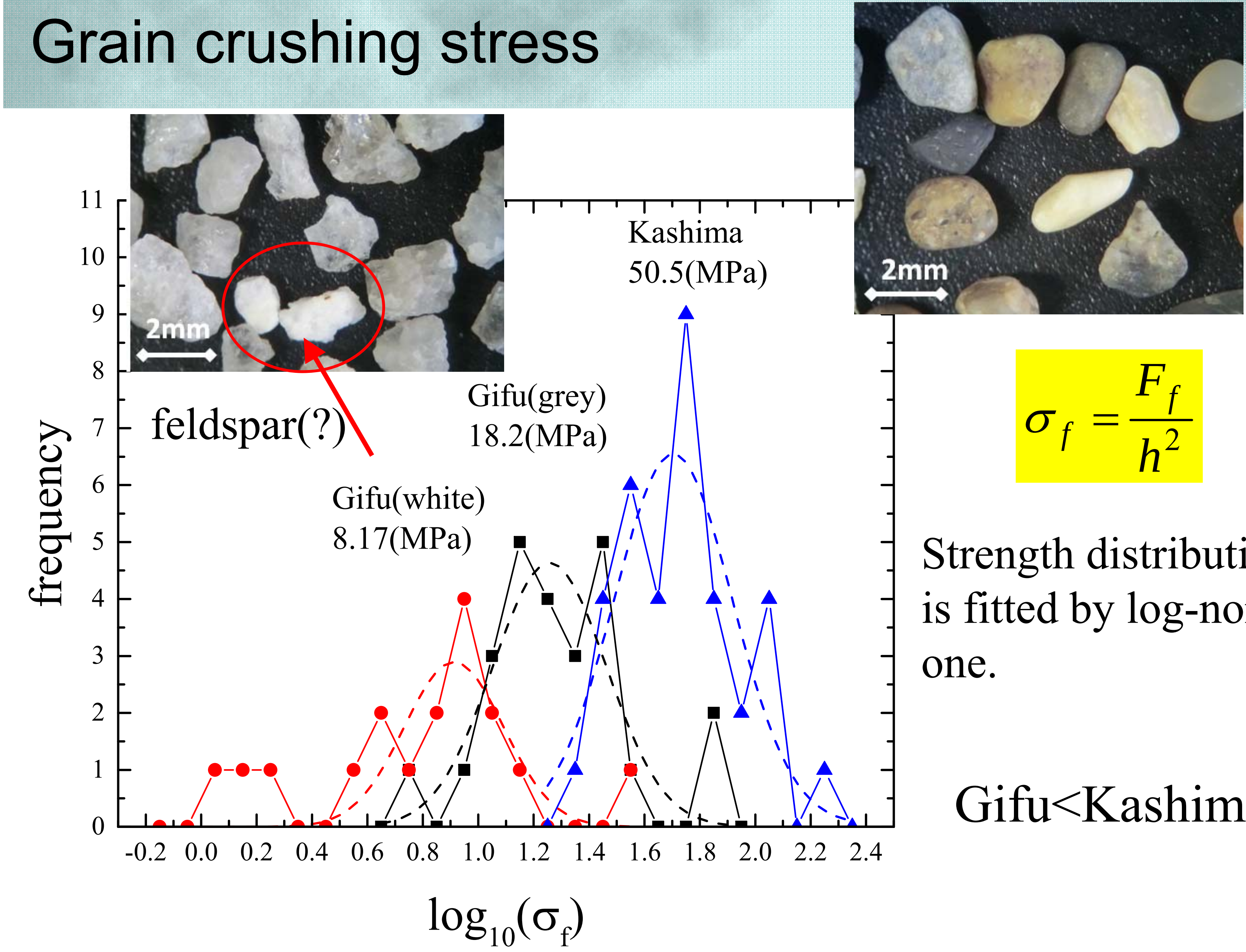
[1] Single grain crushing test





Measure single grain crushing stress

Grain crushing stress



$$\sigma_f = \frac{F_f}{h^2}$$

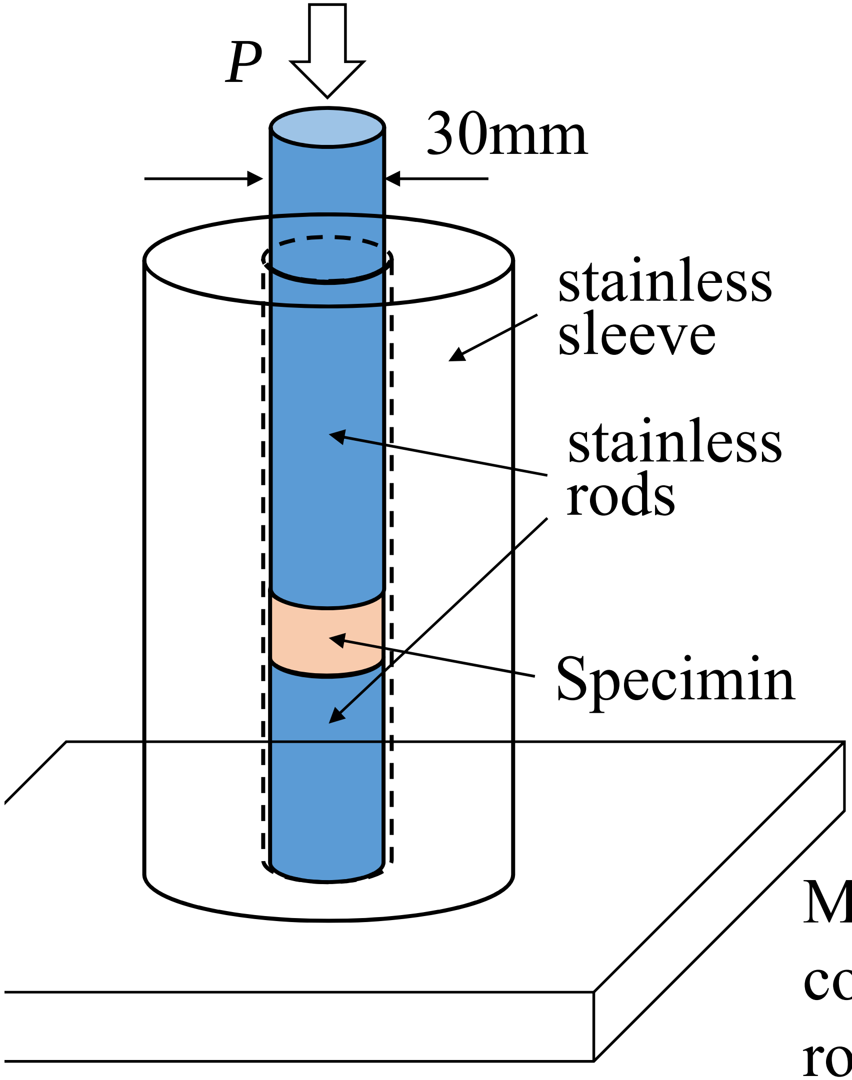
Strength distribution is fitted by log-normal one.

Gifu<Kashima

[2] One-dimensional compression test

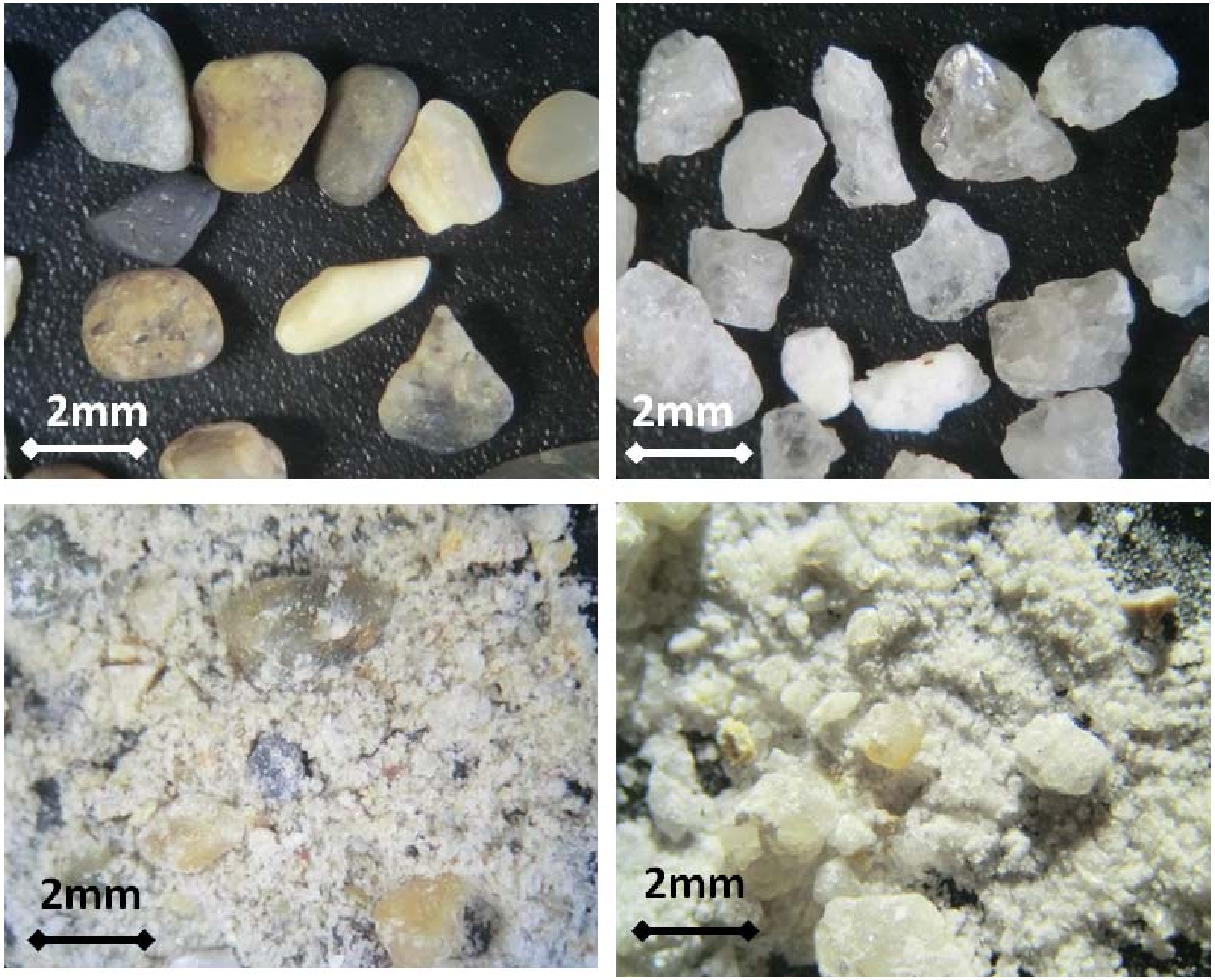
[2] One-dimensional compression test

Sato et al. KKHTCNN, 2015



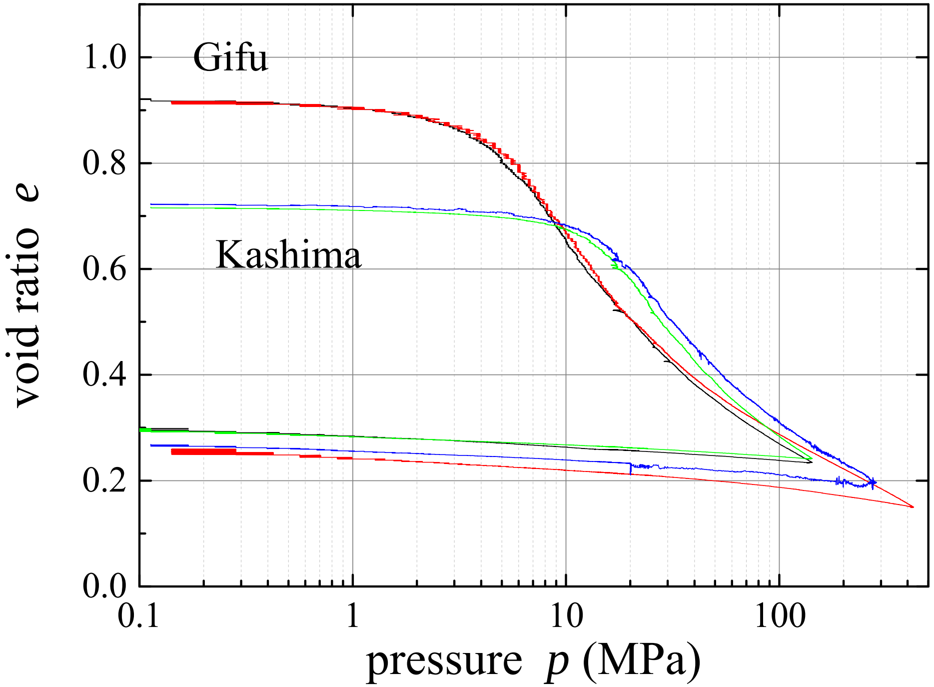
Measured volumetric strain is modified considering the deformation of stainless rods, sleeve and road cell.

[2] One-dimensional compression test

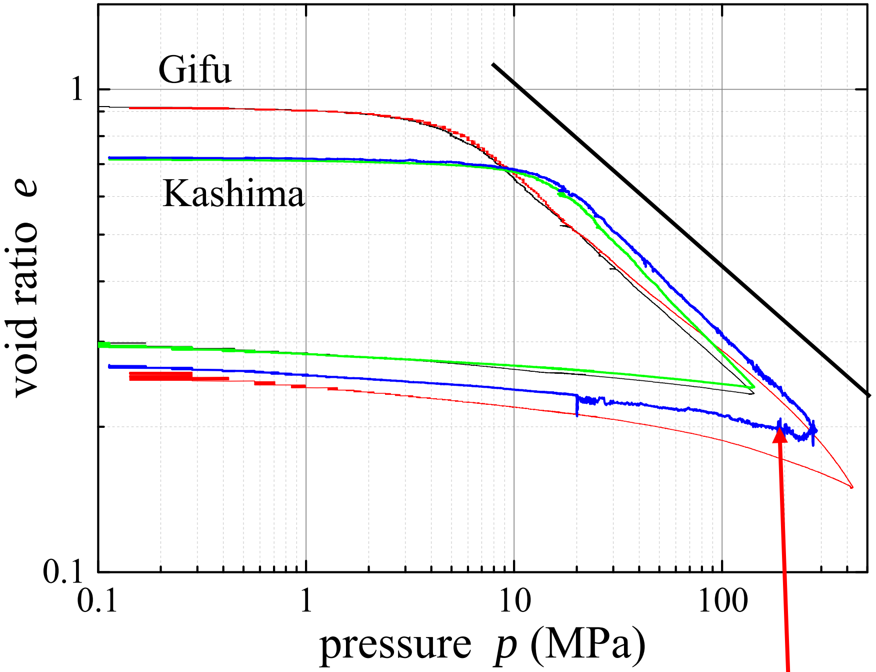


After 400(MPa) loading

Void ratio – Pressure relation



Conventional e-log(p)

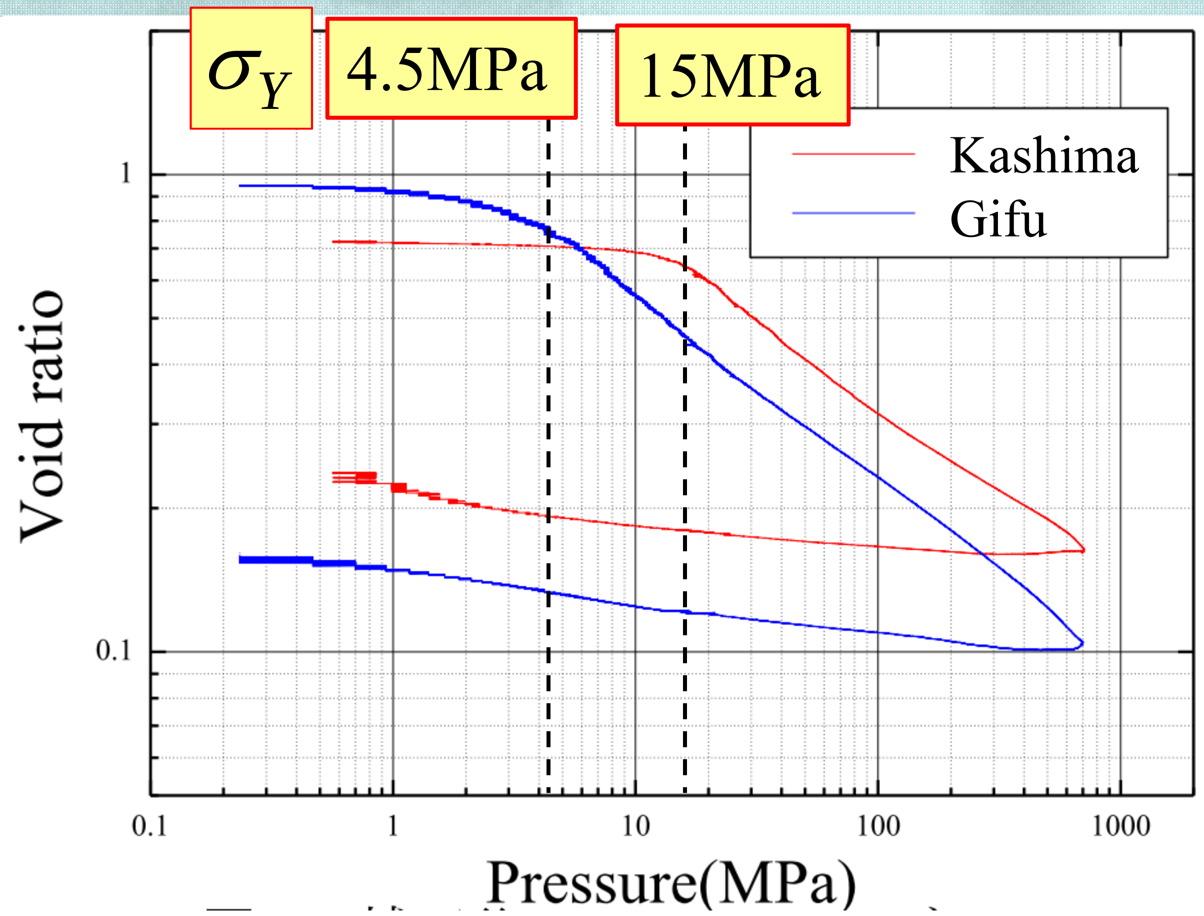


log(e)-log(p)

Plastic compression regime
is well modeled by a straight line

Butterfield, Geotechnique, 1979

log (e) – log (p) relation



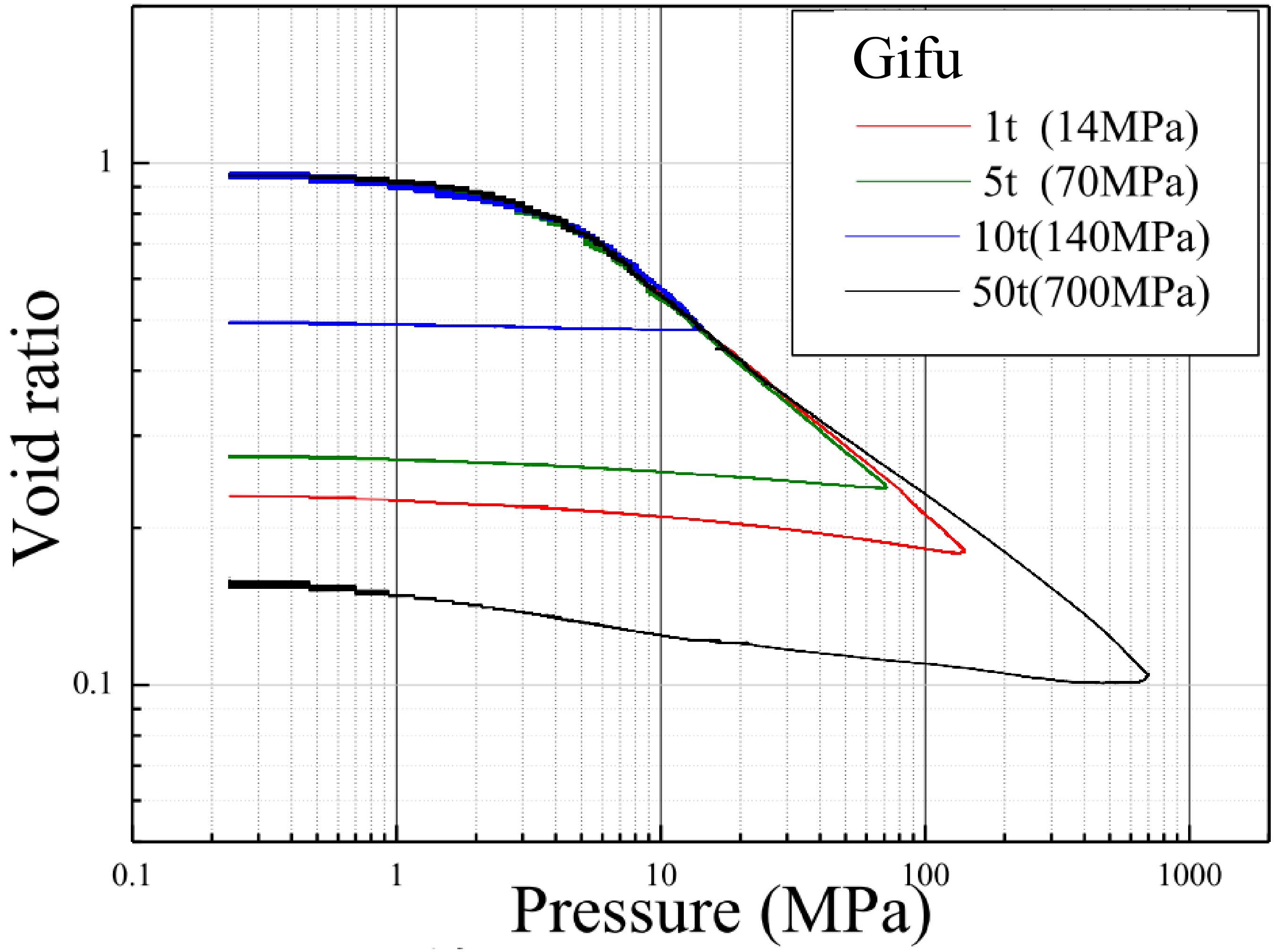
σ_Y 4.5MPa 15MPa

* σ_Y ratio (15MPa : 4.5MPa) ~ σ_f ratio (50MPa : 18MPa)

* The difference of the value itself must be due to heterogeneous stress transmission (force chain).

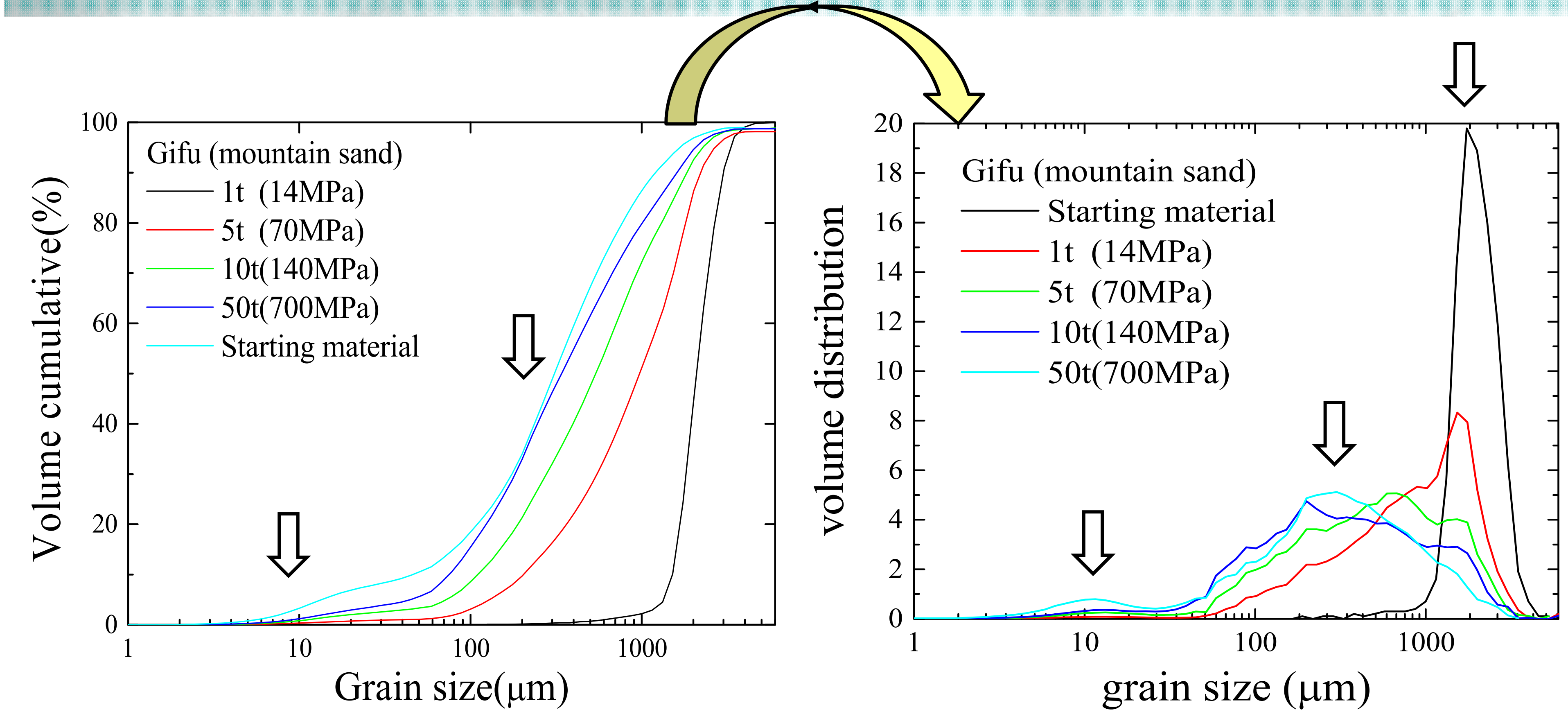
*Both have the similar power in plastic compression regime

log (e) – log (p) relation



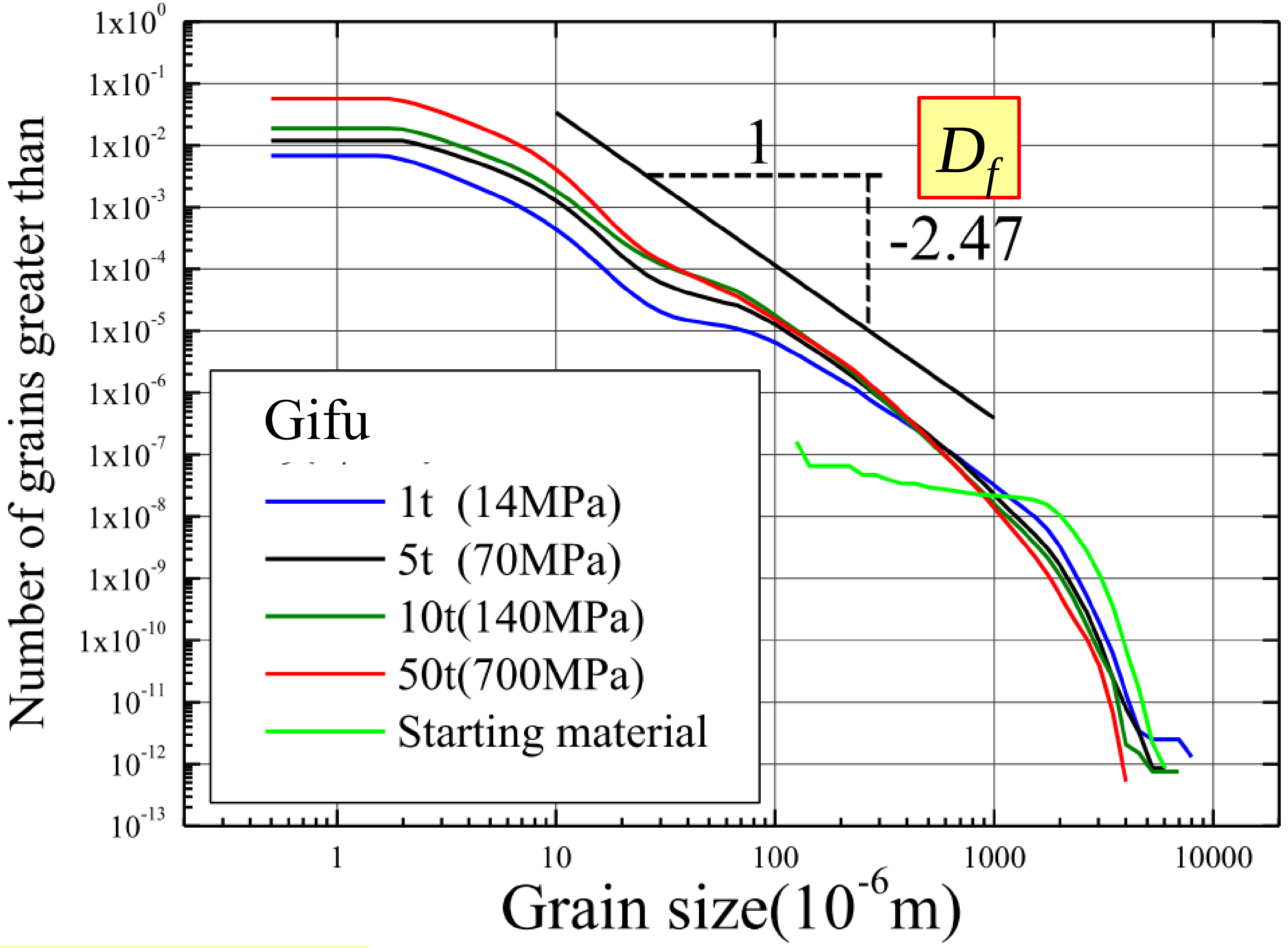
Various loading level → Evolution of grain size distribution

Evolution of grain size distribution



Steady peaks appear during crushing process

Grain size distribution (cumulative number)



$$P(d) = A \cdot d^{-D_f}$$

Fractal (or power law) distribution

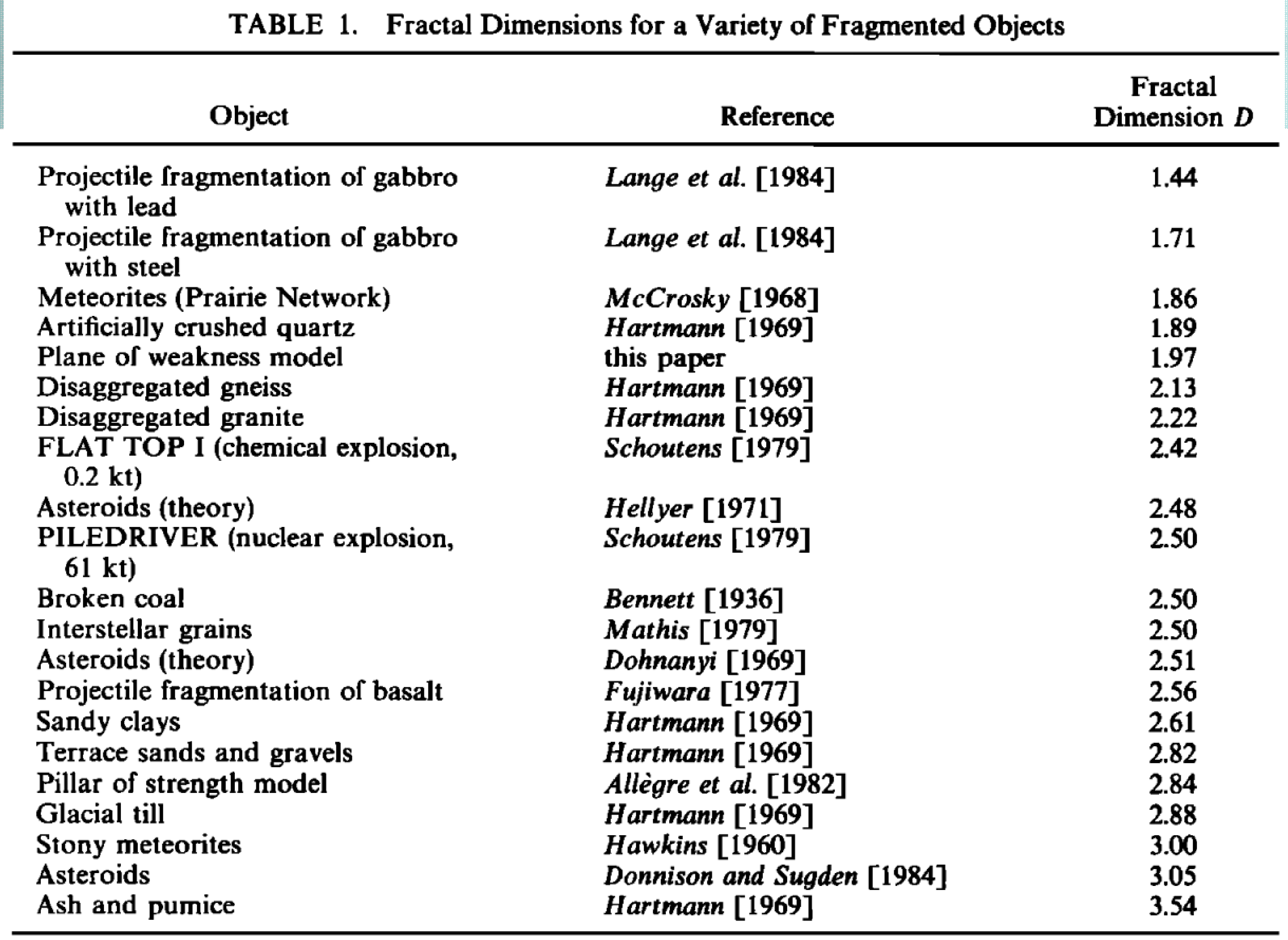


TABLE 1. Fractal Dimensions for a Variety of Fragmented Objects

Object	Reference	Fractal Dimension *D*
Projectile fragmentation of gabbro with lead	*Lange et al.* [1984]	1.44
Projectile fragmentation of gabbro with steel	*Lange et al.* [1984]	1.71
Meteorites (Prairie Network)	*McCrosky* [1968]	1.86
Artificially crushed quartz	*Hartmann* [1969]	1.89
Plane of weakness model	this paper	1.97
Disaggregated gneiss	*Hartmann* [1969]	2.13
Disaggregated granite	*Hartmann* [1969]	2.22
FLAT TOP I (chemical explosion, 0.2 kt)	*Schoutens* [1979]	2.42
Asteroids (theory)	*Hellyer* [1971]	2.48
PILEDRIVER (nuclear explosion, 61 kt)	*Schoutens* [1979]	2.50
Broken coal	*Bennett* [1936]	2.50
Interstellar grains	*Mathis* [1979]	2.50
Asteroids (theory)	*Dohnanyi* [1969]	2.51
Projectile fragmentation of basalt	*Fujiwara* [1977]	2.56
Sandy clays	*Hartmann* [1969]	2.61
Terrace sands and gravels	*Hartmann* [1969]	2.82
Pillar of strength model	*Allègre et al.* [1982]	2.84
Glacial till	*Hartmann* [1969]	2.88
Stony meteorites	*Hawkins* [1960]	3.00
Asteroids	*Donnison and Sugden* [1984]	3.05
Ash and pumice	*Hartmann* [1969]	3.54

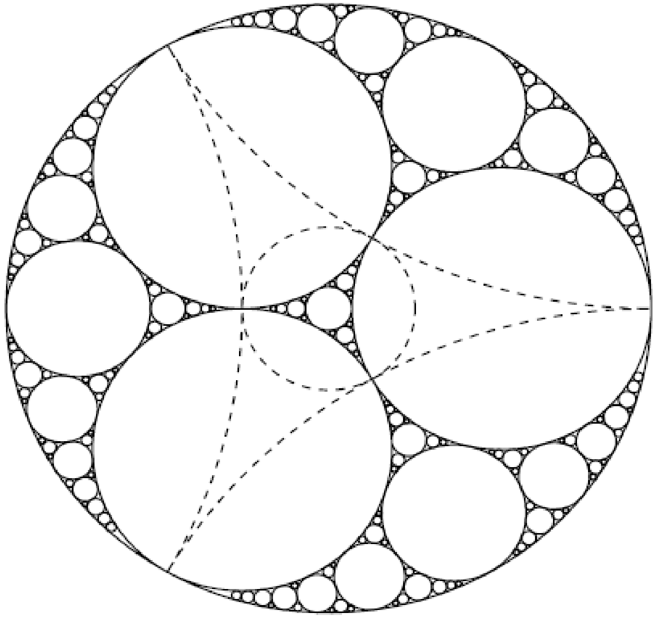
Fractal GSD (Turcotte 1986)

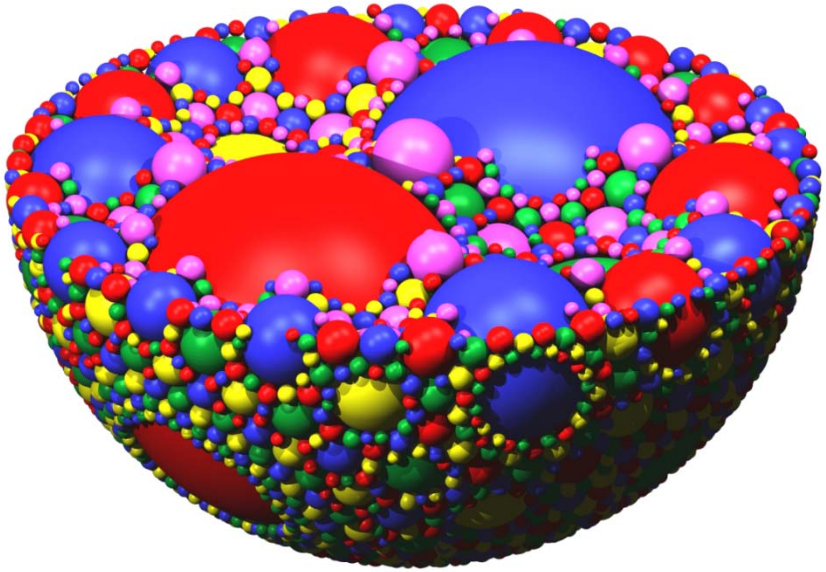
$D_f = 1.4 \sim 3.5$

Apollonian sphere packing

The observed power D is close to that in Apollonian sphere packing (D_f=-2.47)

(*Borkovec, M., De Paris, W., Peikert, R., Fractals, Vol. 2,No.4, 521–526, 1994.*)



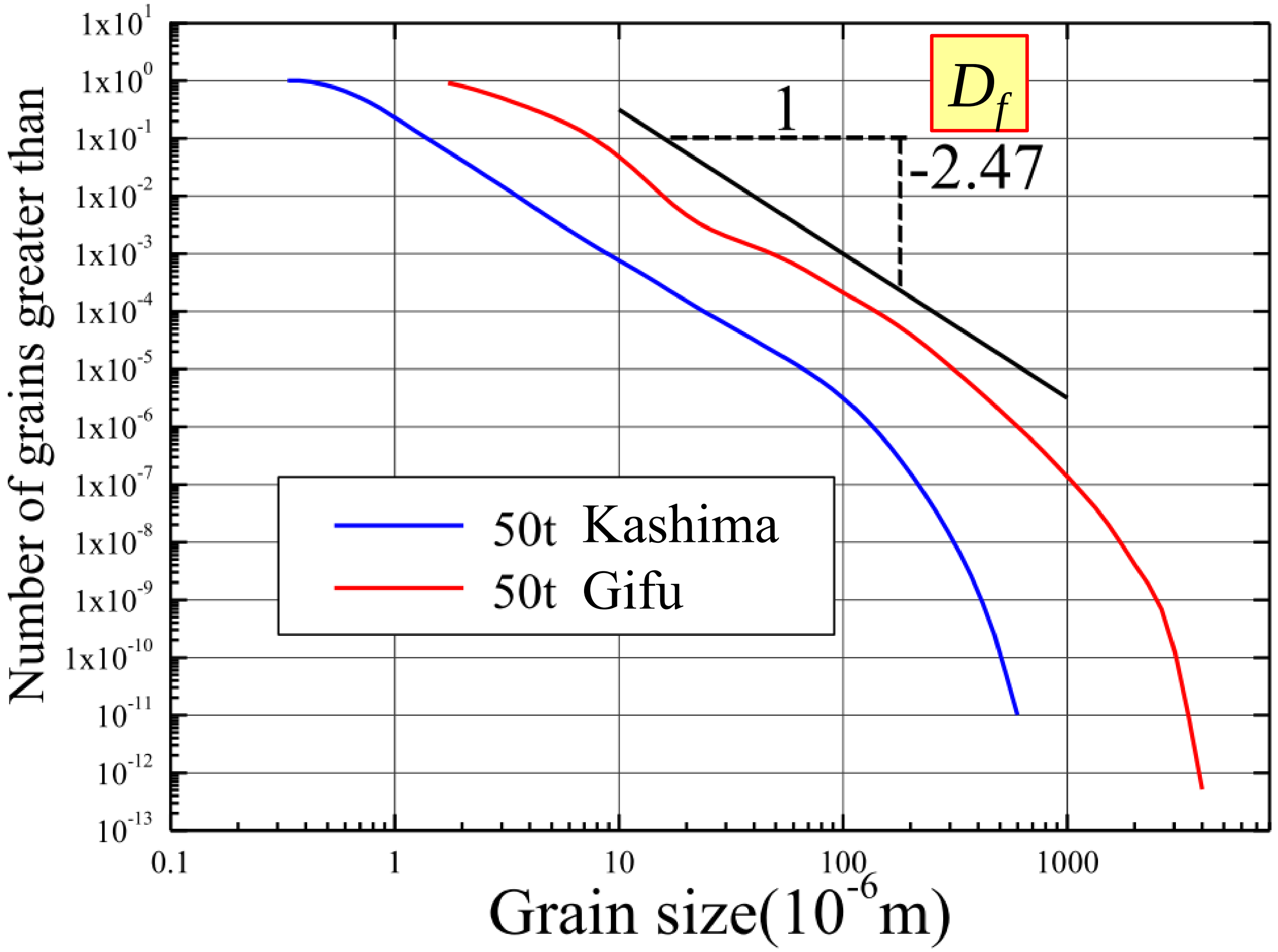


Possible comminution mechanism under confinement

→ The Largest particles fitting in the pore survives

→ Second and third peaks appear

Grain size distribution (cumulative number)



[3] Rotary shear test

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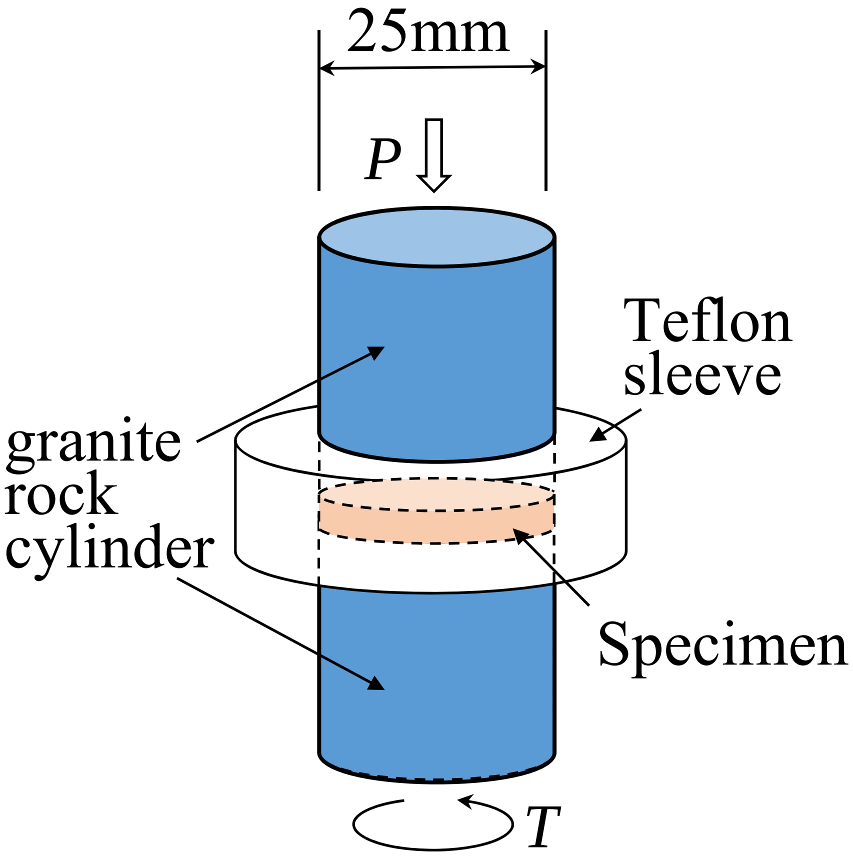
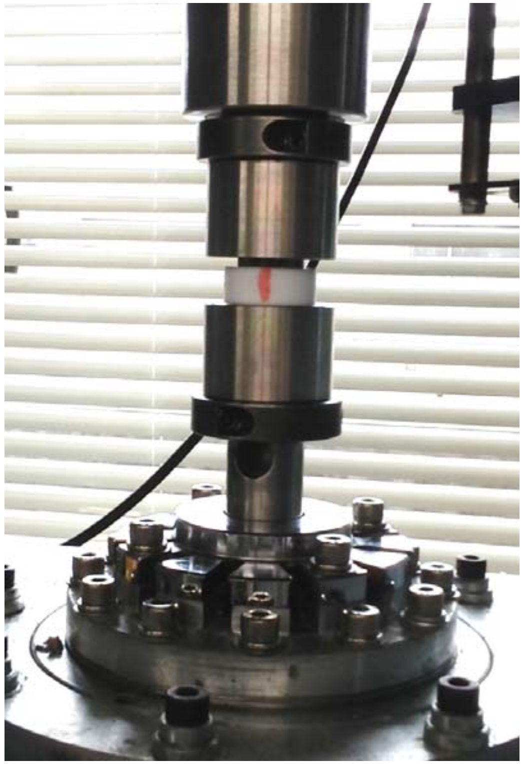
Sato et al. KKHTCNN, 2015

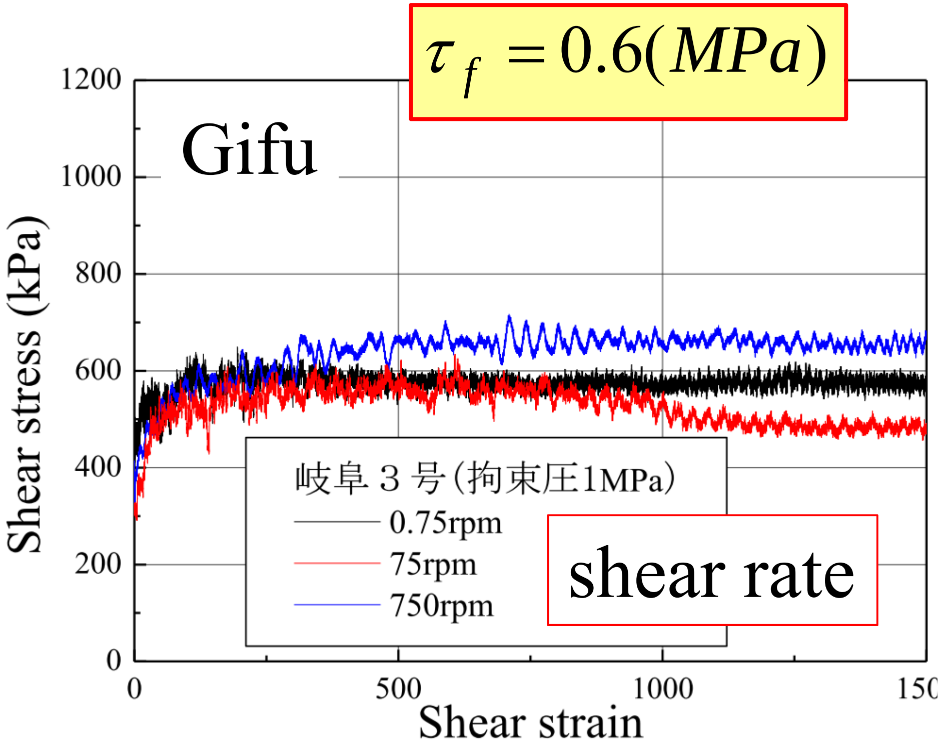


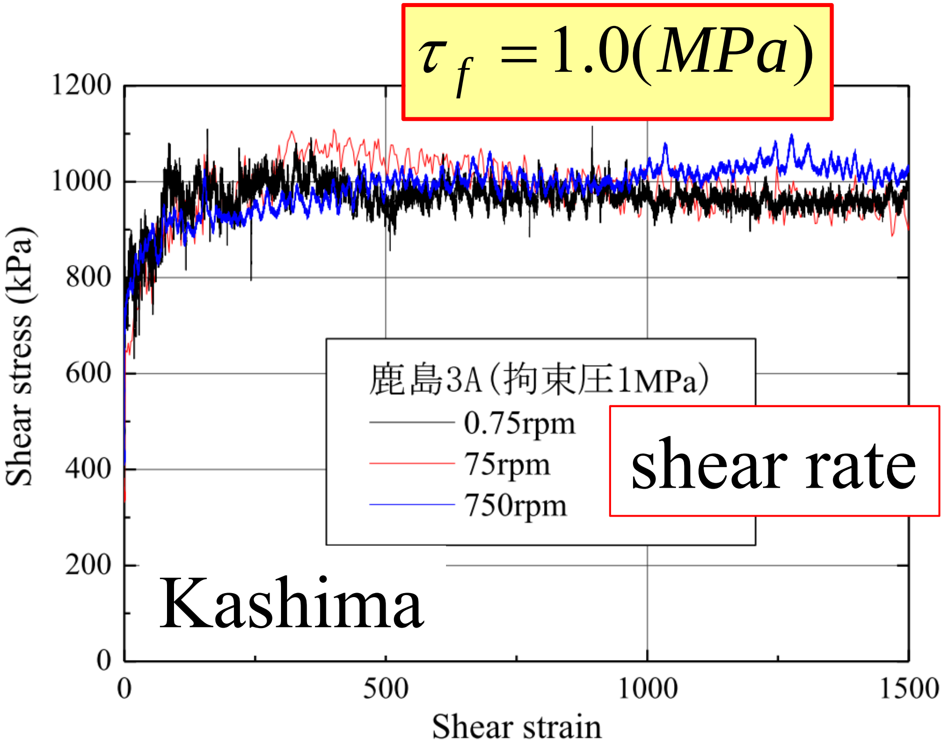


The device can apply wide range of shear rate

0.75(rpm):($\dot{\gamma}$=0.21(1/s)) ～ 750(rpm): ($\dot{\gamma}$=210(1/s))

Shear stress – shear strain relation

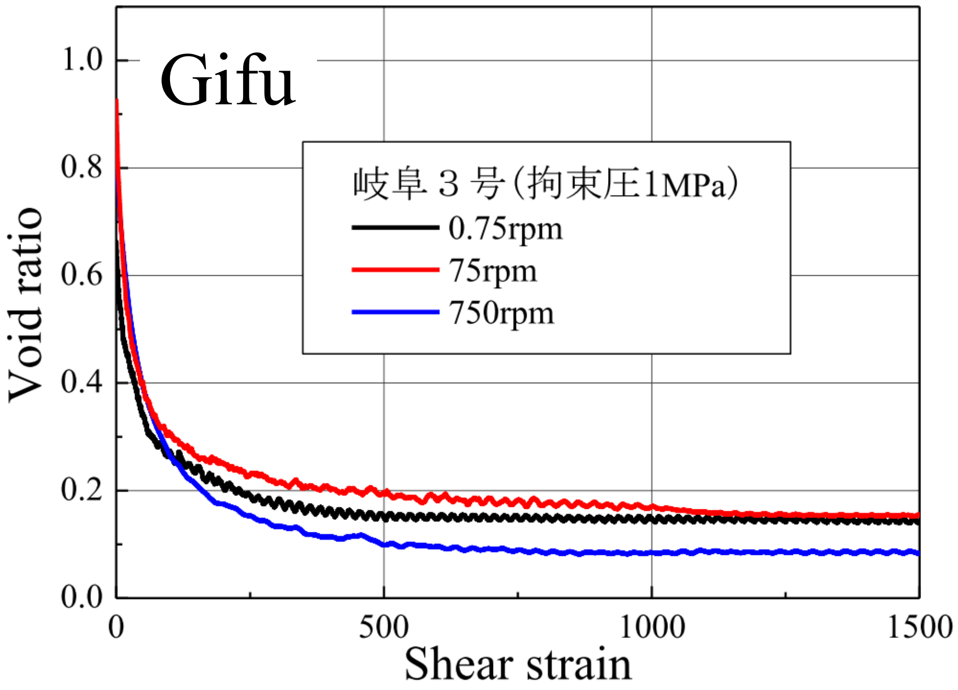


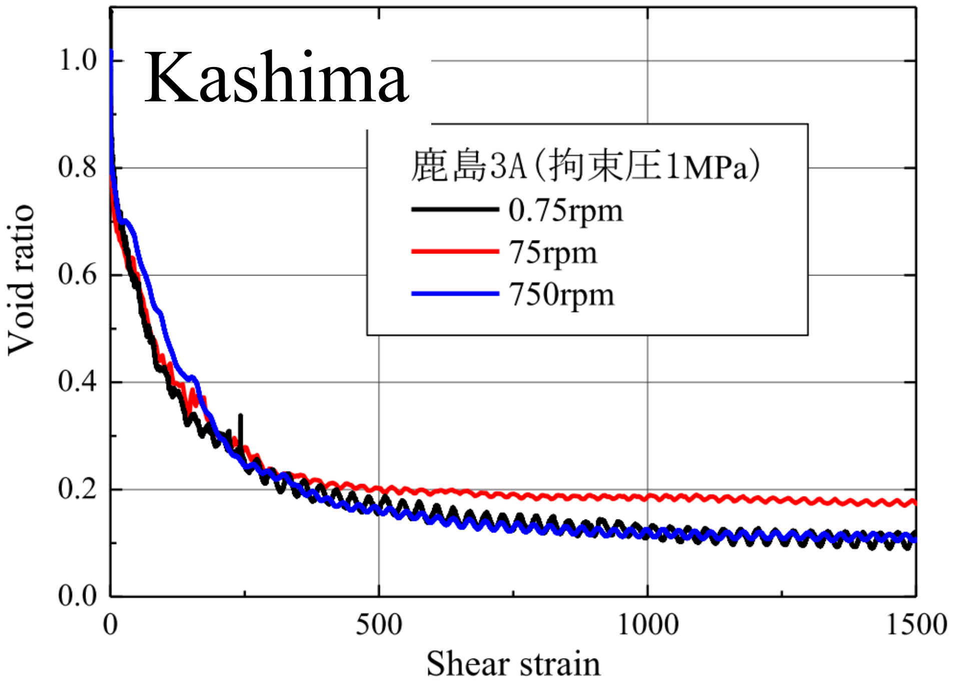


The ratio of peak shear stress is not consistent with the ratio of single grain crushing stress

→Effect of friction (shear without crushing) is included

Void ratio evolution

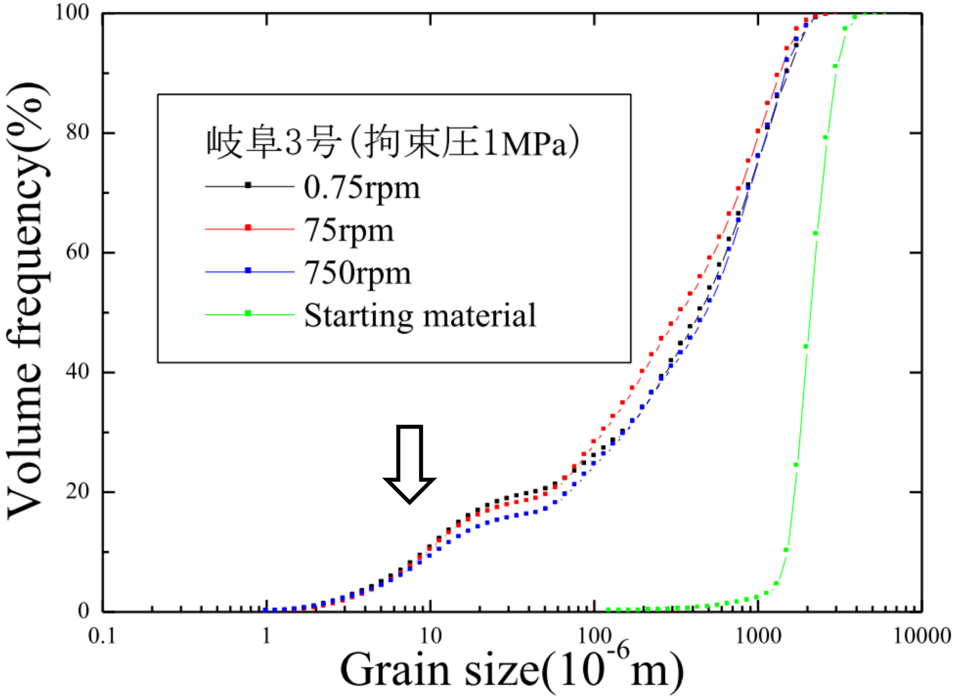




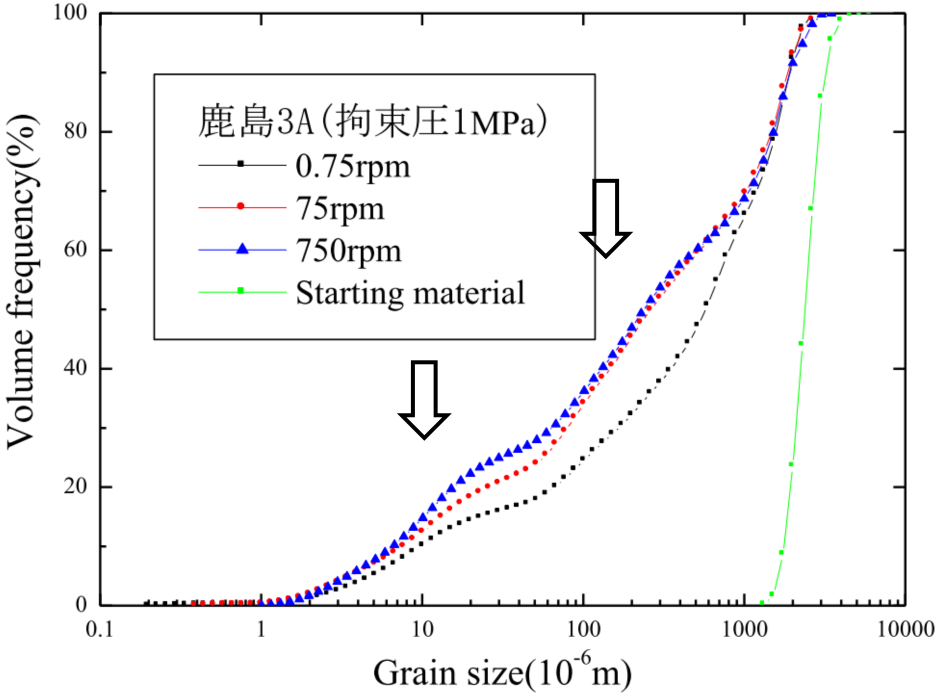
Void ratio reach the residual value (0.1～0.2)
after long shear (shear strain～500)
→Constant fabric change provides constant opportunity
of crushing (Force chain must play an important role)

Grain size distribution (cumulative volume)

Gifu

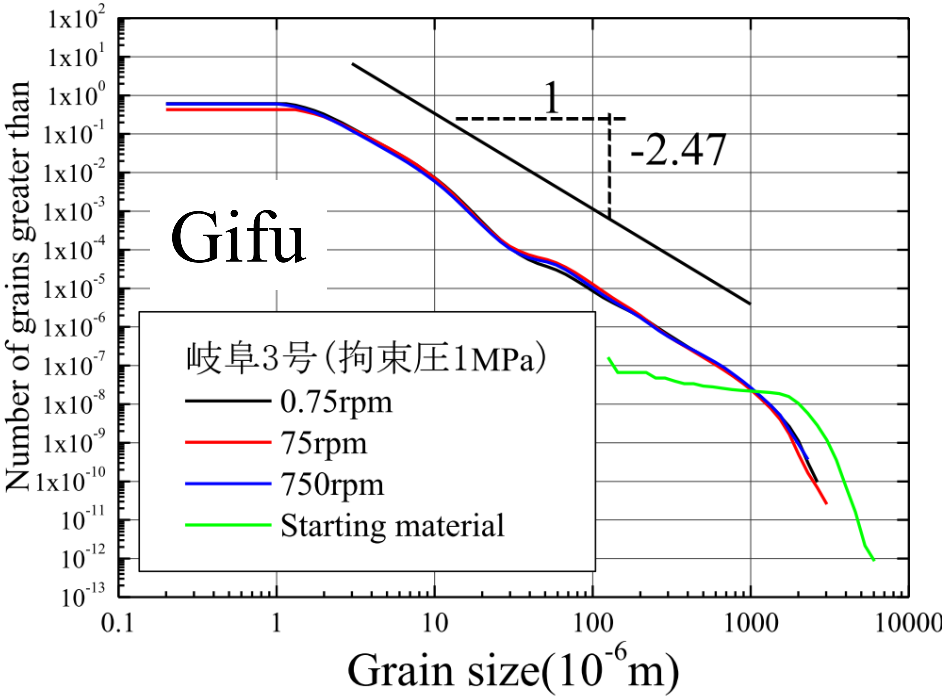


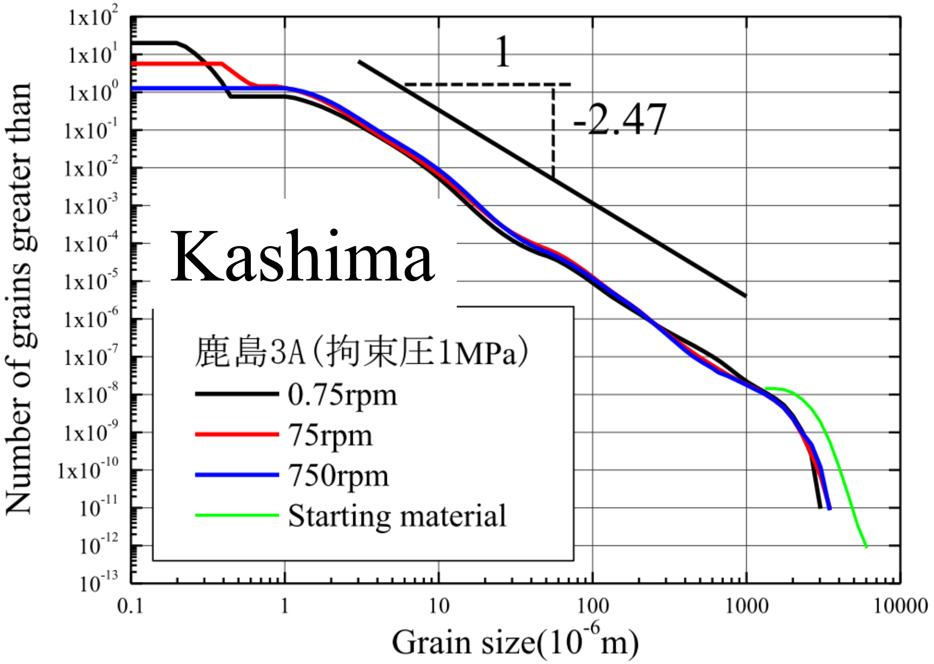
Kashima



The second and third peaks were observed.

Grain size distribution (cumulative number)





The power is similar to that in Apollonian sphere packing

[4] GSD evolution model

GSD evolution model

(1) Mono-disperse granular system (Radius=R)

$$e_0 = \frac{V_{void}}{V_{solid}}$$

(2) Fill the small grains (Radius=r) into the void
with the same void ratio

$$e_0 = \frac{V_{void} - V_{solid}^S}{V_{solid}^S}$$

(3) Number ratio of small grains to large grains

$$\frac{N_S}{N_L} = \frac{V_{Solid}^S}{V_{Solid}^L}\left(\frac{R}{r}\right)^3 = \frac{e_0}{1+e_0}\alpha^{-3}$$

$$\alpha = \frac{r}{R}$$

size ratio

GSD evolution model

$$\frac{N_S}{N_L} = \frac{V_{Solid}^S}{V_{Solid}^L}\left(\frac{R}{r}\right)^3 = \frac{e_0}{1+e_0}\alpha^{-3}$$

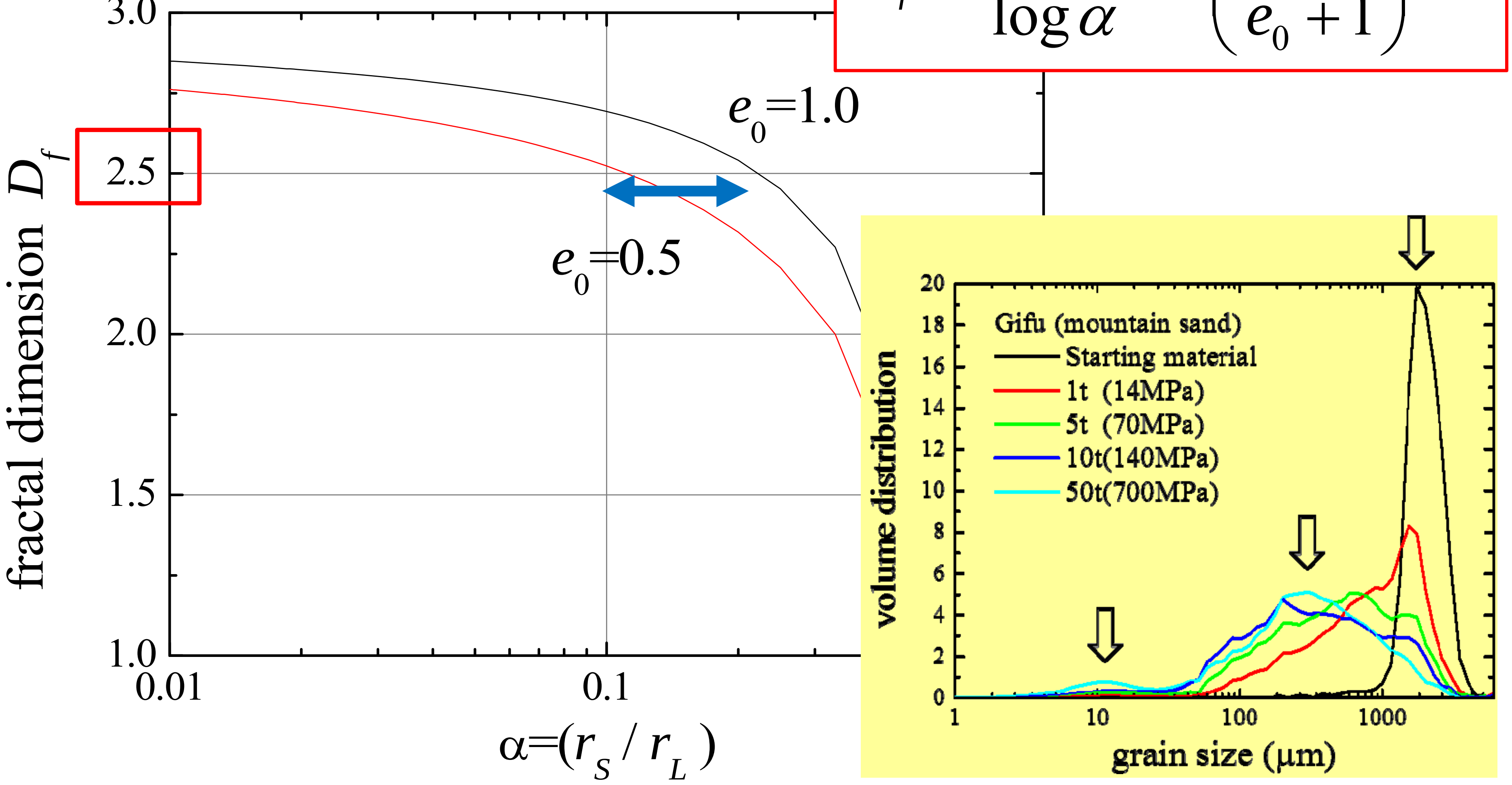
$$\log N_S - \log N_L = \log\left(\frac{e_0}{e_0+1}\right) - 3(\log r - \log R)$$

(4) Fractal dimension

$$D_f \equiv \frac{\log N_S - \log N_L}{\log r - \log R} = \frac{1}{\log \alpha}\log\left(\frac{e_0}{e_0+1}\right) - 3$$

GSD evolution model

$$D_f = \frac{1}{\log\alpha}\log\left(\frac{e_0}{e_0+1}\right) - 3$$



Consistent with Experimental observation on the second and the third peaks

GSD evolution model

(5) Void ratio after adding the small grains

$$e_1 = \frac{e_0}{\frac{e_0+1}{e_0}+1} \quad \Longrightarrow \quad 1+\frac{1}{e_k} = \left(1+\frac{1}{e_0}\right)^{k+1}$$

Recursive equation

(6) Breakage stress for the smallest grain determines the 1D compression stress (McDowell and Bolton 2008)

$$\sigma_0 = C \cdot r_{\min}{}^{-3/m}$$

m: Weibull modulus

$5<m<10$ in various materials

MODELLING

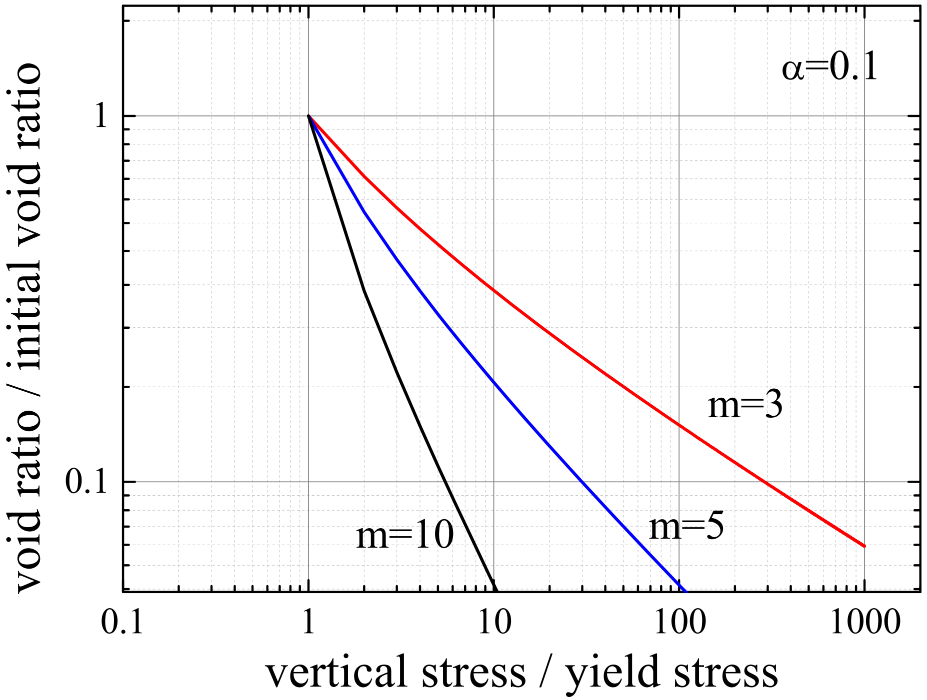
(7) Final relation between e and p

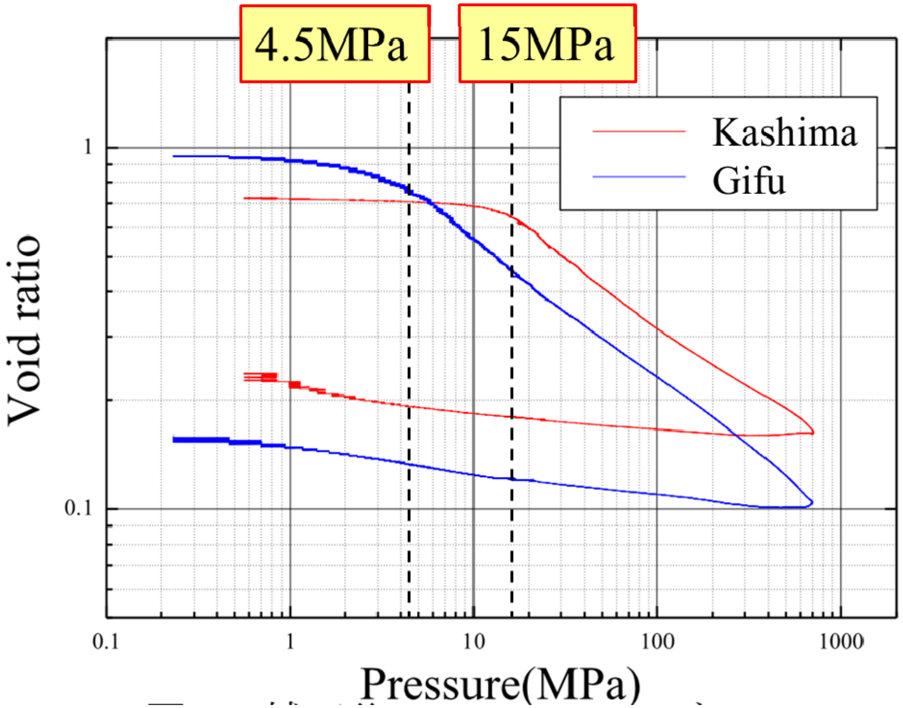
$$r_k = \alpha^k R \qquad \Rightarrow k = \frac{\log \frac{r_i}{R}}{\log \alpha}$$

$$\sigma_Y = C \cdot \left(\frac{r_k}{R}\right)^{-3/m} \qquad \Rightarrow -\frac{3}{m}\log\left(\frac{r_k}{R}\right) = \log C - \log \sigma_Y$$

$$\therefore \frac{\log(1+\frac{1}{\boxed{e_k}})}{\log(1+\frac{1}{e_0})} = k+1 = \frac{\log\frac{r_k}{R}}{\log\alpha}+1 = \frac{-\frac{m}{3}(\log C - \log \boxed{\sigma_Y})}{\log\alpha}+1$$

Validation



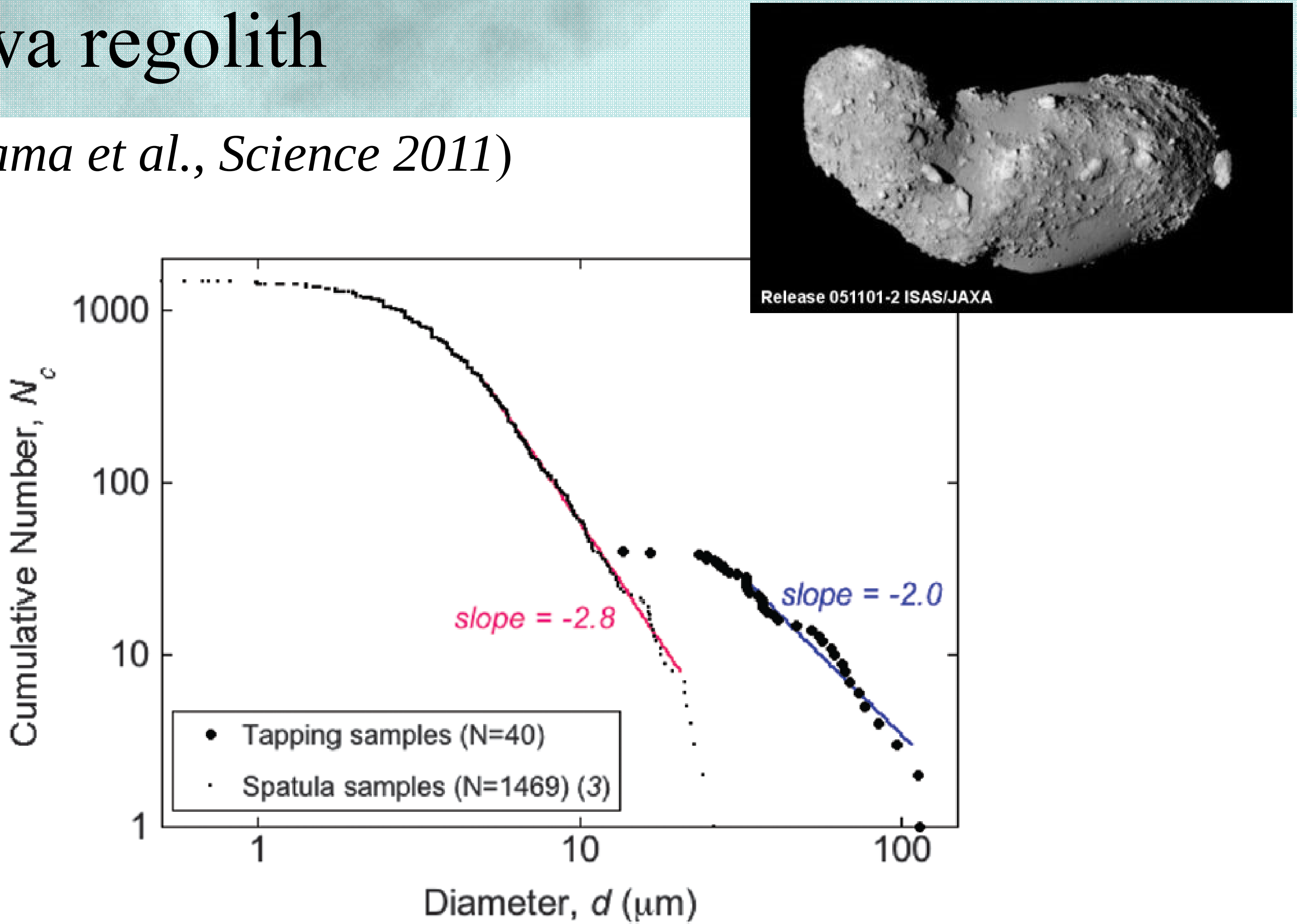


The model can provide more or less linear relation in log(e)-log(p) curve.
The power becomes consistent with the ODC test when *m*=3 (with α=0.1)

[5] Some observation in planetary science

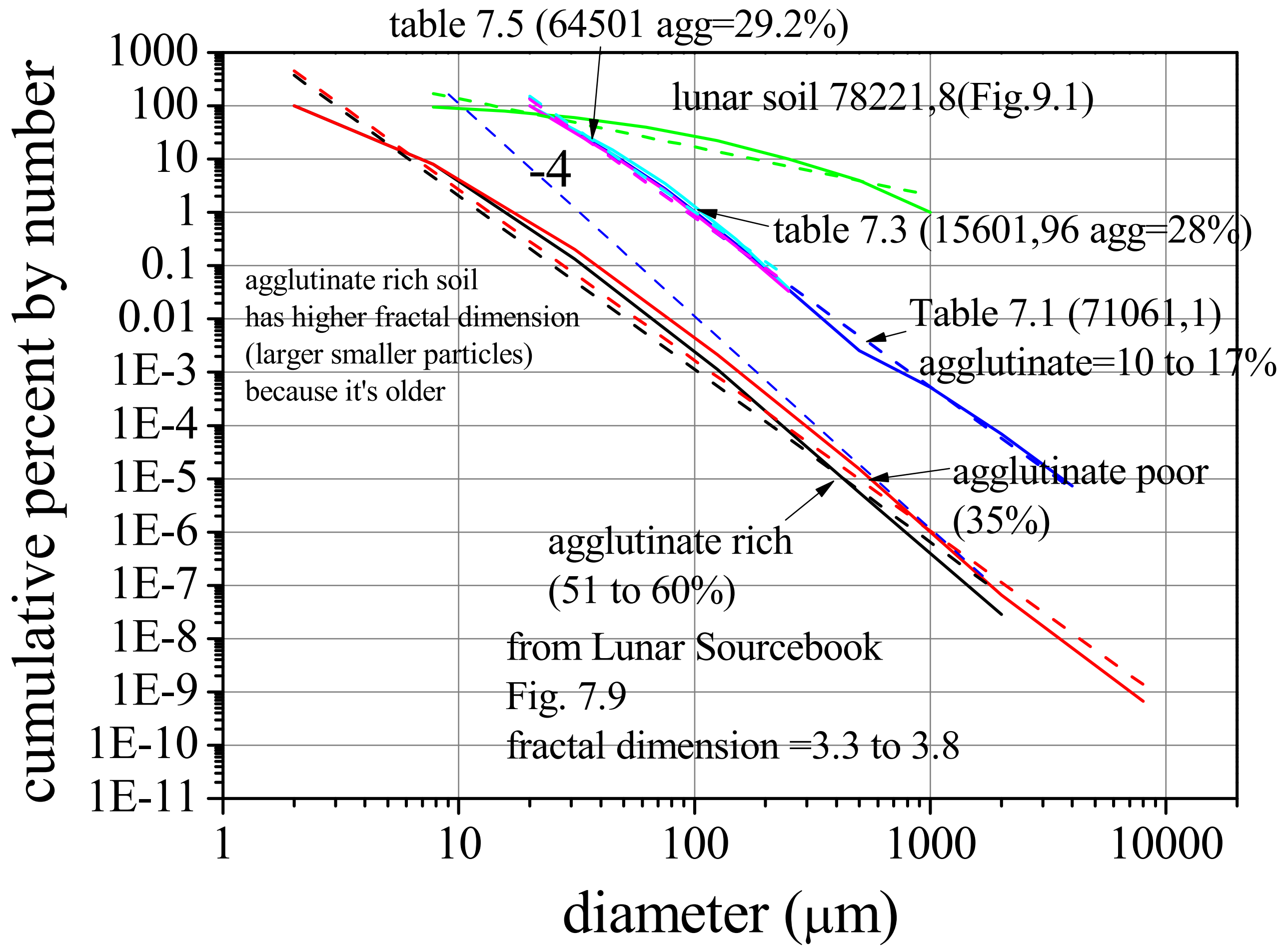
Itokawa regolith

(*Tsuchiyama et al., Science 2011*)



The power D_f=-2.0～-2.8 close to ODC test

Lunar soil GSD (*Lunar Sourcebook*)



D_f=-3.0～-4.0

Higher fractal dimension due to repeated comminution?

or due to unconfined comminution?

Conclusions

(1) Single grain crushing test
(2) One dimensional compression test
(3) Rotary shear test
were performed with the common geomaterials.

We observe
* Linear log(e)-log(p) curve in plastic compression
* Steady peaks and fractal nature in grain size distribution

The proposed model reproduce the above observation.