Mechanical and Numerical Modeling of Gas Hydrate Bearing Sediments

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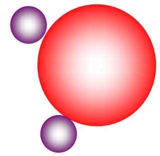
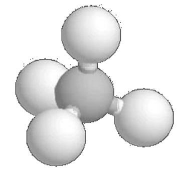
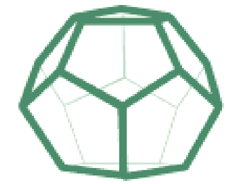
Outline

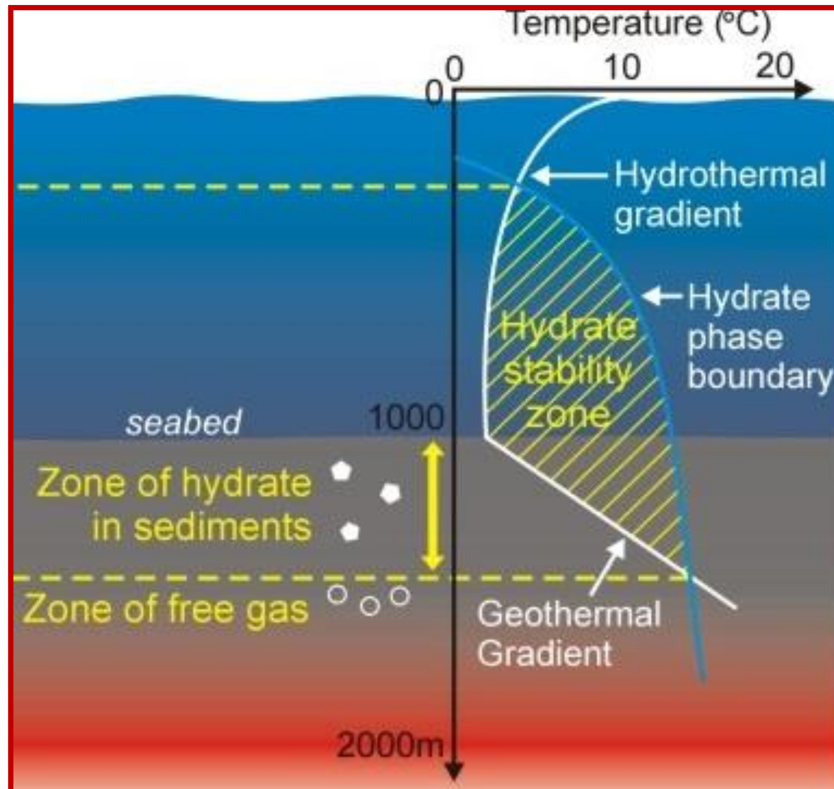
- Brief introduction to Hydrate Bearing Sediments (HBS)
- Basic components of the proposed coupled THCM framework for HBS
- Simple benchmarks involving HBS
- HBS mechanical modeling
- Validation
- Final remarks

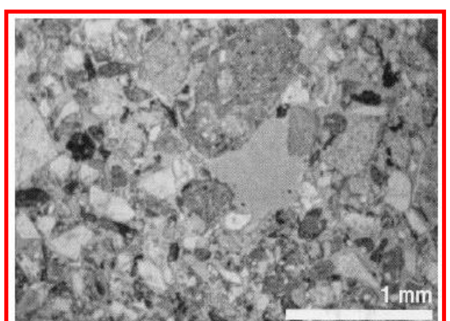
Gas Hydrate Bearing Sediments (HBS)

- Gas hydrates are solid ice-like materials that consists of guest gas molecules encased in a water matrix.
- Hydrate bearing sediments are typically found in:
  - ✓ marine sediments
  - ✓ permafrost
- Methane hydrates sediments are highly compacted (stable) under deposit conditions and are likely to behave as bonded sedimentary soils

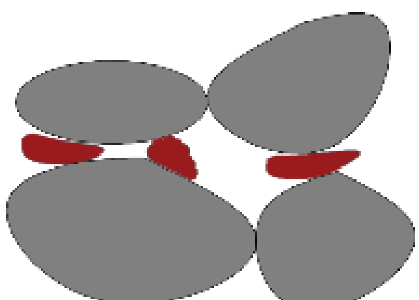




Water + Methane CH_4 = Methane hydrate

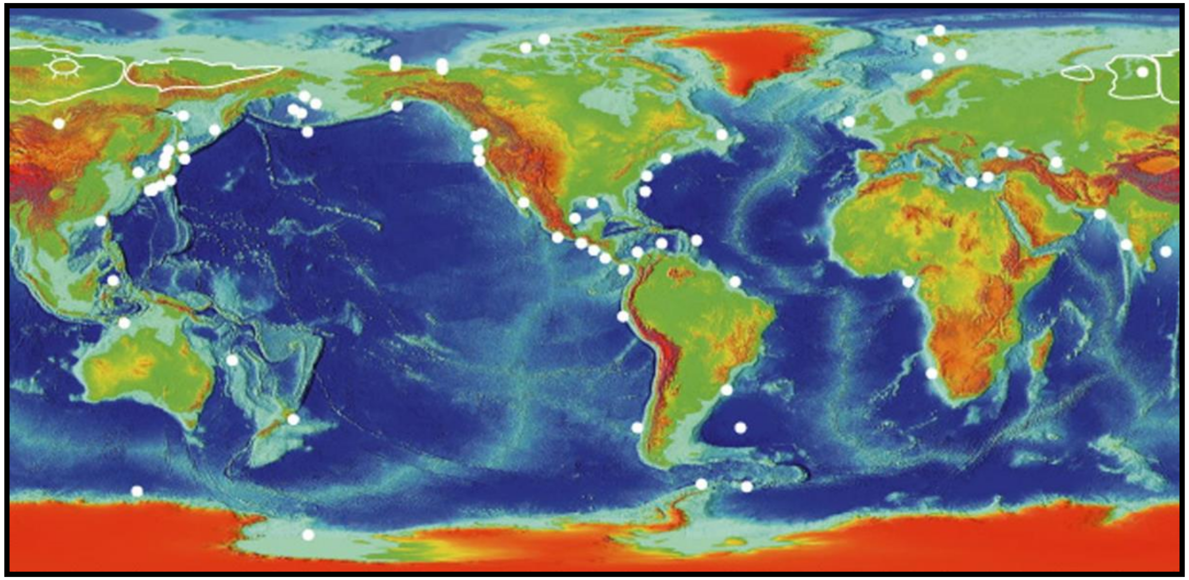


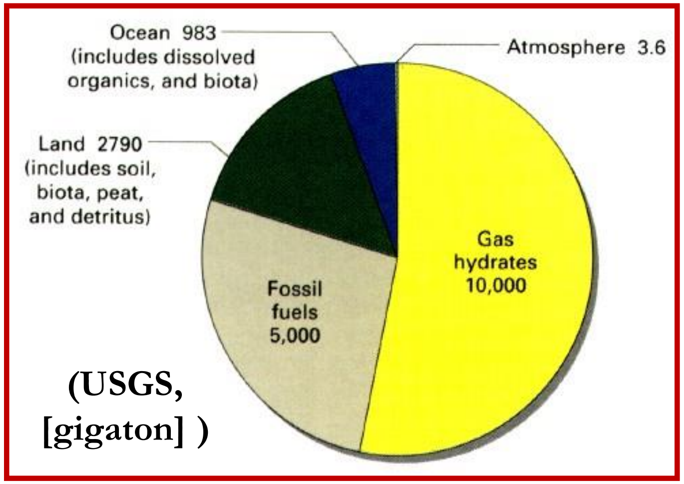


Soga et al. (2006)



Gas Hydrate Bearing Sediments (HBS)

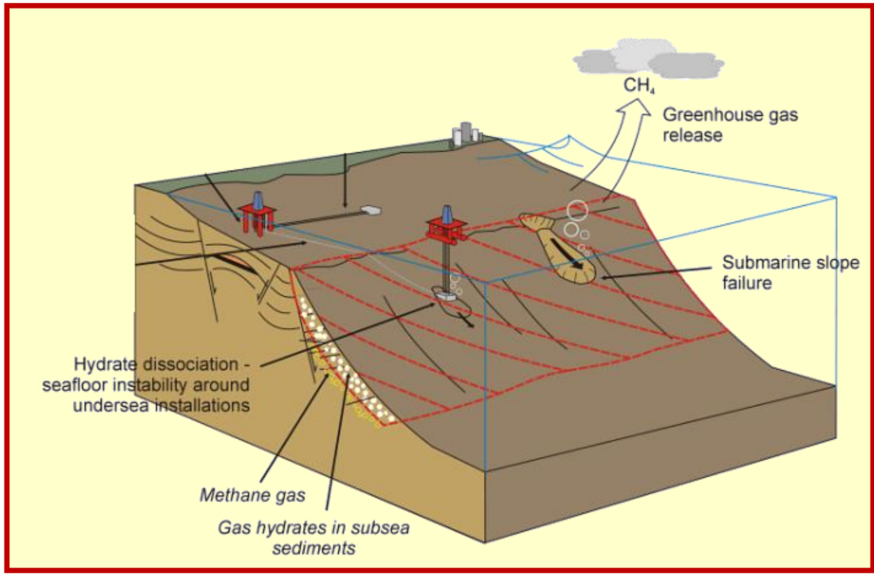




(USGS, [gigaton])

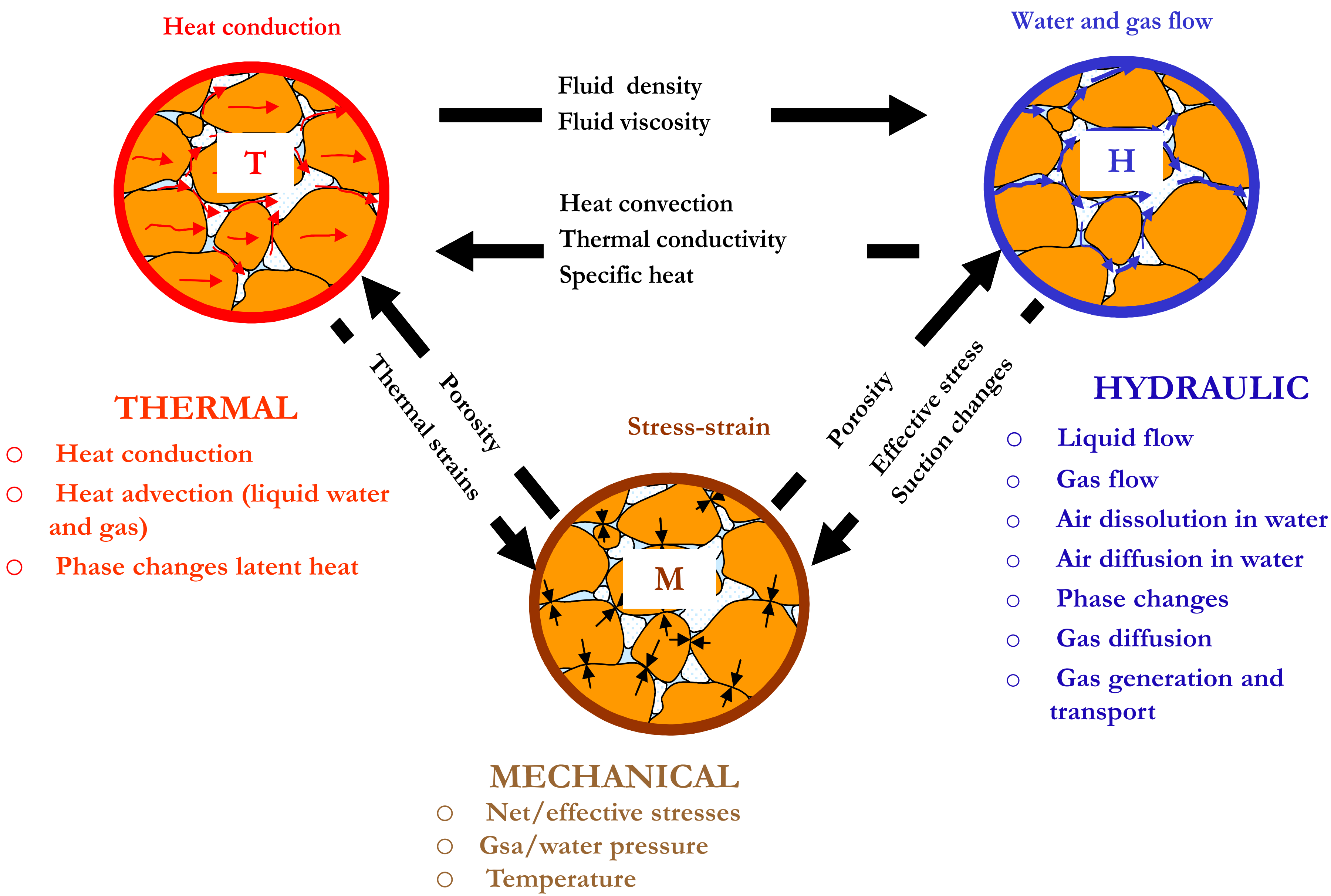
Relevance:

- ✓ methane recovery, energy resource
- ✓ instability (boreholes and slopes)
- ✓ effect on submarine infrastructure
- ✓ climate change
- ✓ CO_2 sequestration



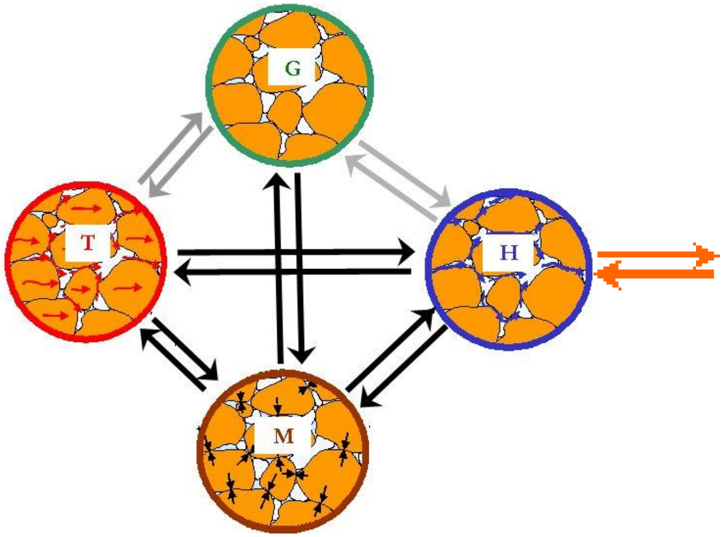
(Kvenvolden and Lorenson, 2001; www.pet.hw.ac.uk; Ballough et al.]

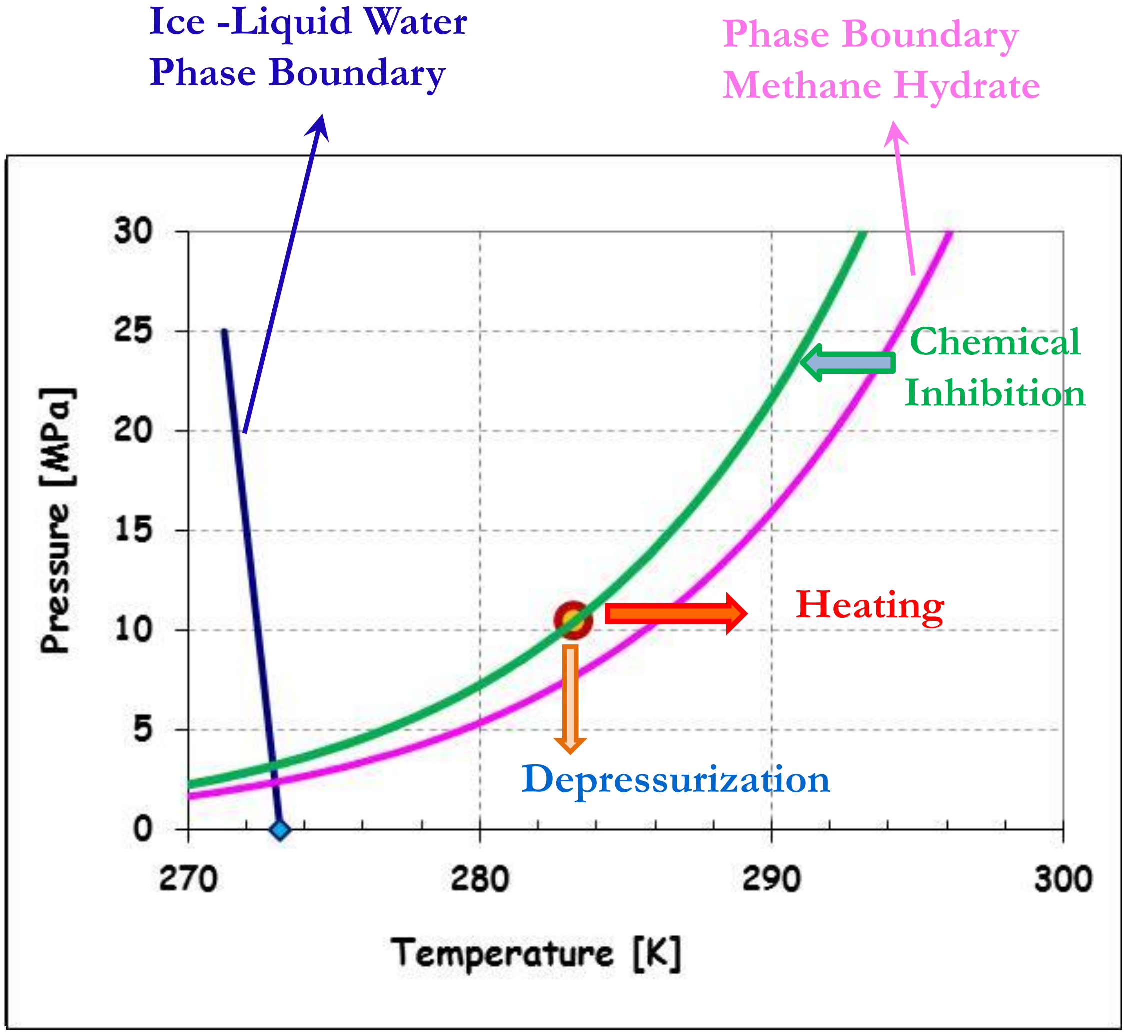
THM Coupled Phenomena



HBS – Coupled Phenomena and Phase Boundaries

Hydrate Dissociation



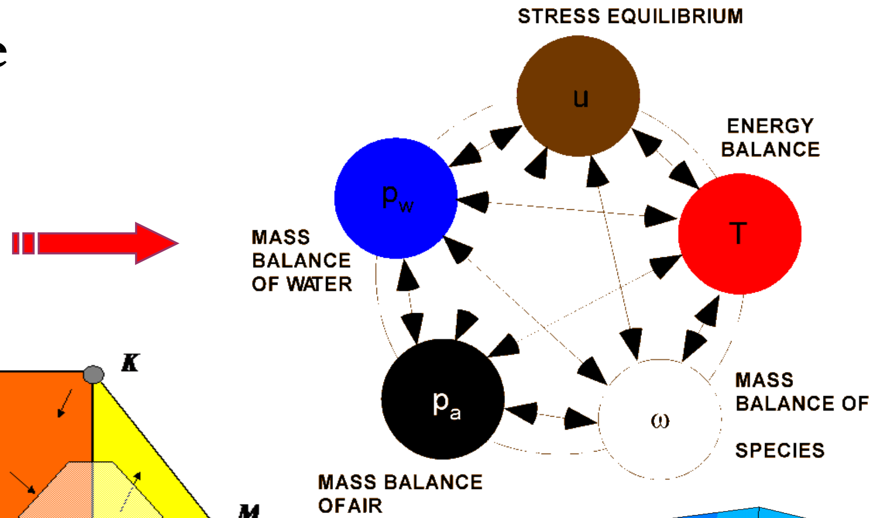


Numerical Code

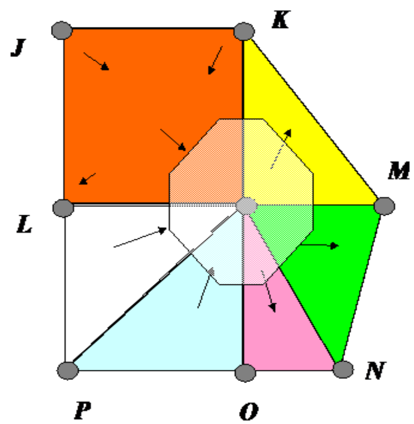
- **CODE_BRIGHT computational code**
  Olivella et al. (1996)

Coupled analysis in geological media

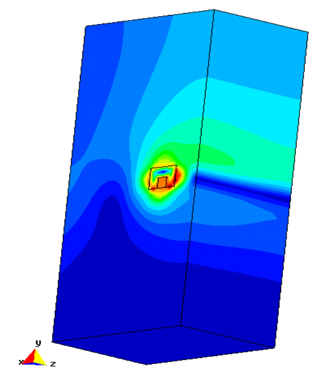
Interaction with the atmosphere

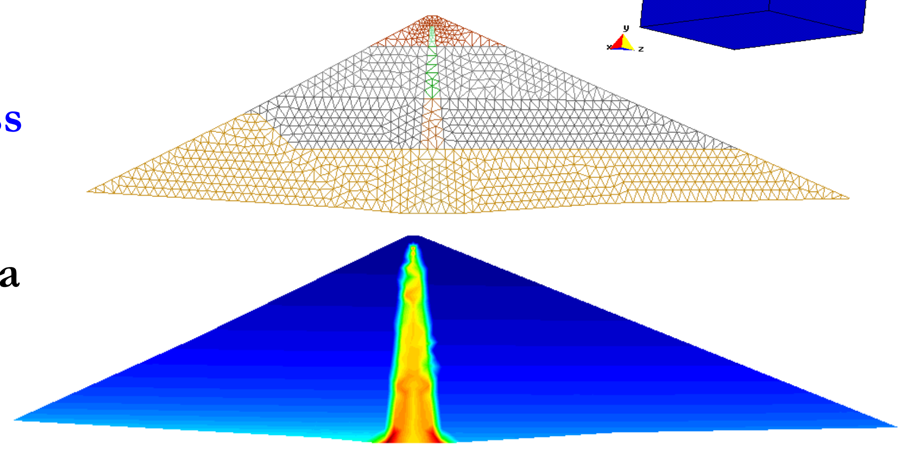


- ✓ Finite element in space
  - 1D, 2D and 3D elements
  - Monolithic coupling
  - Full Newton-Raphson



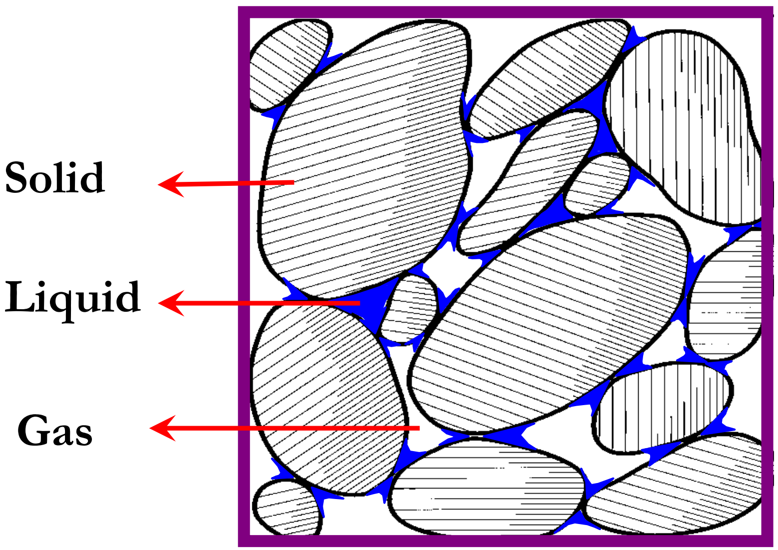
- ✓ Finite difference in time
  - Implicit time discretisation scheme
  - Automatic time advance
  - Mass conservative approach for mass balance equations
- ✓ User-friendly pre/post processing of data





Coupled THM Formulation

➢ Phases and species



Three phases:

- **solid** (*s*) : mineral
- **liquid** (*l*) : water + air dissolved
- **gas** (*g*) : mixture of dry air and water vapour

Three species:

- **mineral** (-) : the mineral is coincident with solid
- **water** (*w*) : as liquid or evaporated in the gas phase
- **air** (*a*) : dry air, as gas or dissolved in the liquid phase

➢ Governing equations

$$\phi = \frac{V_{pores}}{V_{total}} = \frac{V_{liquid} + V_{gas}}{V_{total}}$$

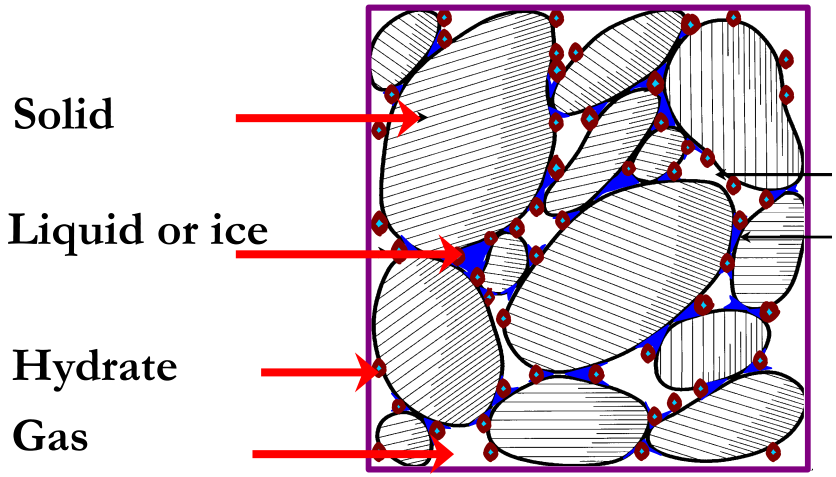
✓ Balance equations

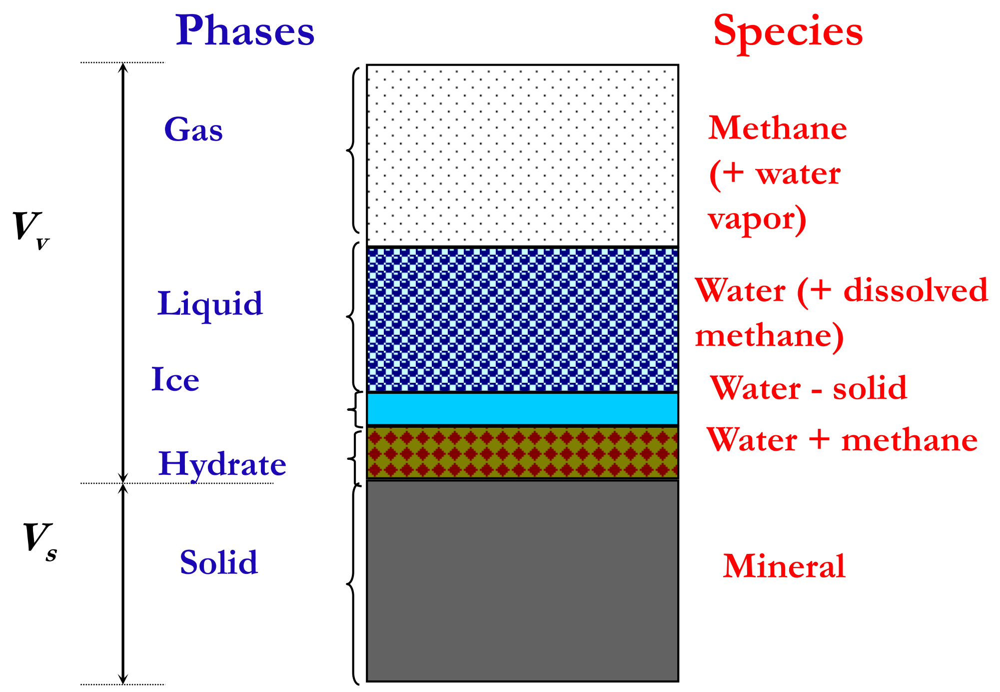
✓ Constitutive equations

$$S_l = \frac{V_{liquid}}{V_{pores}} = \frac{V_{liquid}}{V_{liquid} + V_{gas}} = 1 - S_g$$

✓ Equilibrium restrictions

HBS - Species and Phases





Three main species:

- ✓ mineral
- ✓ water (w)
- ✓ methane (m)

The species are found in five phases:

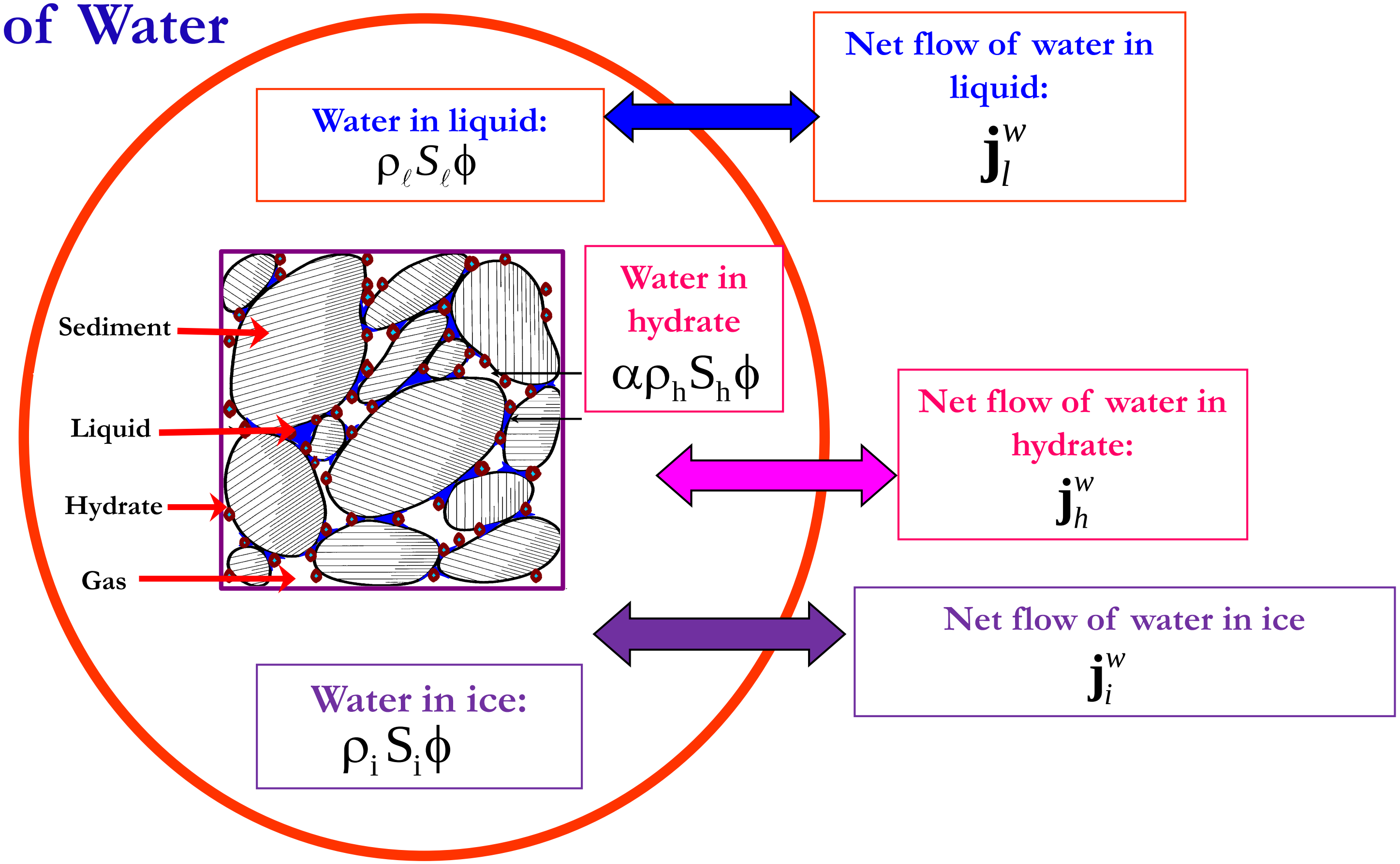
- ✓ solid particles (s)
- ✓ liquid (l)
- ✓ gas (g)
- ✓ hydrates (h)
- ✓ ice (i)

$$\phi = \frac{V_v}{V} = \frac{V_g + V_l + V_h + V_i}{V}$$

$$S_\beta = \frac{V_\beta}{V_v} \qquad \beta = l, g, h, i \qquad S_l + S_g + S_h + S_i = 1$$

HBS - Mass Balance Equations

➢ Mass Balance of Water



$$\frac{\partial}{\partial t}\begin{pmatrix}\text{mass of water in}\\ \text{liquid, hydrate and ice (\& gas) phases}\end{pmatrix} + \text{divergence}\begin{pmatrix}\text{total flows}\\ \text{of water}\end{pmatrix} = \begin{pmatrix}\text{external supply}\\ \text{of water}\end{pmatrix}$$

$$\frac{\partial}{\partial t}[\underbrace{(\rho_\ell S_\ell + \alpha\rho_h S_h + \rho_i S_i)}_{\substack{\text{mass of water in liquid,}\\ \text{hydrate and ice phases}}} \underbrace{\phi}_{\text{porosity}}] + \nabla.[\underbrace{\mathbf{j}_\ell^w + \mathbf{j}_h^w + \mathbf{j}_i^w}_{\substack{\text{total flows}\\ \text{of water}}}] = \underbrace{f^{w}}_{\substack{\text{external supply}\\ \text{of water}}}$$

HBS – Balance Equations

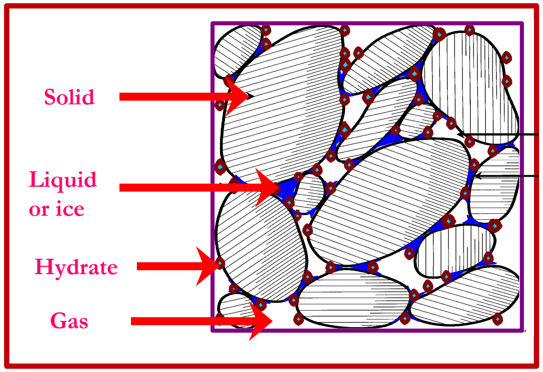
❑ Mathematical Formulation

- **Mass of water (P_l)**

$$\frac{\partial}{\partial t}\underbrace{[(\rho_\ell S_\ell + \alpha\rho_h S_h + \rho_i S_i)\phi]}_{\text{mass water per unit volume}} + \nabla.[\underbrace{\rho_\ell \mathbf{q}_\ell + \rho_\ell S_\ell \phi\,\mathbf{v}}_{\text{w in liquid}} + \underbrace{\alpha\rho_h S_h \phi\,\mathbf{v}}_{\text{w in hydrate}} + \underbrace{\rho_i S_i \phi\,\mathbf{v}}_{\text{w in ice}}] = f^w$$

- **Mass of methane (P_g)**

$$\frac{\partial}{\partial t}\underbrace{\left\{\left[\rho_g S_g + (1-\alpha)\rho_h S_h\right]\phi\right\}}_{\text{mass of methane per unit volume}} + \nabla.[\underbrace{\rho_g \mathbf{q}_g + \rho_g S_g \phi\,\mathbf{v}}_{\text{m in gas}} + \underbrace{(1-\alpha)\rho_h S_h \phi\,\mathbf{v}}_{\text{m in hydrate}}] = f^m$$



- **Mass of solute (c_i)**

$$\frac{\partial}{\partial t}\underbrace{(C_s S_\ell \rho_\ell \phi)}_{\text{mass s per unit volume}} + \nabla.[\underbrace{\mathbf{D}\rho_\ell \nabla C_s}_{\text{non advective flux of s}} + \underbrace{C_s \rho_\ell \mathbf{q}_\ell}_{\text{s in liquid}} + \underbrace{C_s \rho_l S_l \phi\,\mathbf{v}}_{\text{s in liquid}}] = f^s$$

- **Mass of solid (ϕ)**

$$\frac{\partial}{\partial t}\underbrace{[\rho_s(1-\phi)]}_{\text{mass mineral per unit volume}} + \nabla\cdot\underbrace{[\rho_s(1-\phi)\mathbf{v}]}_{\text{m in solid}} = 0$$

- **Energy (T)**

$$\frac{\partial}{\partial t}\overbrace{\left\{\left[e_s\rho_s(1-\phi)\right] + \left(e_\ell\rho_\ell S_\ell + e_g\rho_g S_g + e_h\rho_h S_h + e_i\rho_i S_i\right)\phi\right\}}^{\text{energy per unit volume of the hydrate bearing sediment}} + \nabla.\mathbf{i}_c + \nabla.[\underbrace{e_\ell\rho_\ell(\mathbf{q}_\ell + S_\ell\phi\,\mathbf{v})}_{\text{transport in }\ell} + \underbrace{e_g\rho_g(\mathbf{q}_g + S_g\phi\,\mathbf{v})}_{\text{transport in g}} + \underbrace{e_h\rho_h S_h\phi\,\mathbf{v}}_{\text{transport in h}} + \underbrace{e_i\rho_i S_i\phi\,\mathbf{v}}_{\text{transport in i}} + \underbrace{e_s\rho_s(1-\phi)\,\mathbf{v}}_{\text{transport in s}}] = f^E$$

- **Momentum (u)**

$$\nabla\cdot\boldsymbol{\sigma} + \mathbf{b} = \mathbf{0}$$

HBS - Constitutive Equations and Equilibrium Restrictions

EQUATION	VARIABLE NAME	VARIABLE
Constitutive Equations		
Fourier's law	conductive heat flux	$\mathbf{i}_c$
Darcy´s law	liquid and gas advective flux	$\mathbf{q}_l , \mathbf{q}_g$
Retention curve	liquid degree of saturation	S_l , S_g
Fick's law	vapour and air non-advective fluxes	$\mathbf{i}_g^w , \mathbf{i}_l^m$
Mechanical model	stress tensor	$\boldsymbol{\sigma}$
Phase density	liquid density	ρ_l
Gases law	methane density	ρ_g
Equilibrium Restrictions		
Hydrate dissociation/formation	Hydrate Saturation	S_h
Ice thaw/formation	Ice Saturation	S_i
Henry's law	Methane dissolved mass fraction	ω_l^a
Psychrometric law		

HBS - Phases Properties and Phase Change

Phases	Mass Density ρ [kg/m^{-3}]	Specific Energy			Thermal Conductivity λ [W/(m.K)]
		Expression	Latent heat L [J/g]	Specific heat c [J/(g K)]	
liquid (water)	998	$e_\ell = c_\ell (T - T_o)$	-	4.2	0.58
ice	917	$e_i = -L_{fuse} + c_i (T - T_o)$	334 *fusion*	2.1	2.3
gas (methane)	gas law (see text)	$e_g = c_g (T - T_o)$	-	1.9 V=const 2.5 P=const	0.05 (P-dependent[a])
hydrate	910	$e_h = L_{diss} + c_h (T - T_o)$	339 *dissociation*	2.1	0.6 (T-dependent[b])
mineral	2650	$e_s = c_s (T - T_o)$	-	0.7 *quartz*	8 *quartz*
			-	0.8 *calcite*	3 *calcite*

$$\mu_\ell [Pa.s] = 2.1 \cdot 10^{-6} \exp\left(\frac{1808.5\ ^oK}{T}\right)$$

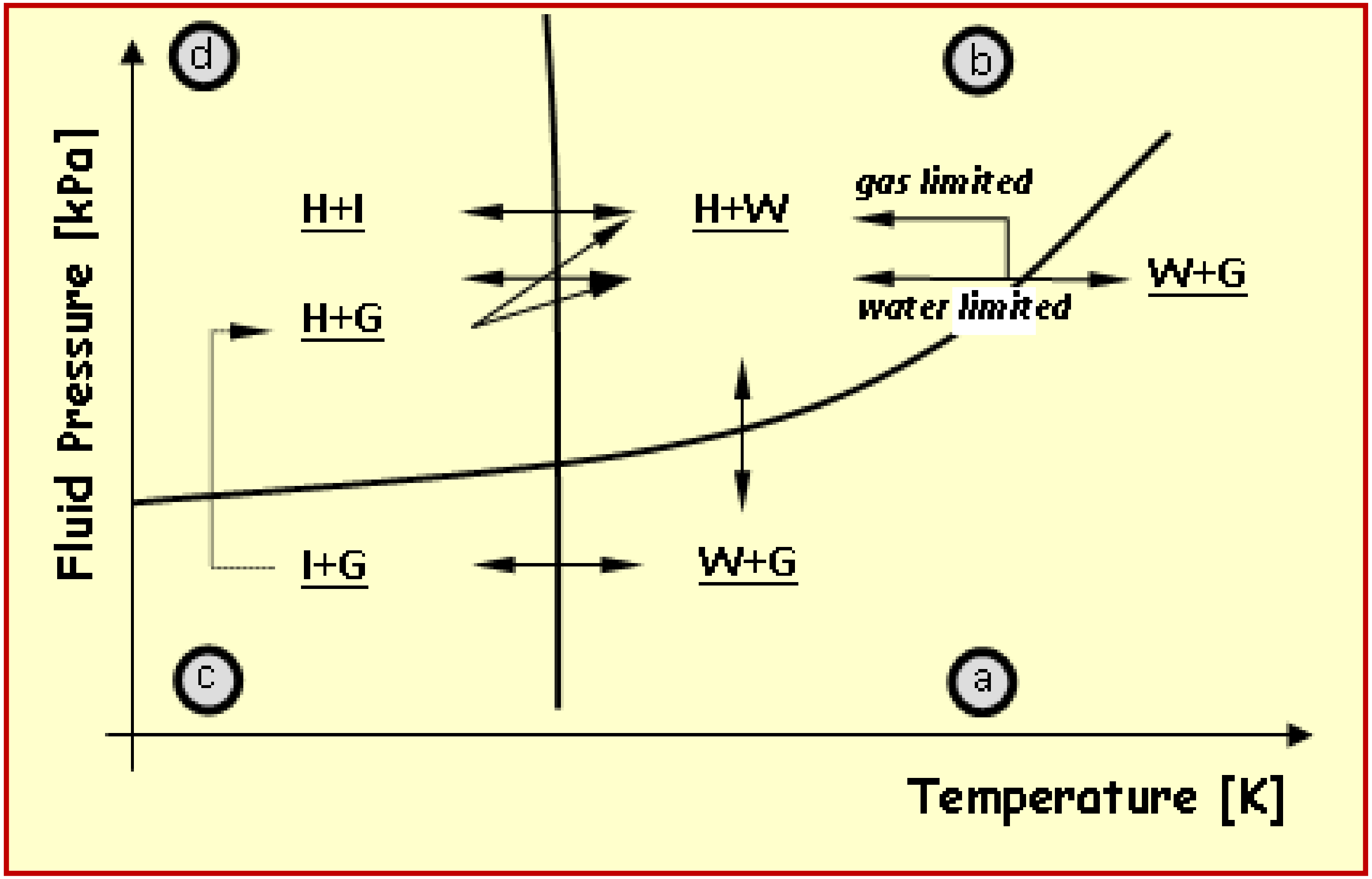
$$\mu_g [Pa.s] = 10.3 \cdot 10^{-6} \left[1 + 0.053 \frac{P_g}{MPa} \left(\frac{280\ ^\circ K}{T}\right)^3\right]$$

$$\rho_\ell = \rho_{\ell o} \left(1 + \frac{P_\ell}{B_\ell}\right) \left[1 - \beta_{T\ell} \left(\frac{T - 277\ ^\circ K}{5.6}\right)^2\right]$$

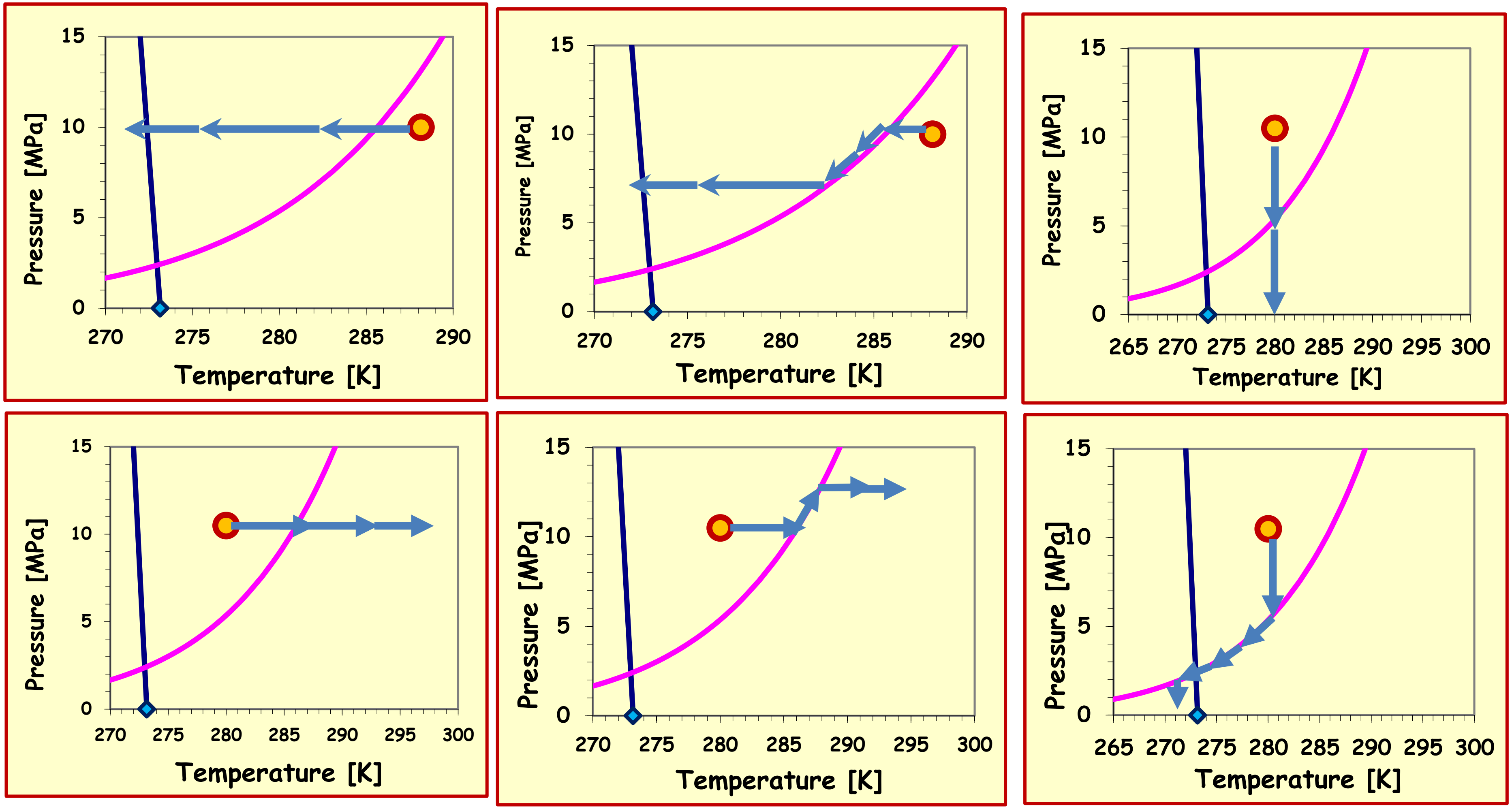
$$\lambda_{hbs} = \left[(1-\phi)\lambda_s^\beta + \phi\left(S_h \lambda_h^\beta + S_i \lambda_i^\beta + S_g \lambda_g^\beta + S_\ell \lambda_\ell^\beta\right)\right]^{\frac{1}{\beta}}$$

HBS – Coupled Phenomena and Phase Boundaries

- **PT Paths: Four Regions**
  - ✓ Phase boundaries for methane hydrate (H), gas (G), water (W) & ice (I).

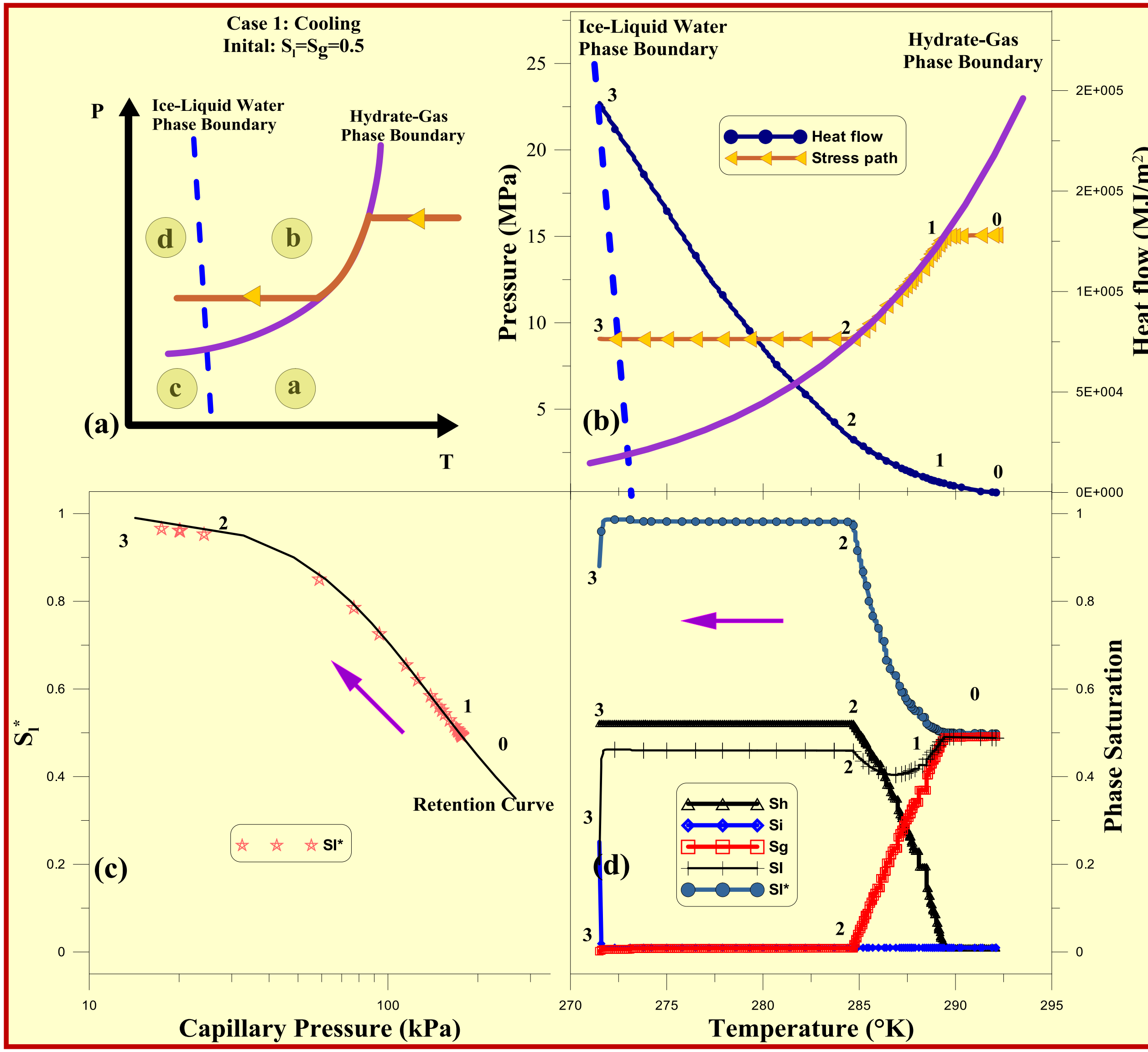


Pressure-Temperature paths



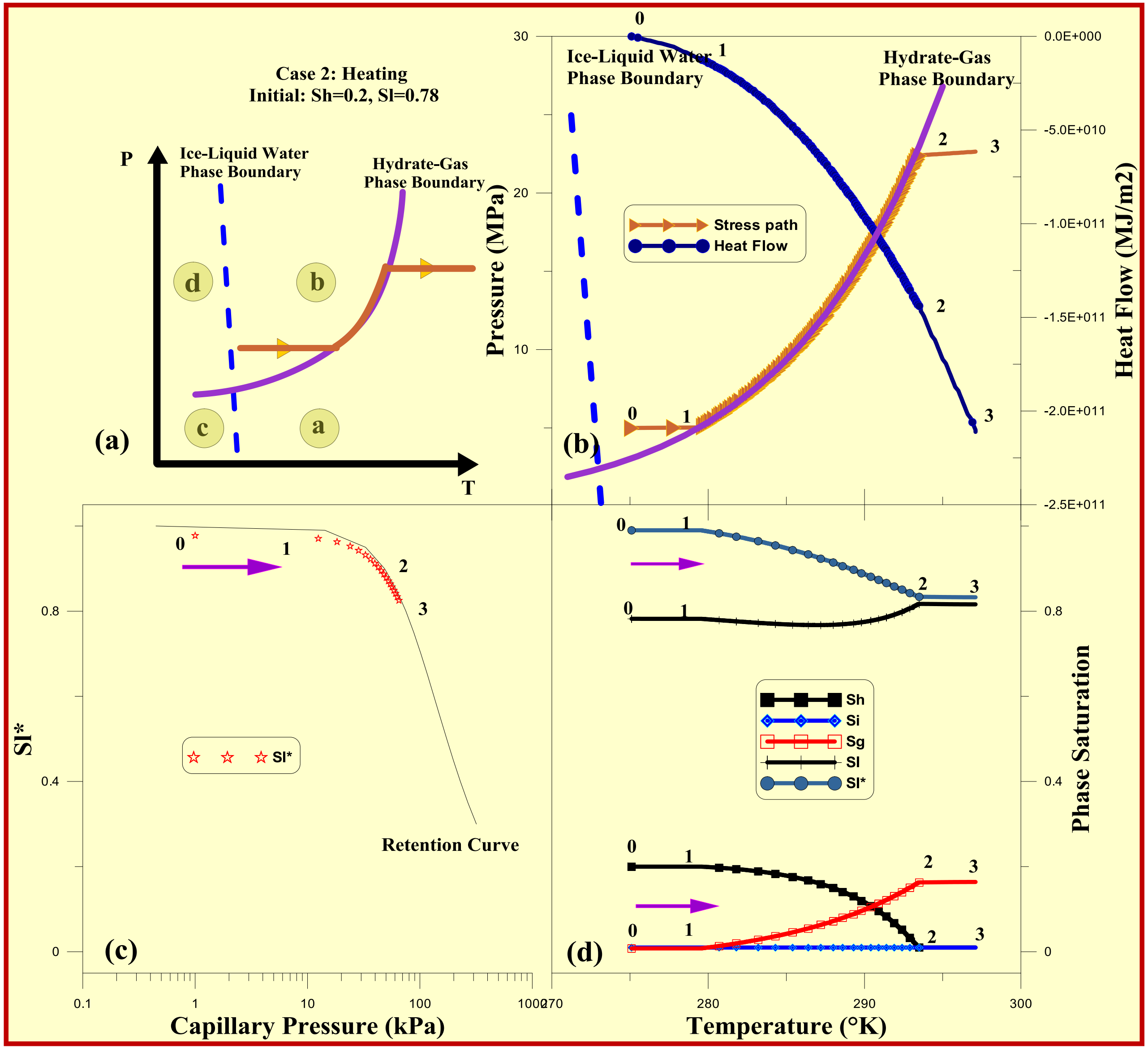
Pressure-Temperature paths

- Hydrate formation

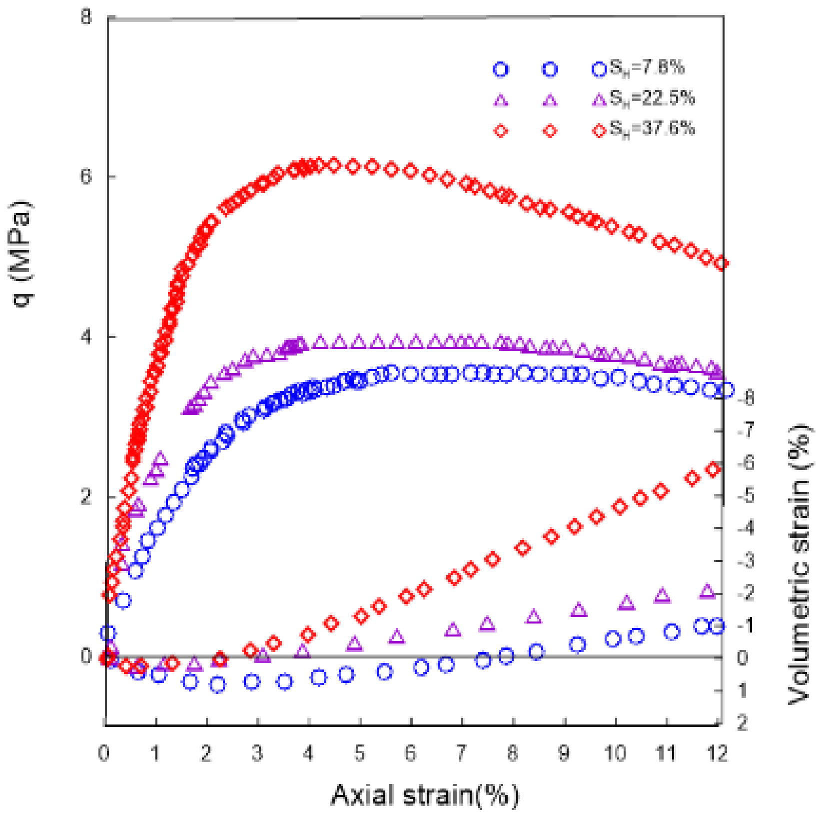


Pressure-Temperature paths

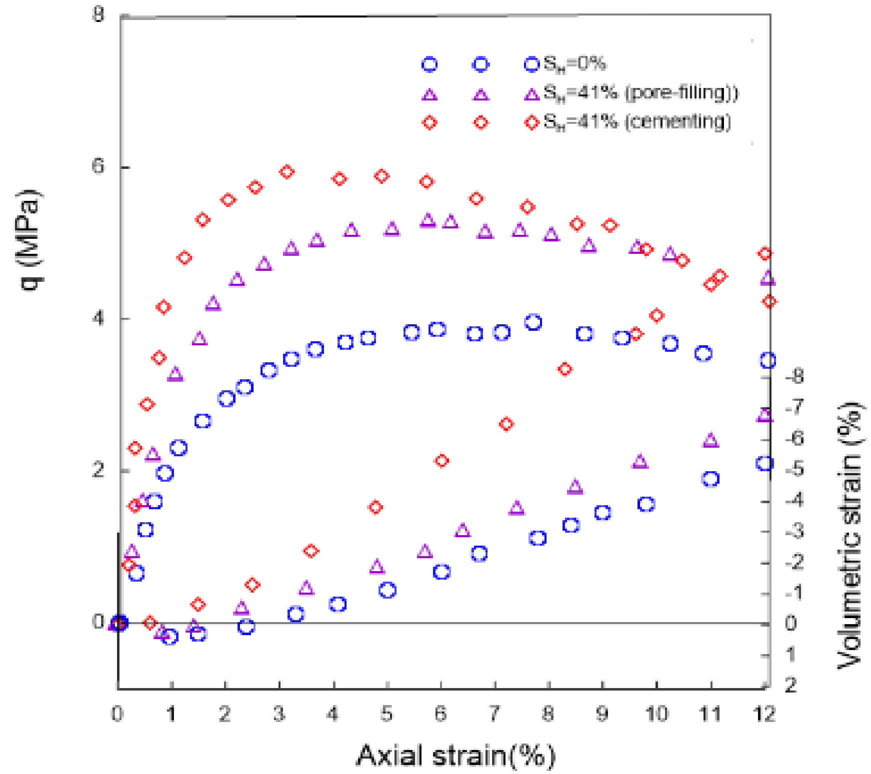
- **Hydrate Dissociation - Heating**



HBS – Mechanical Behavior at Constant S_h

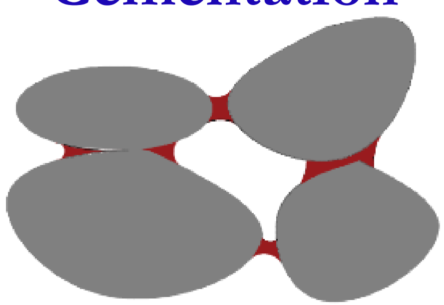


Masui et al. (2008)

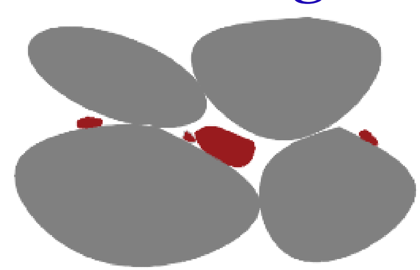


Masui et al. (2005)

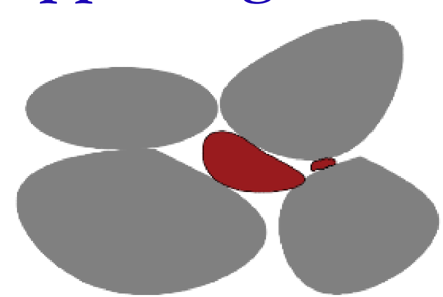
Cementation



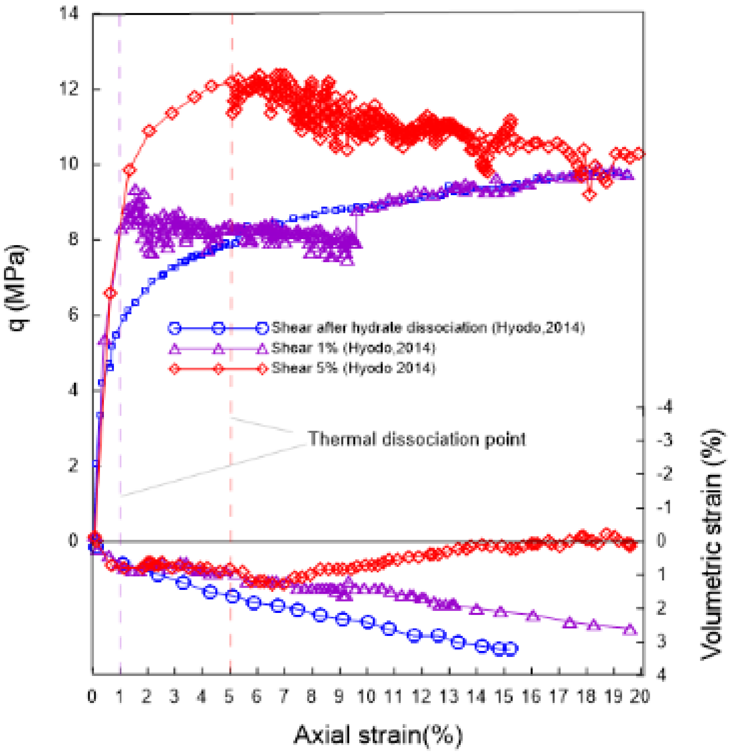
Pore Filling



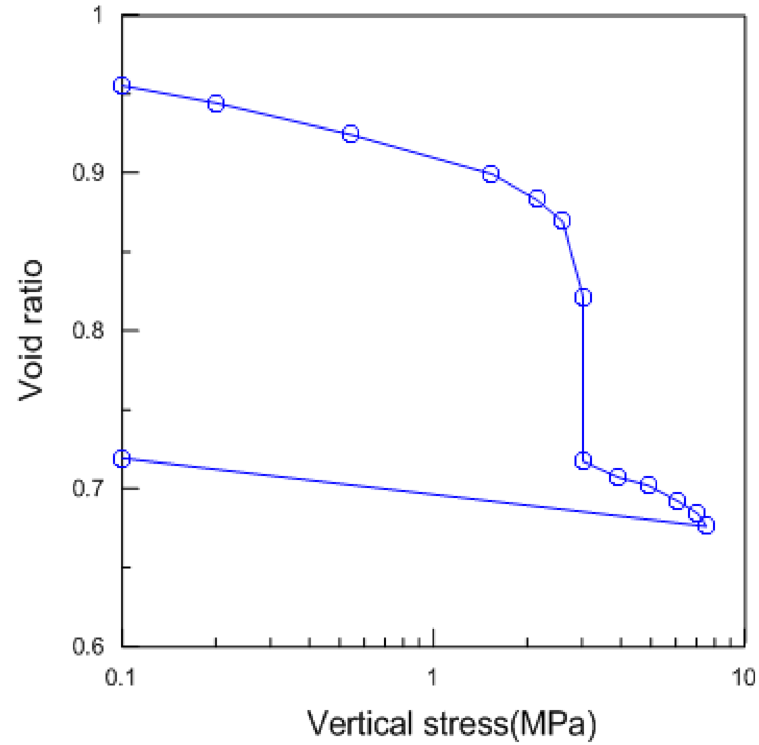
Supporting Matrix



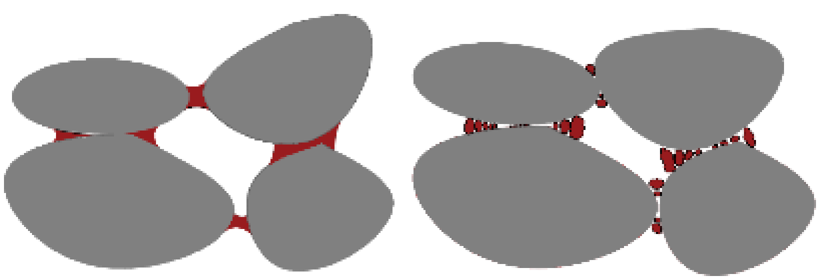
HBS – Mechanical Behavior During Dissociation Under Stress



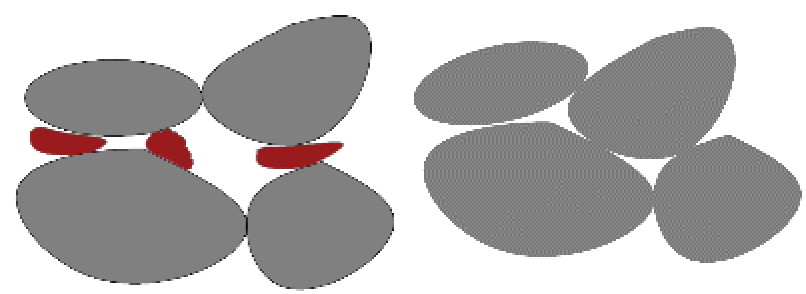
Hyodo et al. (2014)



Santamarina et al. (2015)



Shearing ➔ Hydrate damage



Hydrate dissociation ➔ Sediment collapse

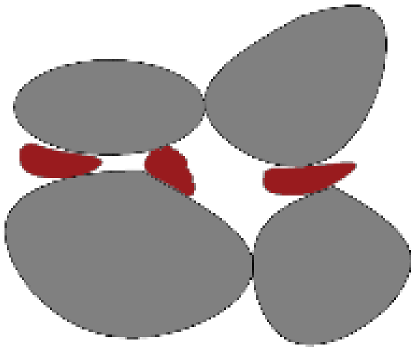
HBS – Mechanical Behavior

Some previous developments

- Mohr–Coulomb based model
  - Rutqvist and Moridis (2007)
  - Klar, Soga and Ng (2010)
- Based on an elasto–viscoplastic framework
  - Kimoto, Oka, Fushita and Fujiwaki (2007).
  - Kimoto, Oka, Fushita (2010)
- Modified Cam-Clay based model
  - Sultan and Garziglia (2011)
  - Uchida, Soga and Yamamoto (2012)

HBS – Mechanical Behavior

- The mechanical behavior of HBS depends on
  - Hydrate concentration
  - Pore habit
  - Stress level
  - Stress history



- Hydrates in soils
  - contribute to support the external applied stresses,
    - ✓ the strain partition concept is used to compute this contribution;
  - alter the mechanical behavior of sediments, e.g. provide hardening and dilation enhancement
    - ✓ a critical state model for the sediment to account for these effects.

HBS – Mechanical Model

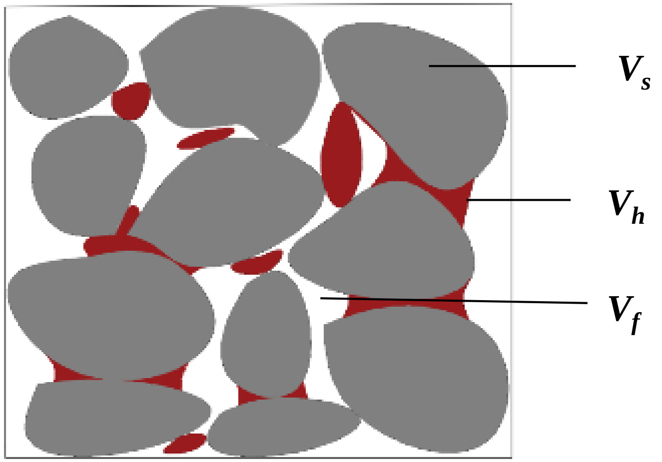
❑ Strain partition concept

Proposed by Pinyol et al. (2007) for clayed cemented materials

$$\varepsilon^{v} = \varepsilon_{ss}^{v} + C_h \varepsilon_h^{v} \qquad \varepsilon^{q} = \varepsilon_{ss}^{q} + C_h \varepsilon_h^{q}$$

$$\varepsilon_h^{v} = \chi \varepsilon_{ss}^{v} \qquad \varepsilon_h^{q} = \chi \varepsilon_{ss}^{q}$$

$$\boldsymbol{\varepsilon}_h = \frac{\chi}{1 + C_h \chi} \boldsymbol{\varepsilon}$$



❑ Hydrate Model

$$C_h = \phi S_h$$

$$\boldsymbol{\sigma}_h = e^{-L} \mathbf{D}_{h0} \boldsymbol{\varepsilon}_h = \mathbf{D}_h \boldsymbol{\varepsilon}_h$$

$$\chi = \chi_0 e^{-\frac{L}{2}}$$

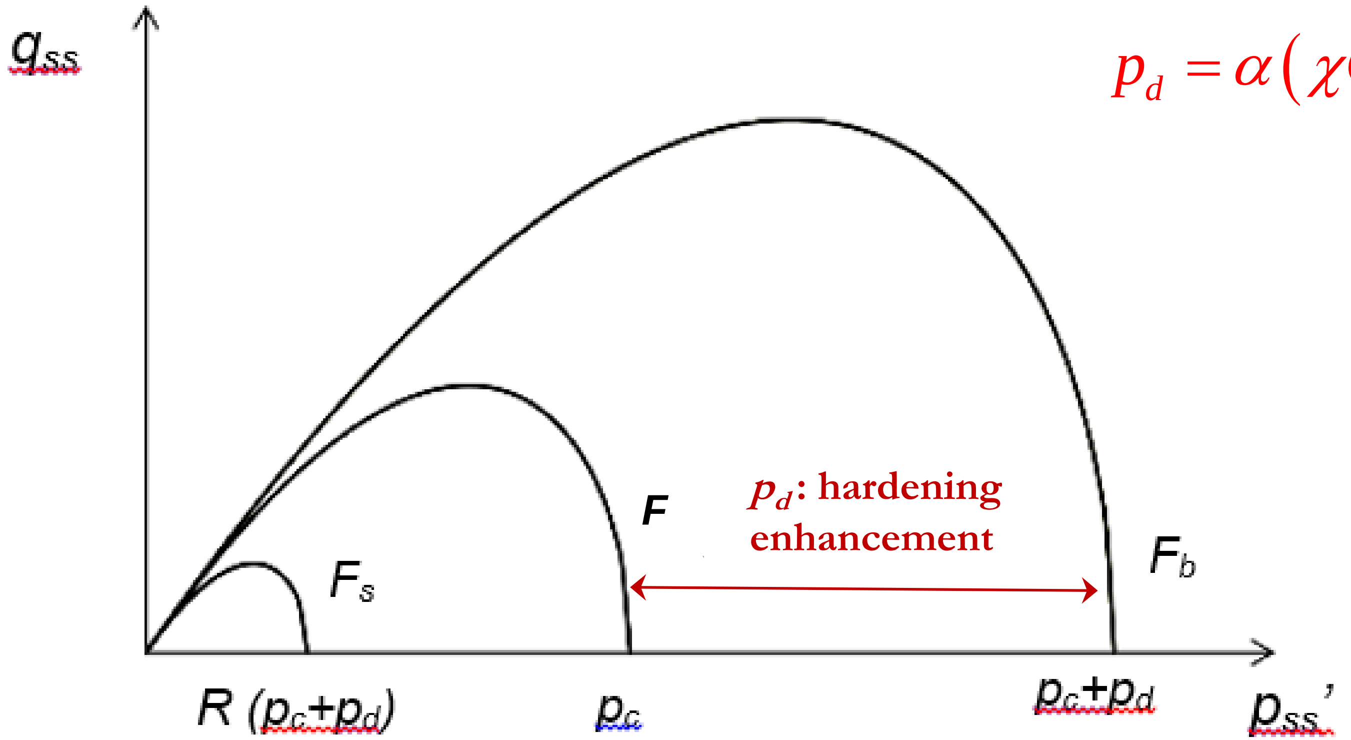
$$r_{(L)} = r_0 e^{r_1 L} = u_h$$

HBS – Mechanical Model

❑ Sediment Model

$$F_b = \frac{a}{M^2} q_{ss}^{\ 2} - 9\gamma \left\{ \left(p_{ss}^{'}\right)^2 - \left(p_{ss}^{'}\right)^n \left[\left(p_c + p_d\right)\right]^{2-n} \right\}$$

$$p_d = \alpha \left(\chi C_h\right)^\beta$$



❑ Final Stress-Strain Relationships

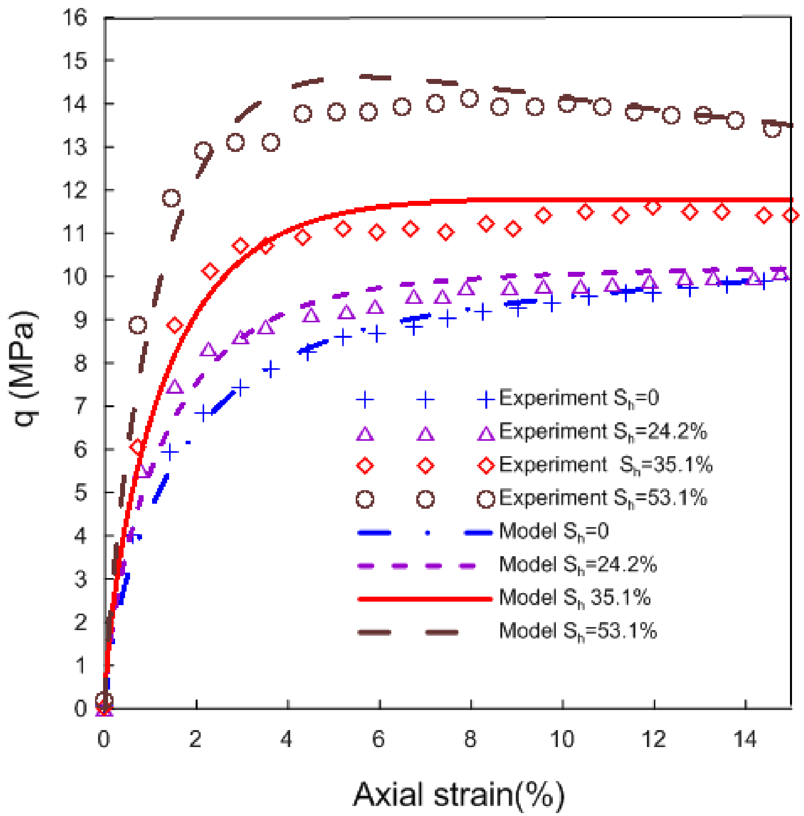
$$d\boldsymbol{\sigma}^{'} = d\boldsymbol{\sigma}_{ss}^{'} + \frac{C_h \chi}{1 + C_h \chi} d\boldsymbol{\sigma}_h$$

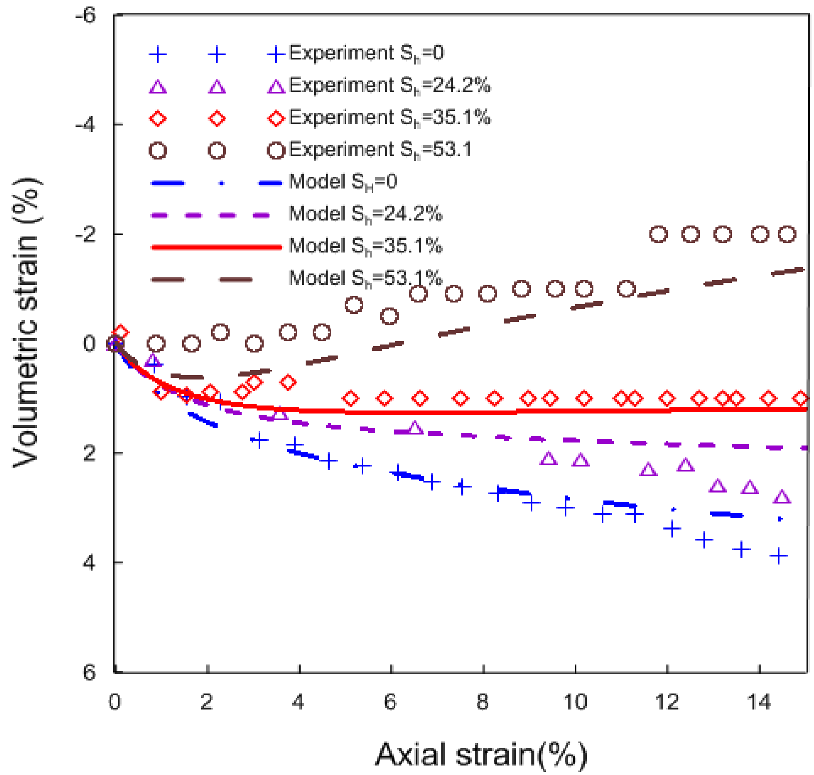
$$d\boldsymbol{\sigma}^{'} = \left[\mathbf{D}_{ss} + \left(\frac{C_h \chi}{1 + C_h \chi}\right)^2 \mathbf{D}_h\right] d\boldsymbol{\varepsilon} + \left[\mathbf{d}_{C_h} + \boldsymbol{\sigma}_h \left(\frac{C_h \chi}{1 + C_h \chi} - \left(\frac{C_h \chi}{1 + C_h \chi}\right)^2\right)\right] dC_h$$

HBS – Mechanical Model Validation

❑ Effect of Hydrate Saturation – Constant S_h

Synthetic Samples – Triaxial Conditions



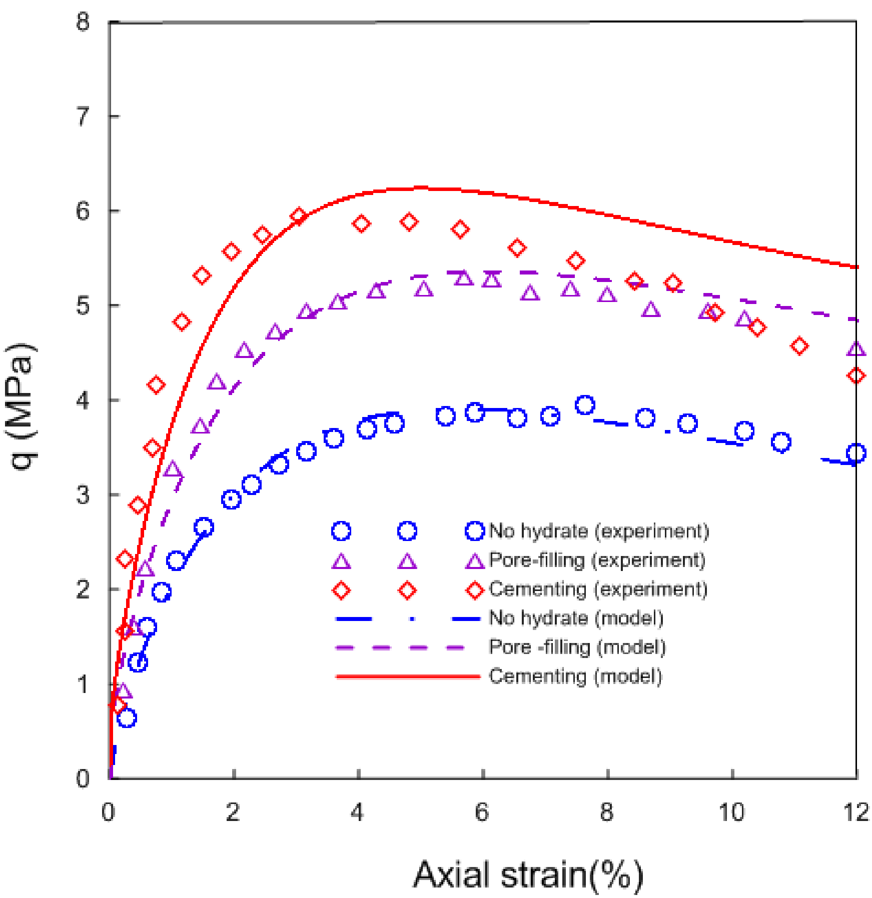


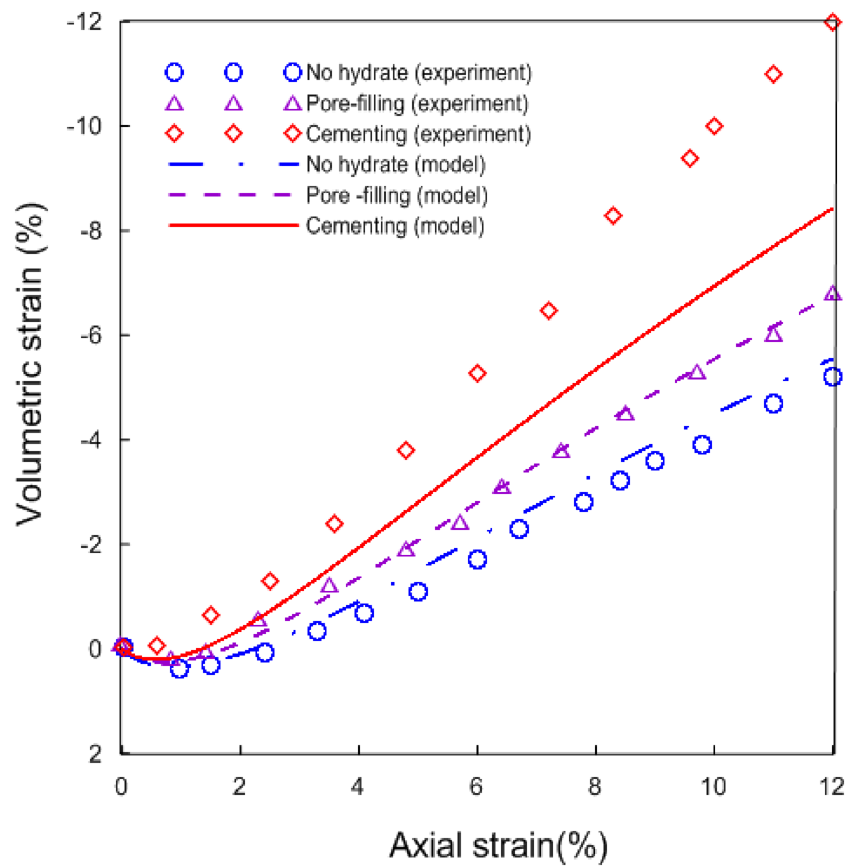
Experimental data from Hyodo et al. (2013)

HBS – Mechanical Model Validation

❑ Effect of Pore Habit – Constant S_h

Synthetic Samples – Triaxial Conditions



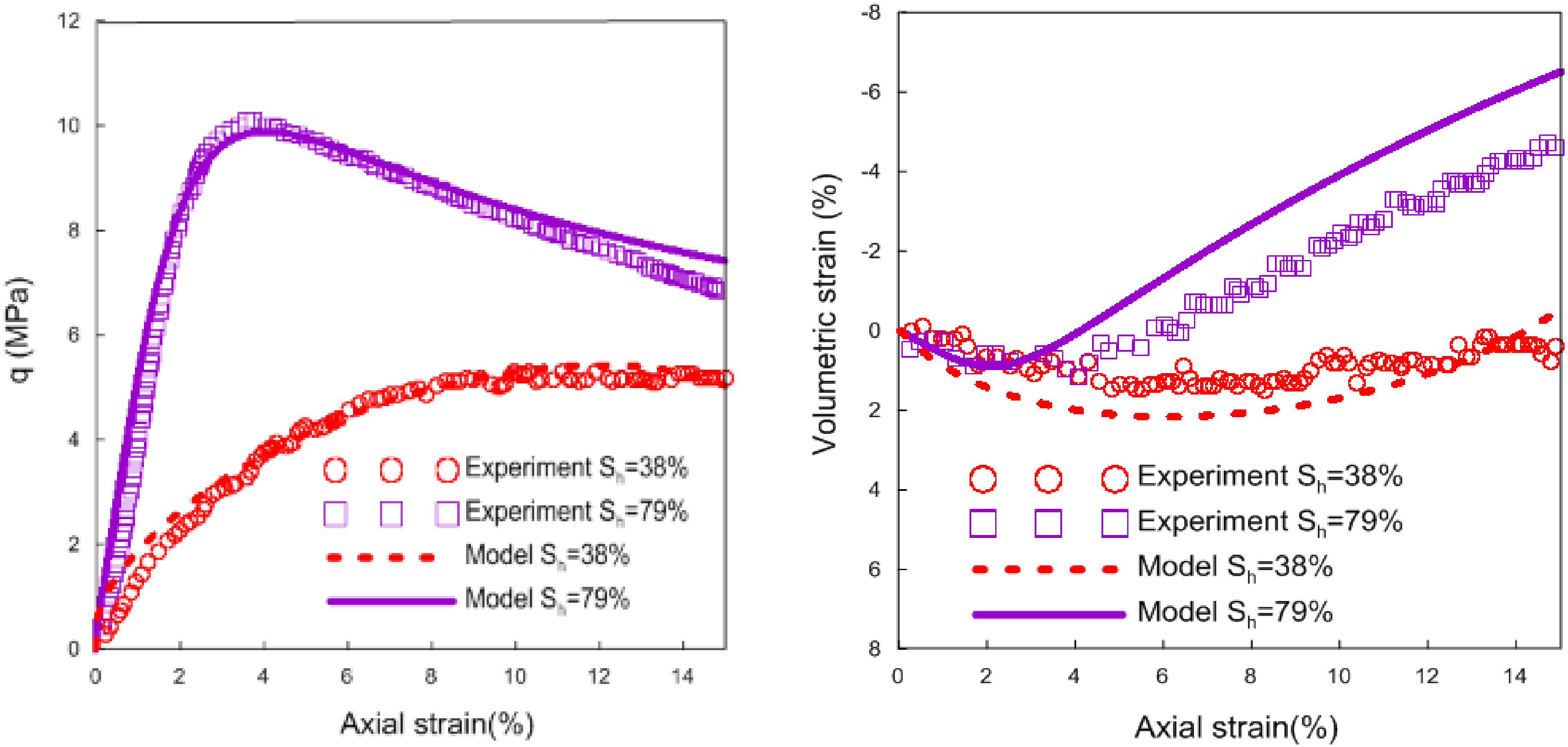


Experimental data from Masui et al. (2005)

HBS – Mechanical Model Validation

❑ Effect of Hydrate Saturation – Constant S_h

Natural Samples – Triaxial Conditions

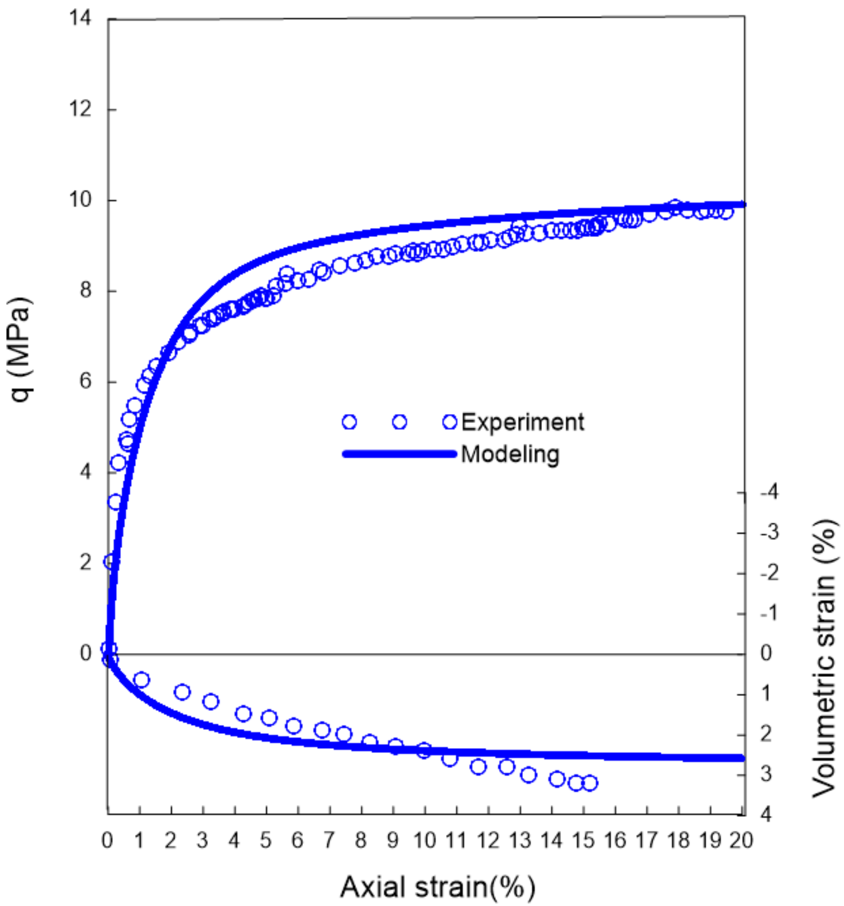


Experimental data from Joneda et al. (2015)

HBS – Mechanical Model Validation

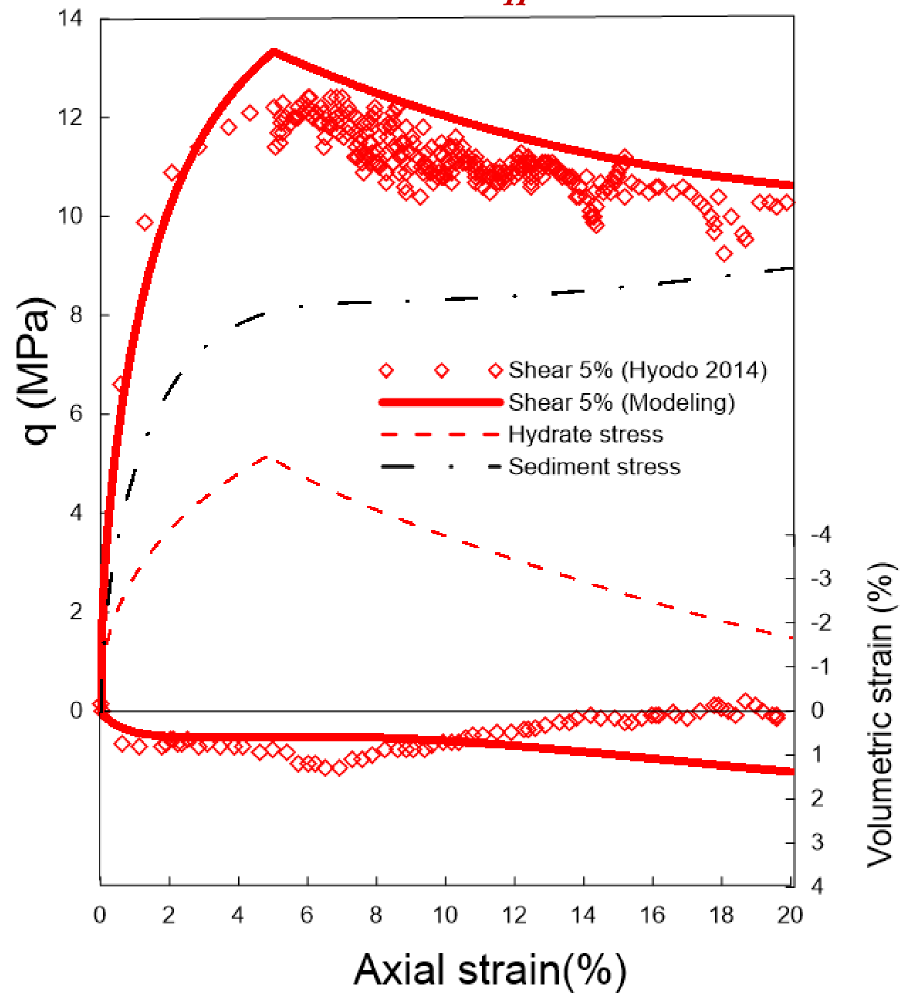
❑ Effect of Hydrate Dissociation

Synthetic Samples – Triaxial Conditions - S_h~48%



Already dissociated sediment

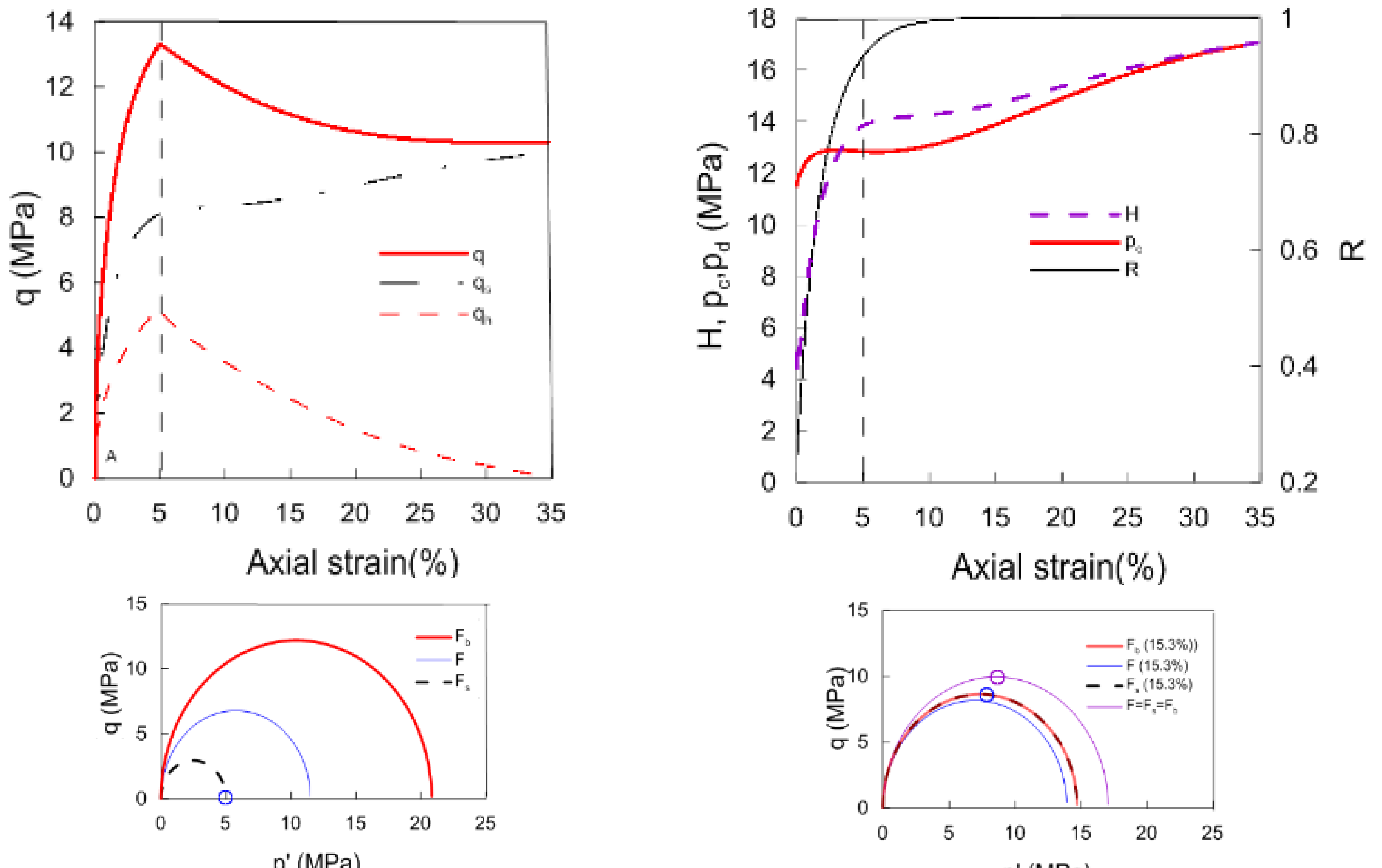
Experimental data from Hyodo et al. (2014)



Dissociation induced at ε_a=5%.

HBS – Mechanical Model Validation

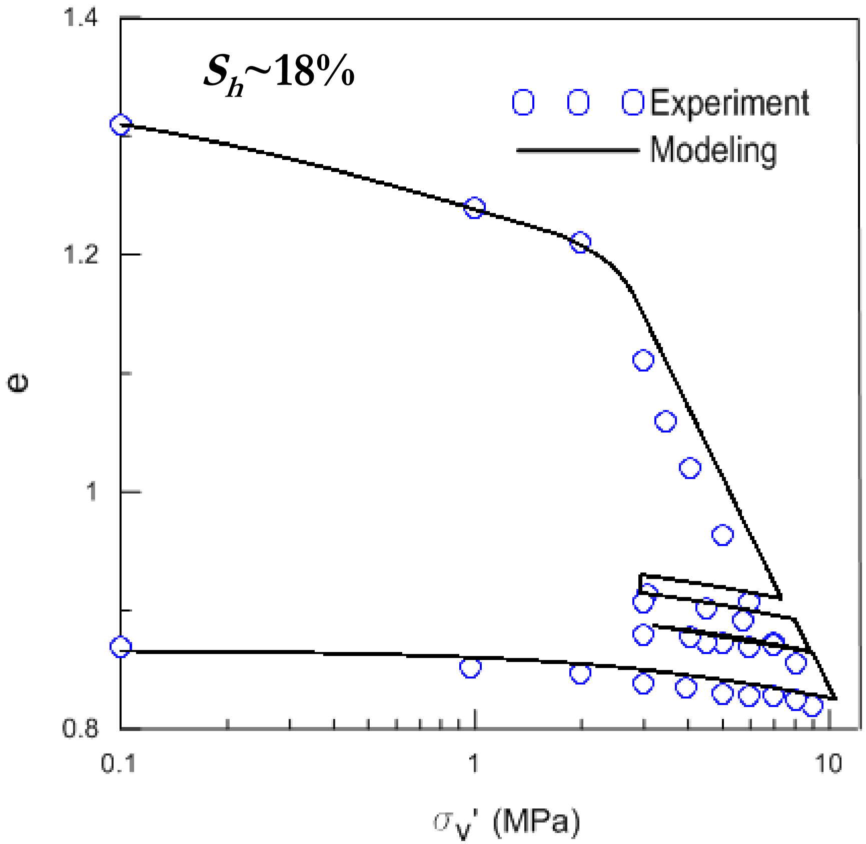
❑ Effect of Hydrate Dissociation

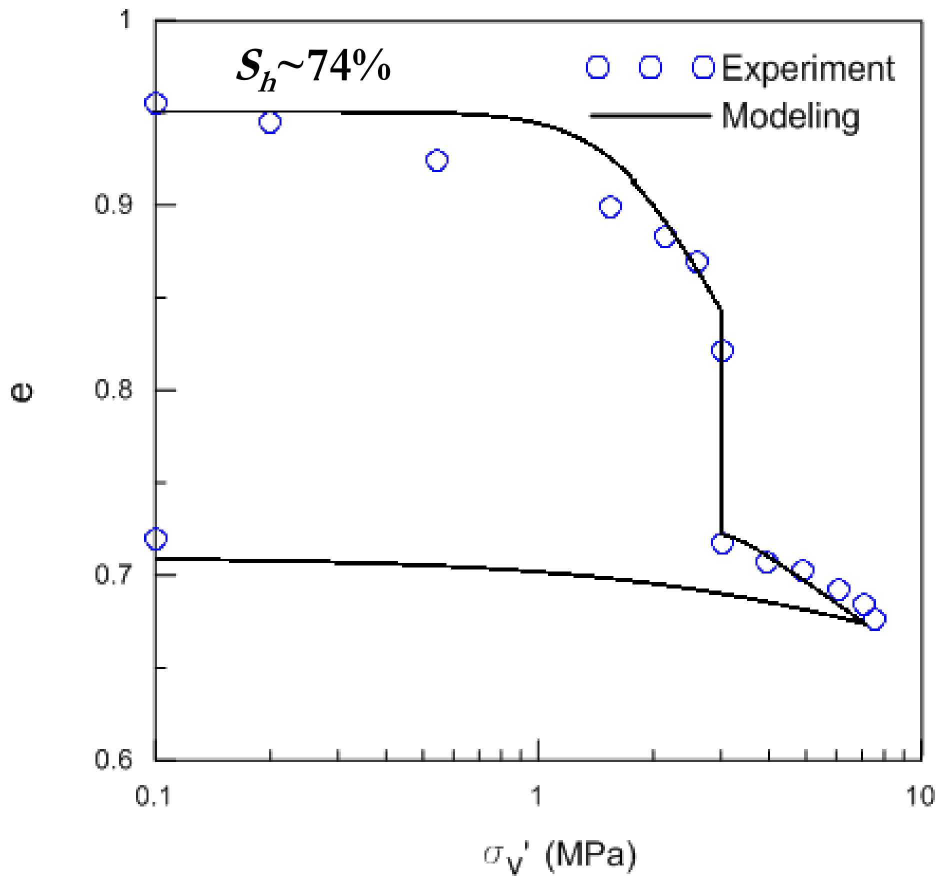


HBS – Mechanical Model Validation

❑ Effect of Hydrate Dissociation

Natural Samples – Oedometric Conditions



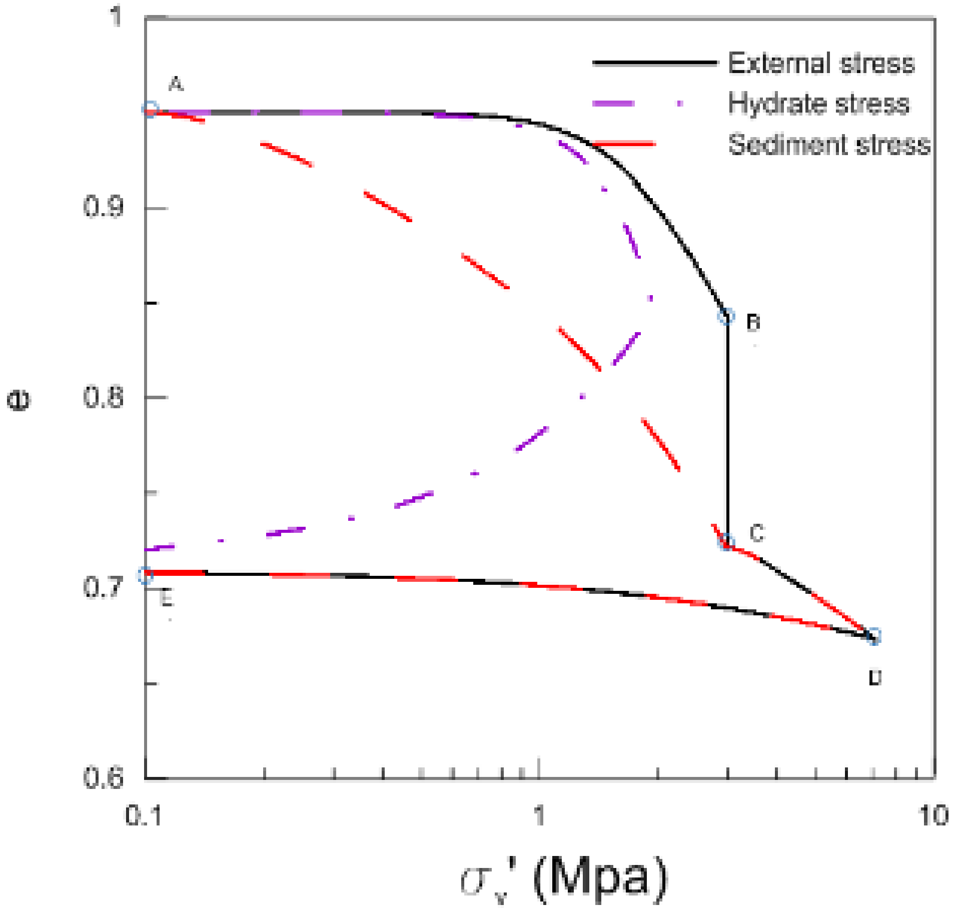


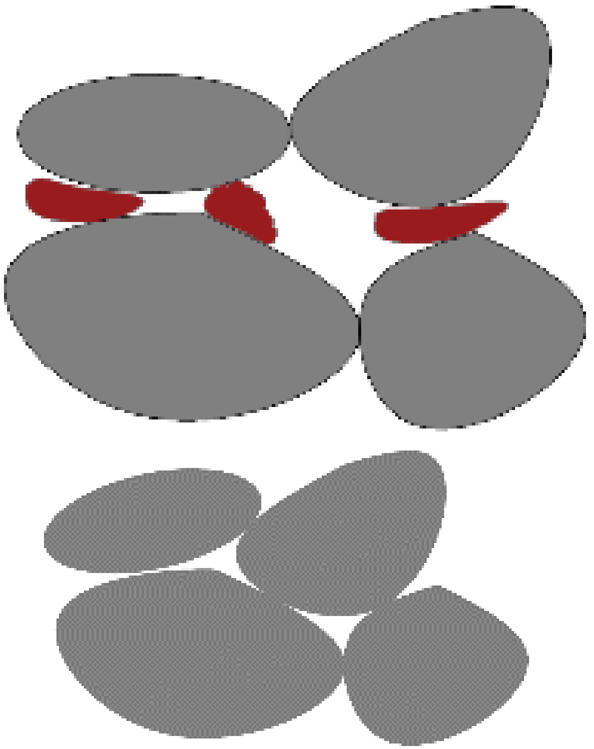
Experimental data from Santamarina et al. (2015)

HBS – Mechanical Model Validation

❑ Effect of Hydrate Dissociation

Oedometric Conditions



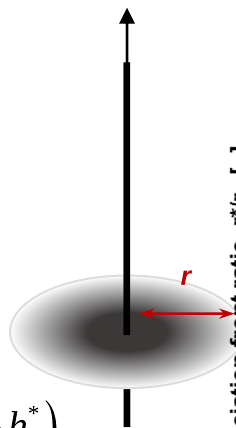


Code verification

➢ Maximum gas production from HBS by depressurization

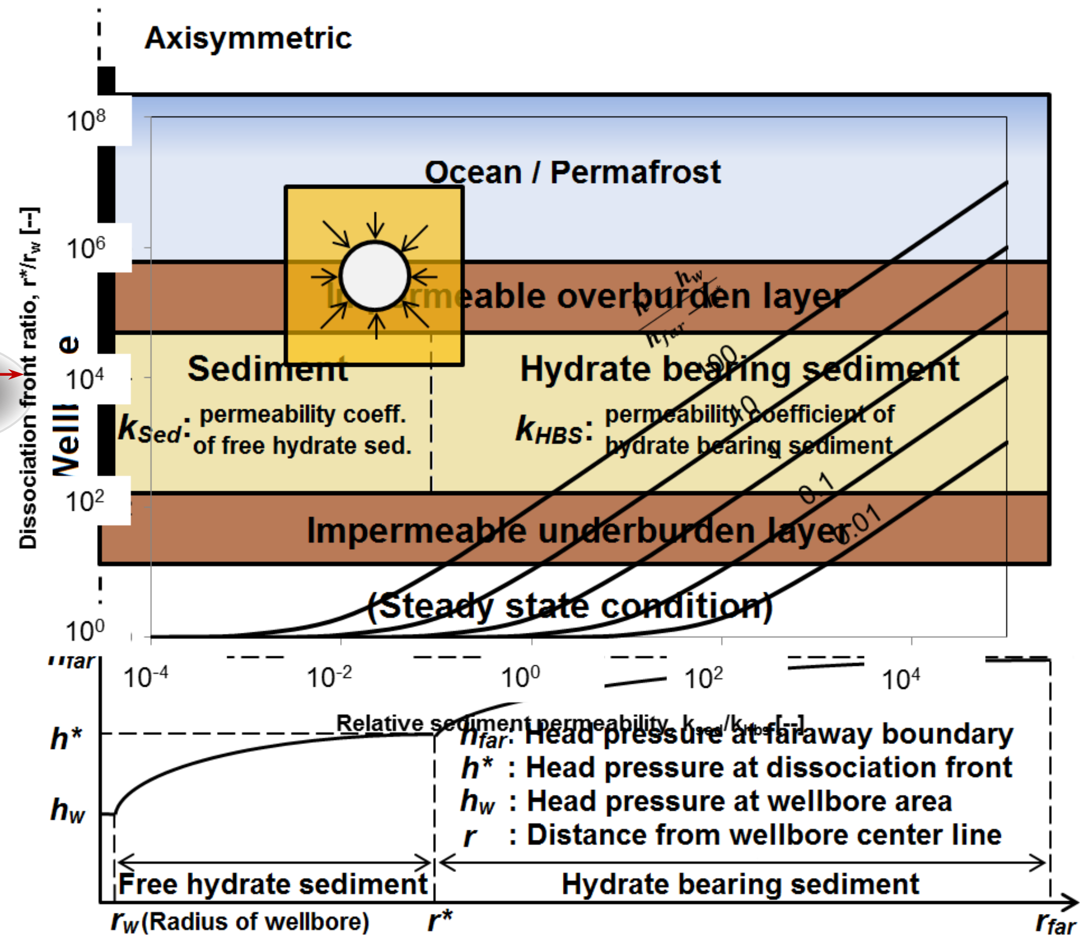
✓ Analytical solution – Cylindrical radial flow - Steady state conditions

$$q = -\frac{2\pi k H (h_2 - h_1)}{\ln(\frac{r_2}{r_1})}$$



$$\frac{2\pi k_{Sed} H \left(h^* - h_w\right)}{\ln\left(\frac{r^*}{r_w}\right)} = \frac{2\pi k_{HBS} H \left(h_{far} - h^*\right)}{\ln\left(\frac{r_{far}}{r^*}\right)}$$

$$r^* = \left(r_w r_{far}^{\left(\frac{k_{Sed}}{k_{HBS}}\right)\left(\frac{h^* - h_w}{h_{far} - h^*}\right)} \right)^{\left(1 + \left(\frac{k_{Sed}}{k_{HBS}}\right)\left(\frac{h^* - h_w}{h_{far} - h^*}\right)\right)^{-1}}$$



Code verification

➢ Maximum gas production from HBS by depressurization

✓ Analytical solution – Cylindrical radial flow - Steady state conditions

- 2D axisymmetric model
- A single vertical well producing
- Very fine grid (2503 elements)

$$k_{HBS} = k_{sed}\left(1 - S_h\right)^N$$

k_{HBS}=1x10^{-12} m^2

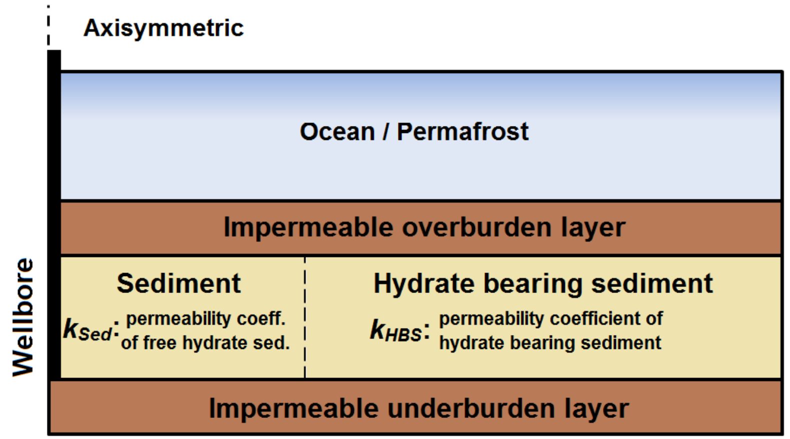
S_h=0.5

ϕ=0.40

L= 1.20 km

h=0.40 m

r_w=0.1m

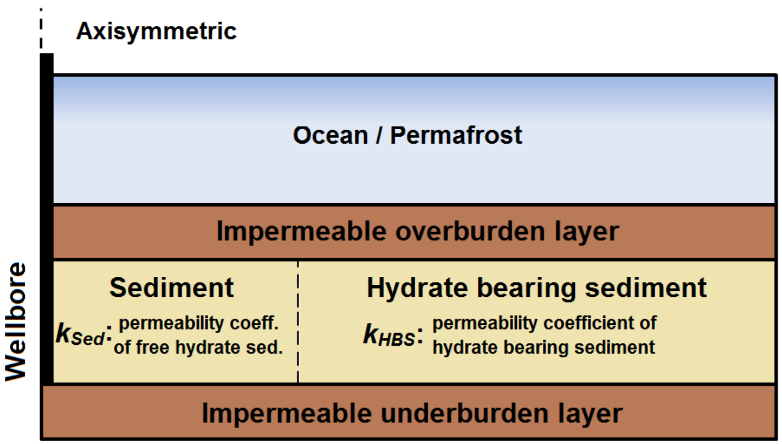


Case	h_{far} (m)	h_w (m)	T (°C)	$\frac{h^* - h_w}{h_{far} - h^*}$
A	1020	306	12	7.14
B	1224	306	12	2.14
C	1224	510	12	1.44
D	1224	306	10	0.91

Code verification

➢ Maximum gas production from HBS by depressurization

✓ Analytical solution – Cylindrical radial flow - Steady state conditions



Case	h_{far} (m)	h_w (m)	T (°C)	$\frac{h^*-h_w}{h_{far}-h^*}$
A	1020	306	12	7.14
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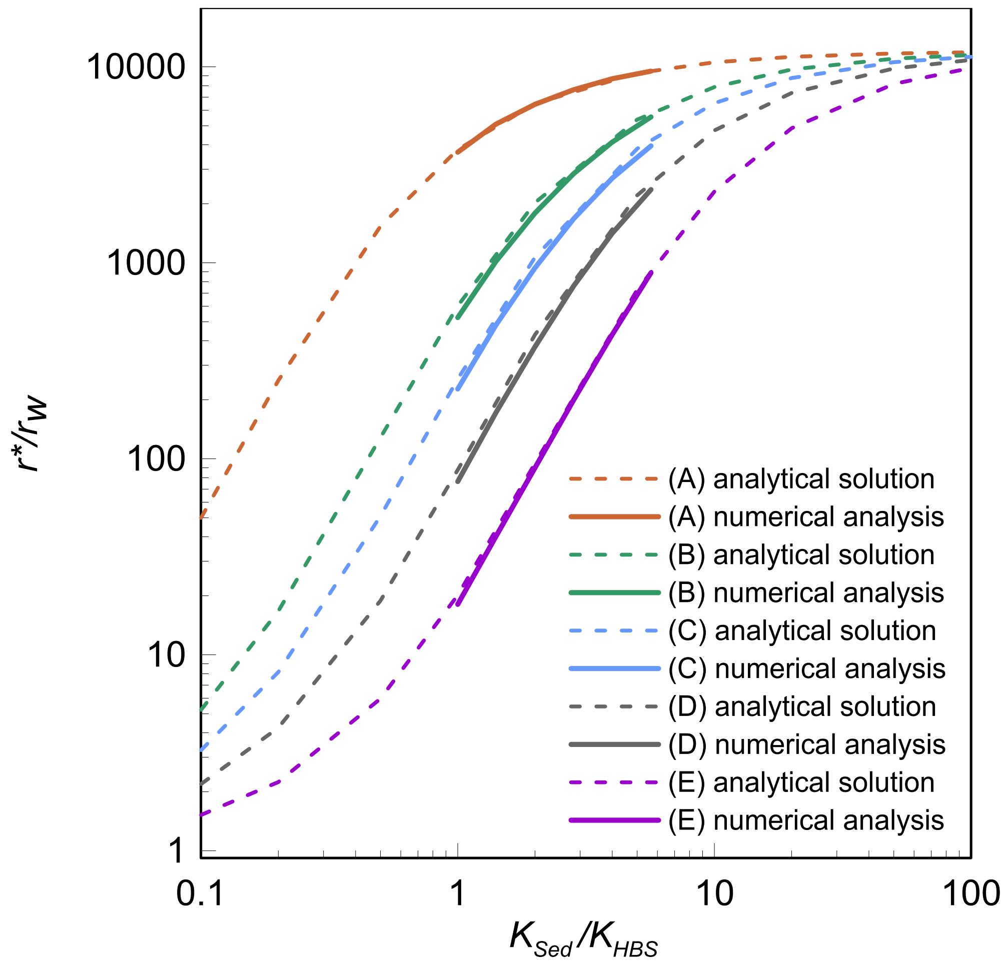
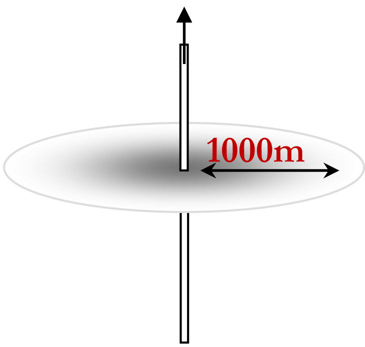
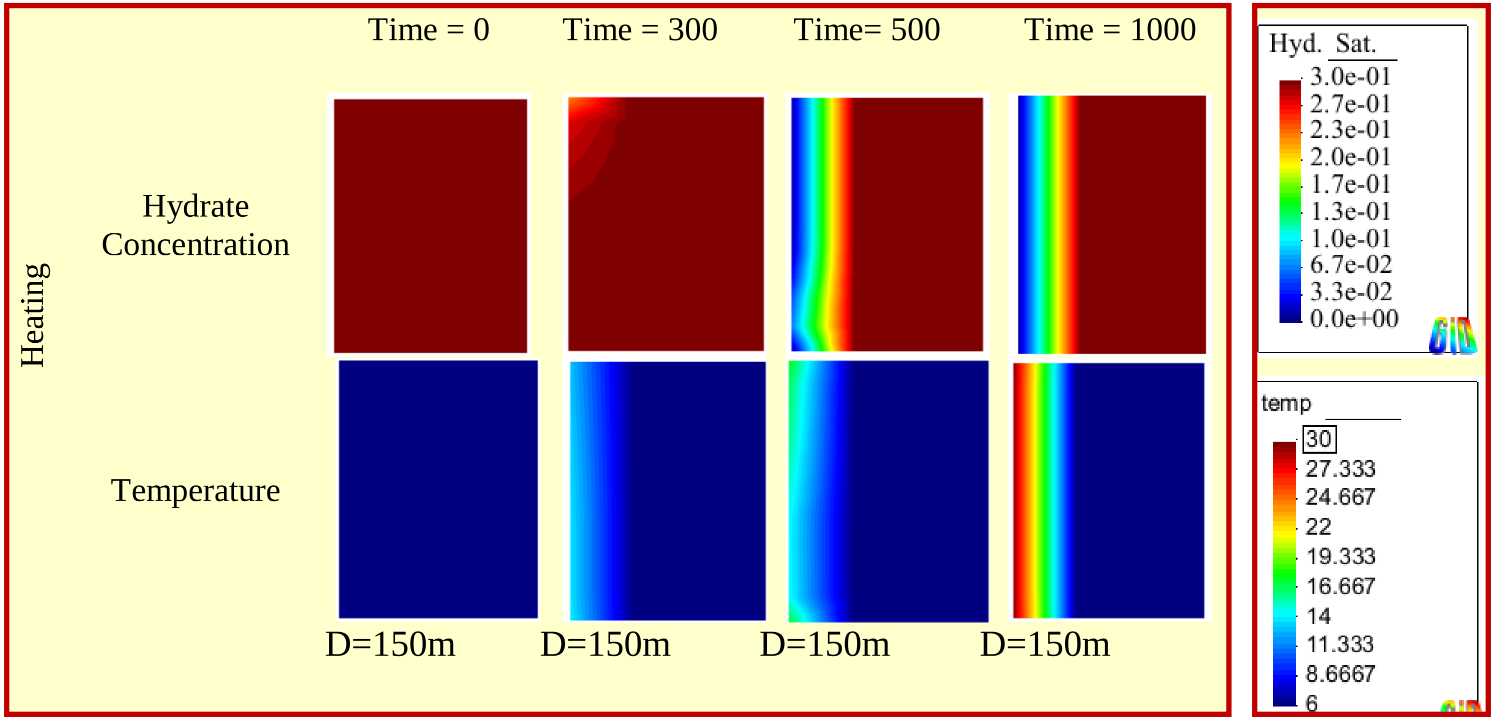
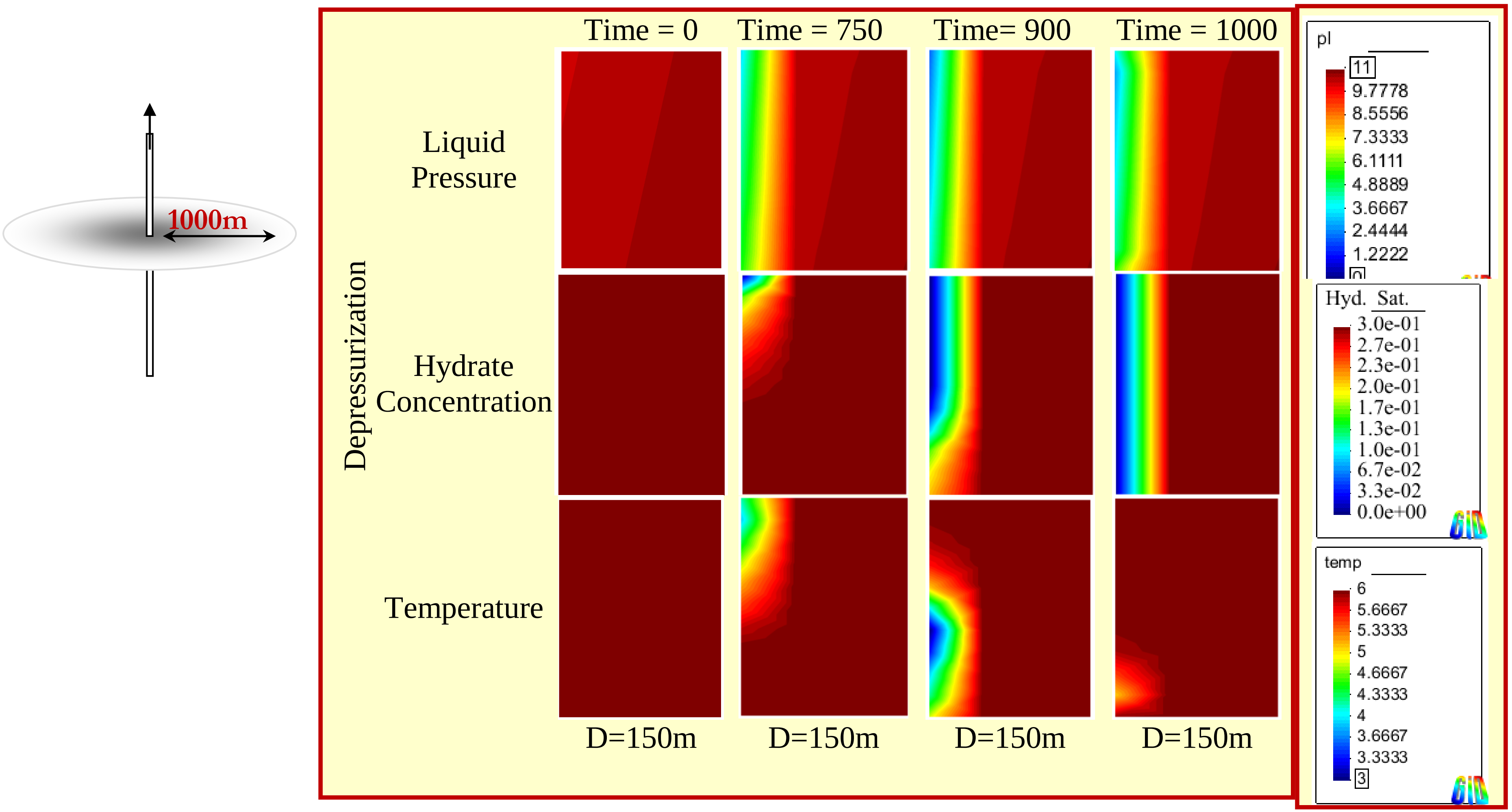


Gas Production in-situ by Heating





Gas Production in-situ by Depressurization



Summary and conclusions

- In this work we present a coupled THCM formulation for modeling the behavior of gas hydrates bearing sediments.
- The proposed approach incorporates the fundamental physical and chemical phenomena that control de behavior of gas hydrates bearing sediments.
- The FE program CODE_BRIGHT has been adapted to incorporate the main balance and constitutive equations related to problems involving gas hydrate sediments.
- An advanced mechanical model for HBS has been proposed and validated.
- Cases studies, at actual scale, modeling the different strategies for gas (methane) production has been analyzed, showing the potential of the proposed approach to model these kinds of problems

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Project: THCM Coupled Model For Hydrate-Bearing Sediments: Data Analysis and Design of New Field Experiments (Marine and Permafrost Settings)

Award No.: DE-FE0013889.