MTiG 4, Modern Trends in Geomechanics, Assisi, Italy, 16.-18. May 2016

Numerical and experimental study of debris flow

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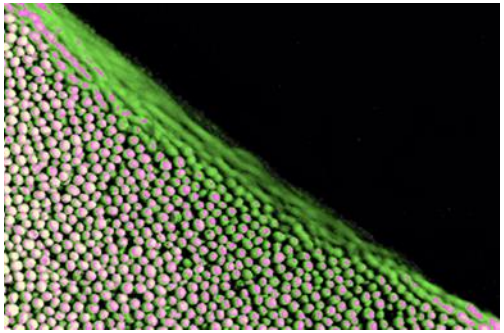
Landslides and debris flows

Content

- Constitutive model: solid and fluid
- SPH: numerical model for large deformation
- LBM-DEM: model for debris flow
- Outlook

Granular flow: from slow to fast



- Stress decomposition (static part+dynamic part):

$$\boldsymbol{\sigma} = \boldsymbol{\sigma_h} + \boldsymbol{\sigma_d} \qquad \dot{\boldsymbol{\sigma}} = \dot{\boldsymbol{\sigma}}_{\boldsymbol{h}} + \dot{\boldsymbol{\sigma}}_{\boldsymbol{d}}$$

- Static part:

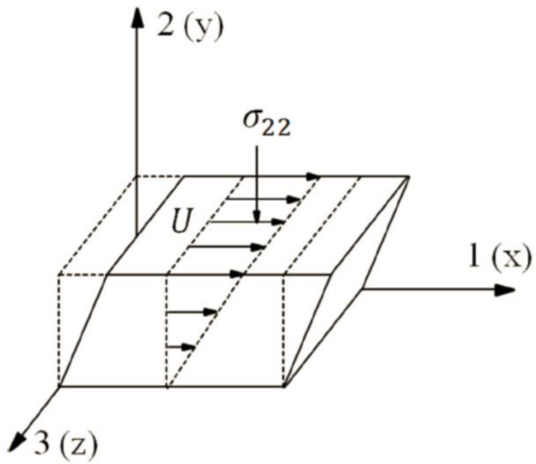
$$\dot{\boldsymbol{\sigma}}_{\boldsymbol{h}} = \boldsymbol{L}(\boldsymbol{\sigma}, \dot{\boldsymbol{\epsilon}}) + f(e)\boldsymbol{N}(\boldsymbol{\sigma})||\dot{\boldsymbol{\epsilon}}||$$

- Dynamic part:

$$\dot{\boldsymbol{\sigma}}_{\boldsymbol{d}} = \boldsymbol{H}(\boldsymbol{\sigma}, \dot{\boldsymbol{\epsilon}}, \ddot{\boldsymbol{\epsilon}}, e)$$

- Some remarks

Bagnold's findings:



- Macro viscous regime:

$$T_v = k_1 \frac{\mathrm{d}U}{\mathrm{d}y},$$

$$k_1 = 2.25\lambda^{\frac{3}{2}}\mu,$$

- Grain inertia regime:

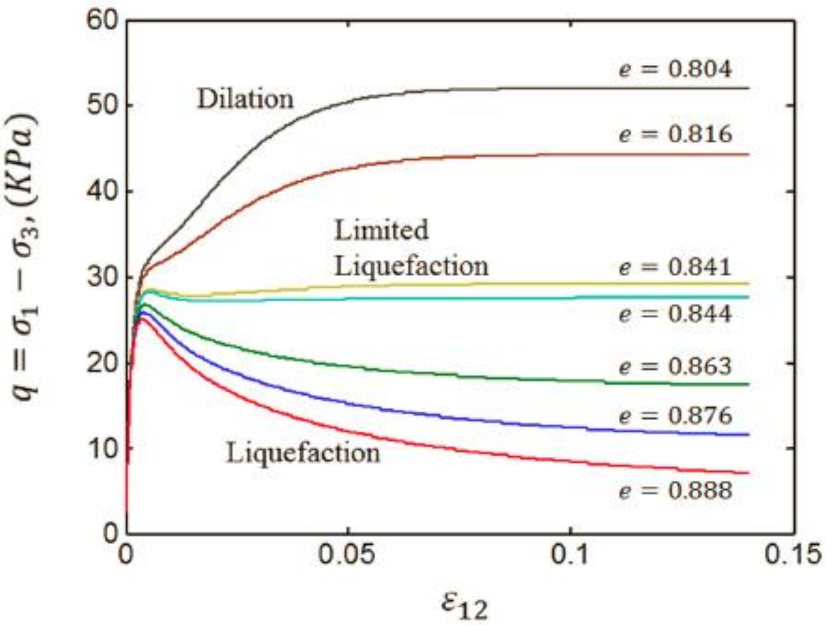
$$T_i = k_2 \left(\frac{\mathrm{d}U}{\mathrm{d}y}\right)^2,$$

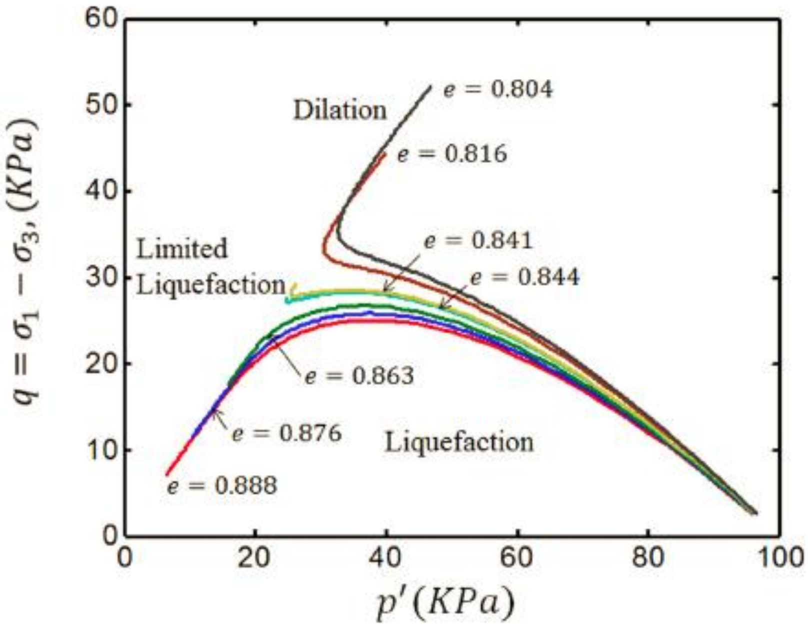
$$k_2 = 0.042\rho_s(\lambda d)^2 \sin\alpha_i,$$

- Bagnold number:

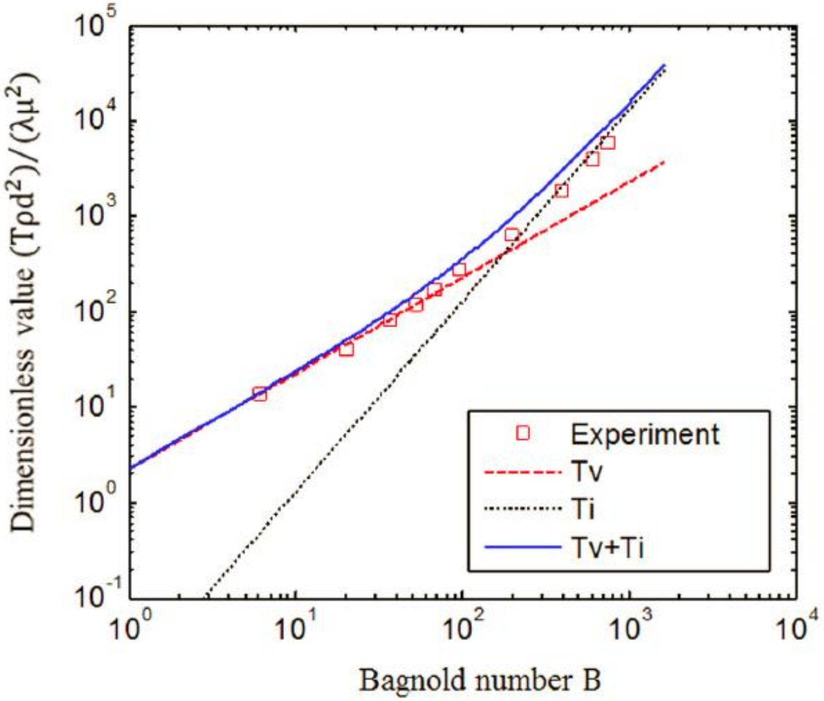
$$B = \frac{\lambda^{\frac{1}{2}}\rho_s d^2(\mathrm{d}U/\mathrm{d}y)}{\mu},$$

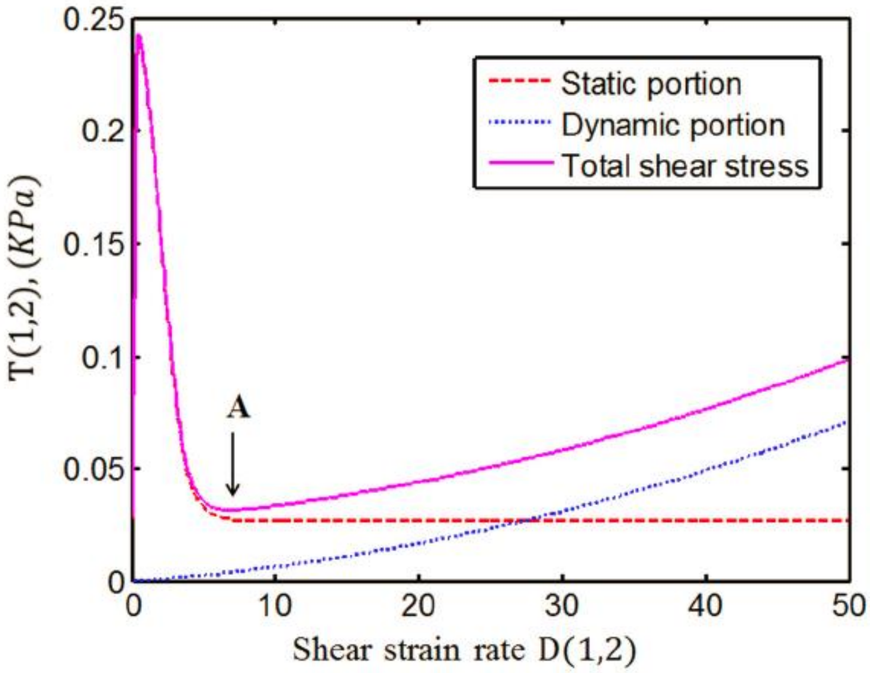
Model performance (SS)





Numerical simulation of simple shear

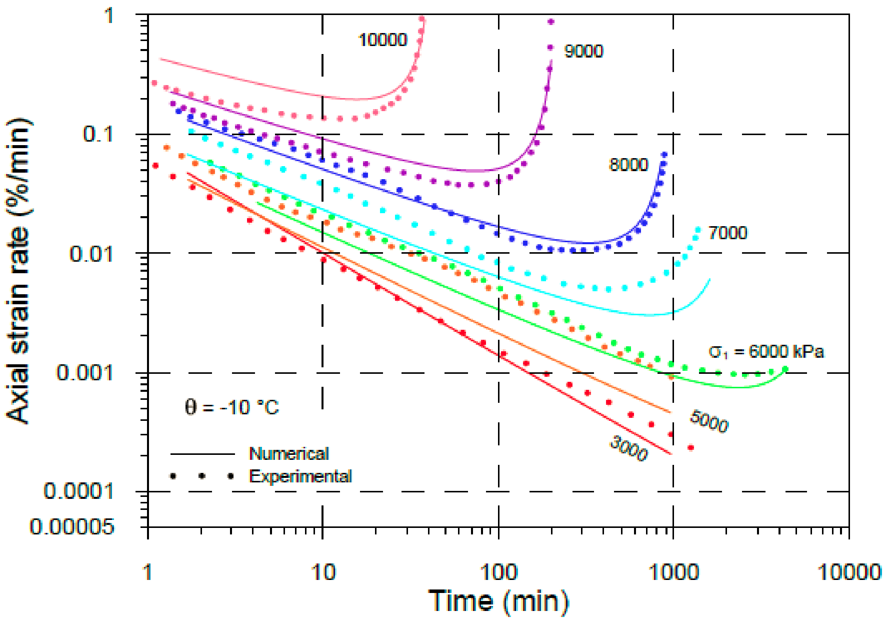


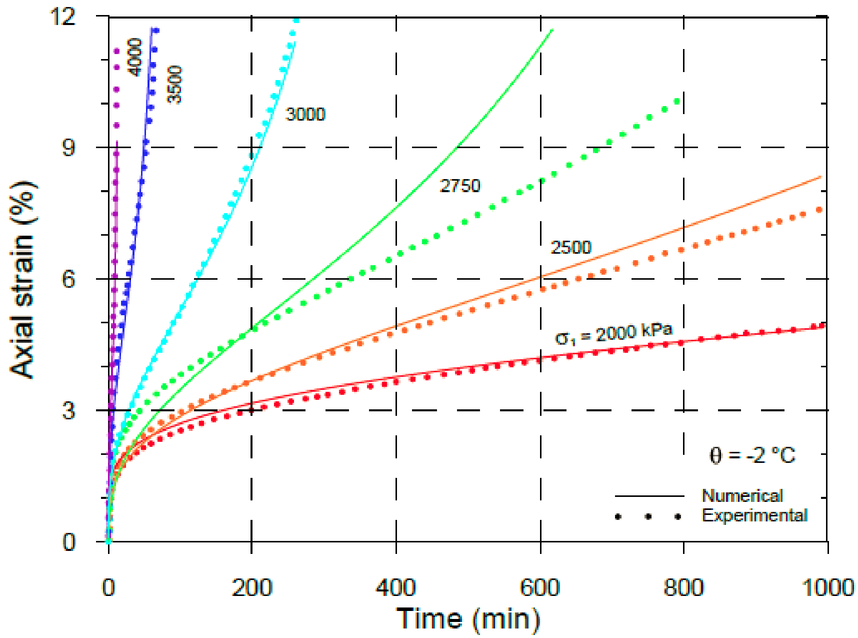


Guo, XG, Peng, Ch, Wu, W, Wang, YQ, Hypoplastic constitutive model for debris material, *Granular Matter*, submitted, 2016

Modelling creep

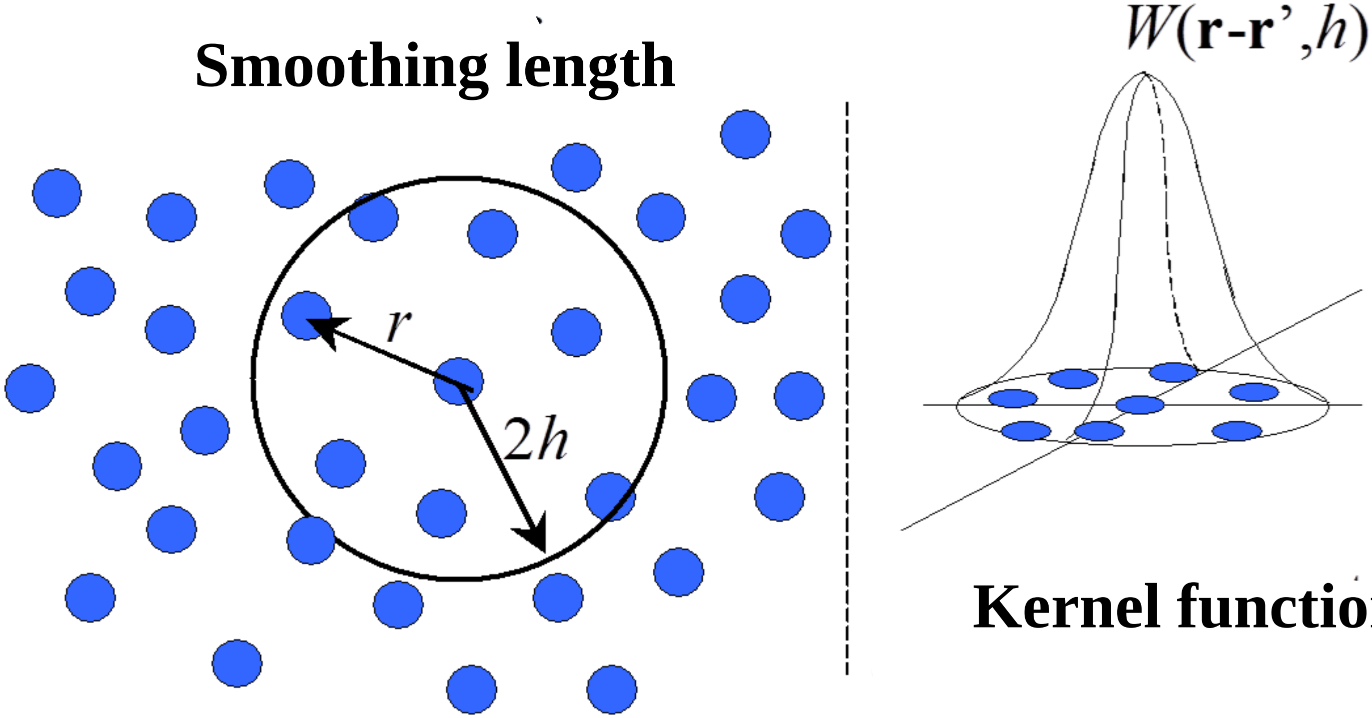
$$\dot{\mathbf{T}} = \dot{\mathbf{T}}_h + \dot{\mathbf{T}}_d = 0$$





Xu, GF, Wu, W, Qi, JL, Modeling the viscous behavior of frozen soil with hypoplasticity, Int. J.Numer. Anal. Methods Geomech., in press, 2016

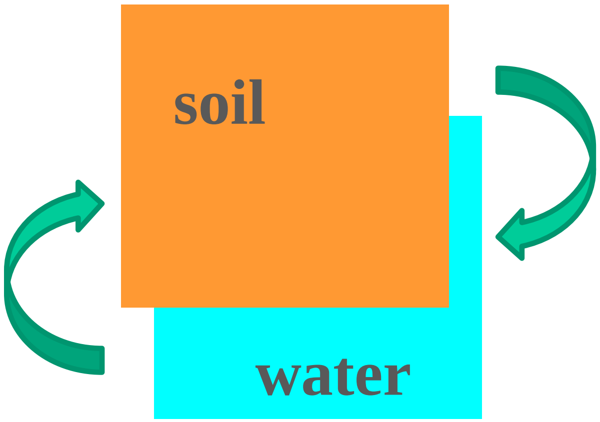
SPH Principle



Field equations: Soil-water mixture

Mathematical model – Mixture theory

- **Soil and water occupy the whole domain simultaneously;**
- **Each constituent satisfies its own balance equations;**
- **Interactions are modelled by buoyance force and drag force.**



$$\partial_t(\tilde{\rho}_s\phi_s) + \nabla\cdot(\tilde{\rho}_s\phi_s\boldsymbol{v}_s) = 0$$

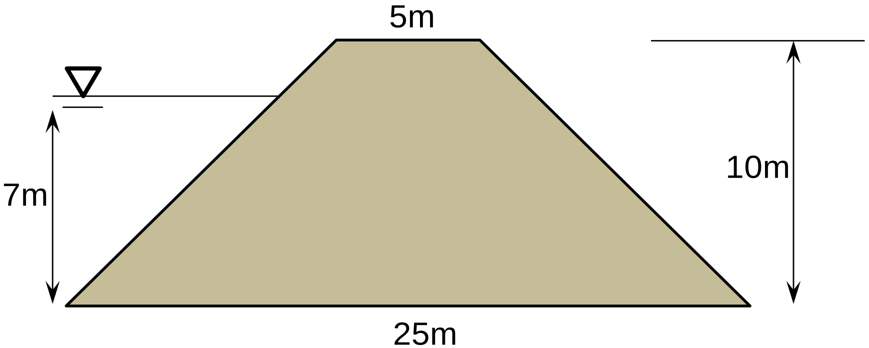
$$\partial_t(\tilde{\rho}_s\phi_s\boldsymbol{v}_s) + \nabla\cdot(\tilde{\rho}_s\phi_s\boldsymbol{v}_s\boldsymbol{v}_s) = \nabla\cdot(\phi_s\boldsymbol{\sigma}_s) + \tilde{\rho}_s\phi_s\boldsymbol{g} - \phi_s\nabla p + \boldsymbol{f}_d$$

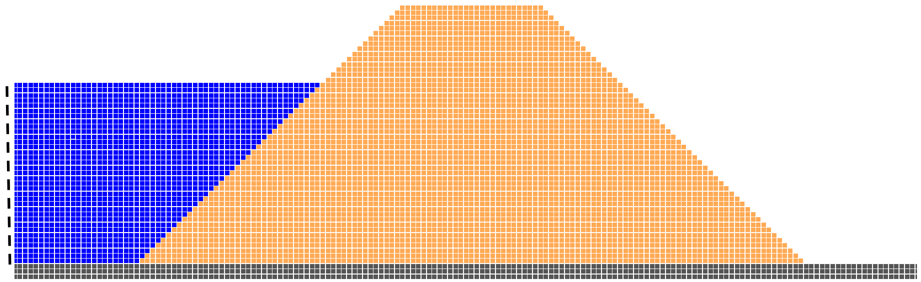
$$\partial_t(\tilde{\rho}_f\phi_f) + \nabla\cdot(\tilde{\rho}_f\phi_f\boldsymbol{v}_f) = 0$$

$$\partial_t(\tilde{\rho}_f\phi_f\boldsymbol{v}_f) + \nabla\cdot(\tilde{\rho}_f\phi_f\boldsymbol{v}_f\boldsymbol{v}_f) = -\phi_f\nabla p + \nabla\cdot(\phi_f\boldsymbol{\tau}_f) + \tilde{\rho}_f\phi_f\boldsymbol{g} - \boldsymbol{f}_d$$

Numerical example

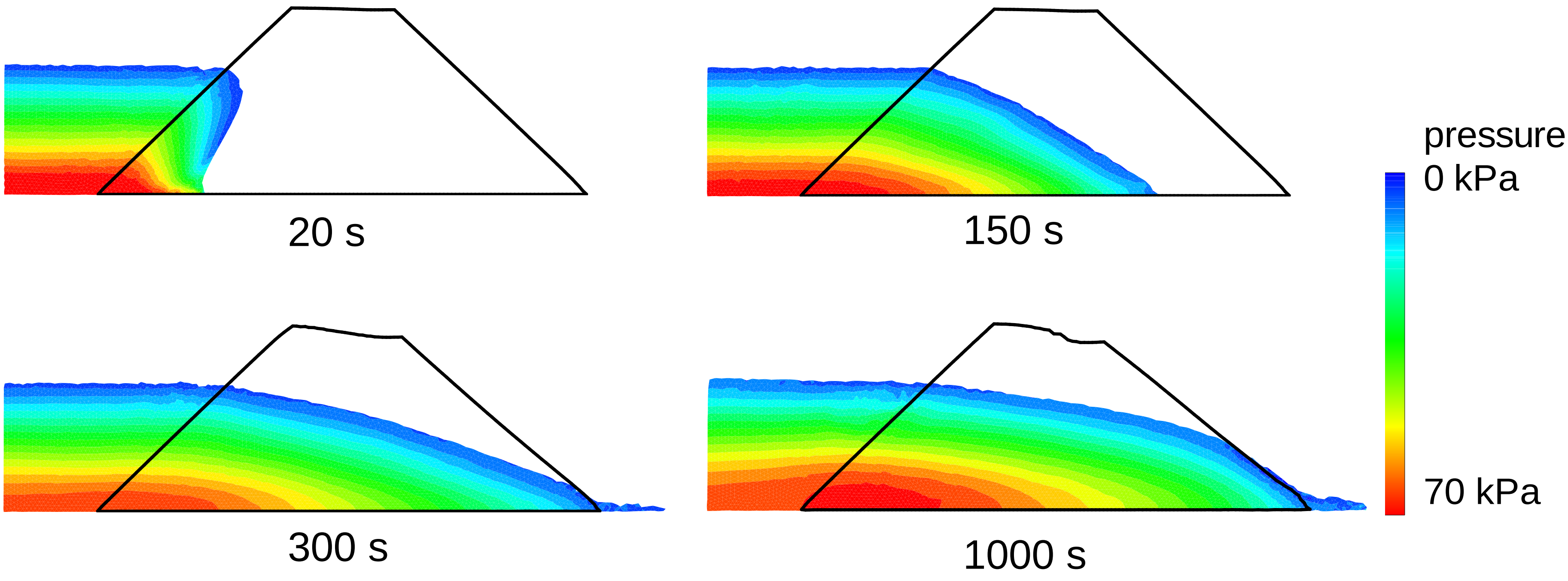
Seepage failure of an embankment





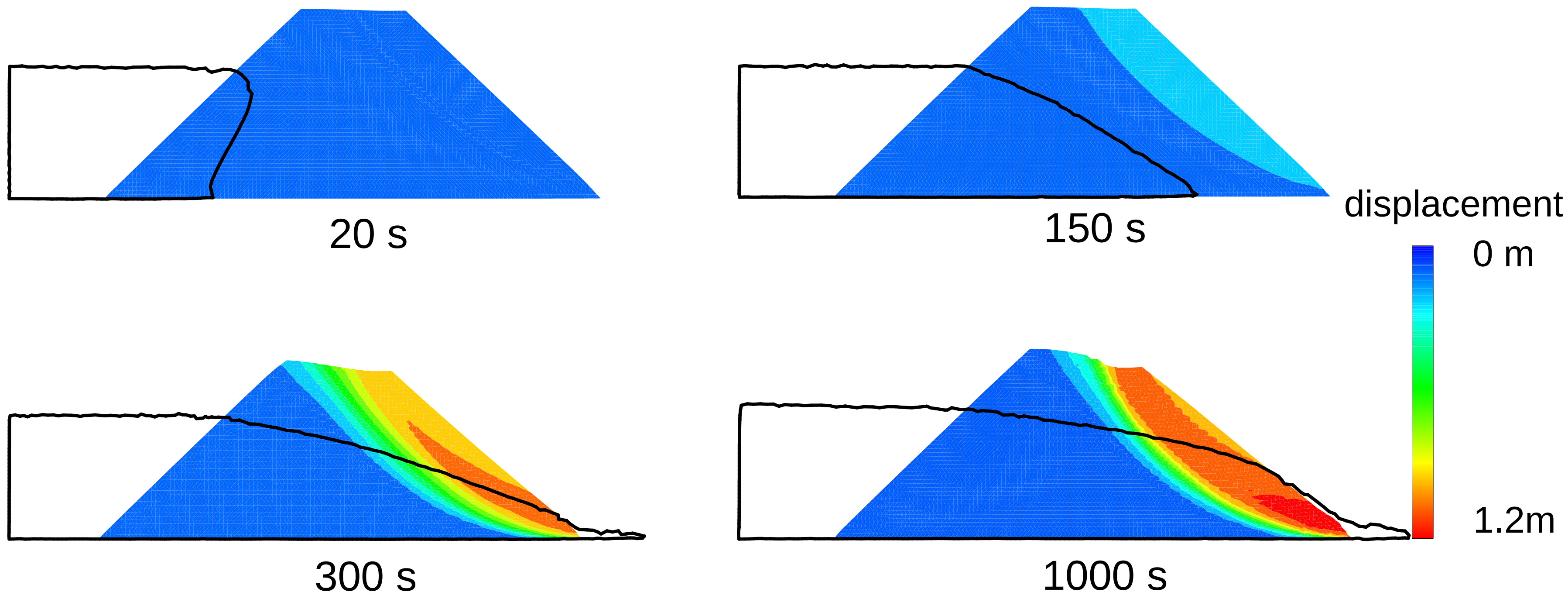
Numerical example

Problem 2: Seepage failure of an embankment



Numerical examples

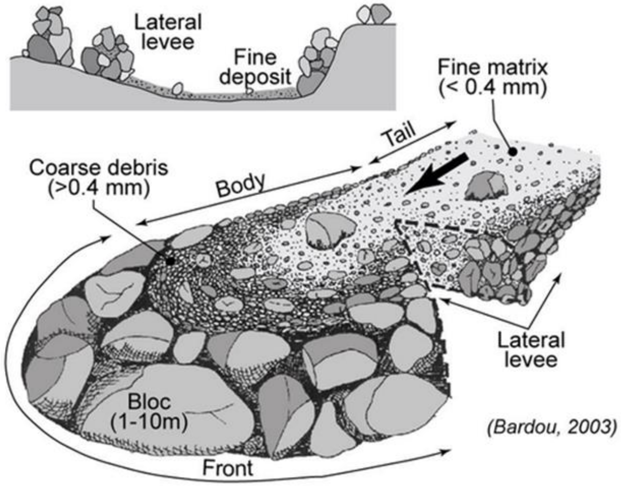
Seepage failure of an embankment



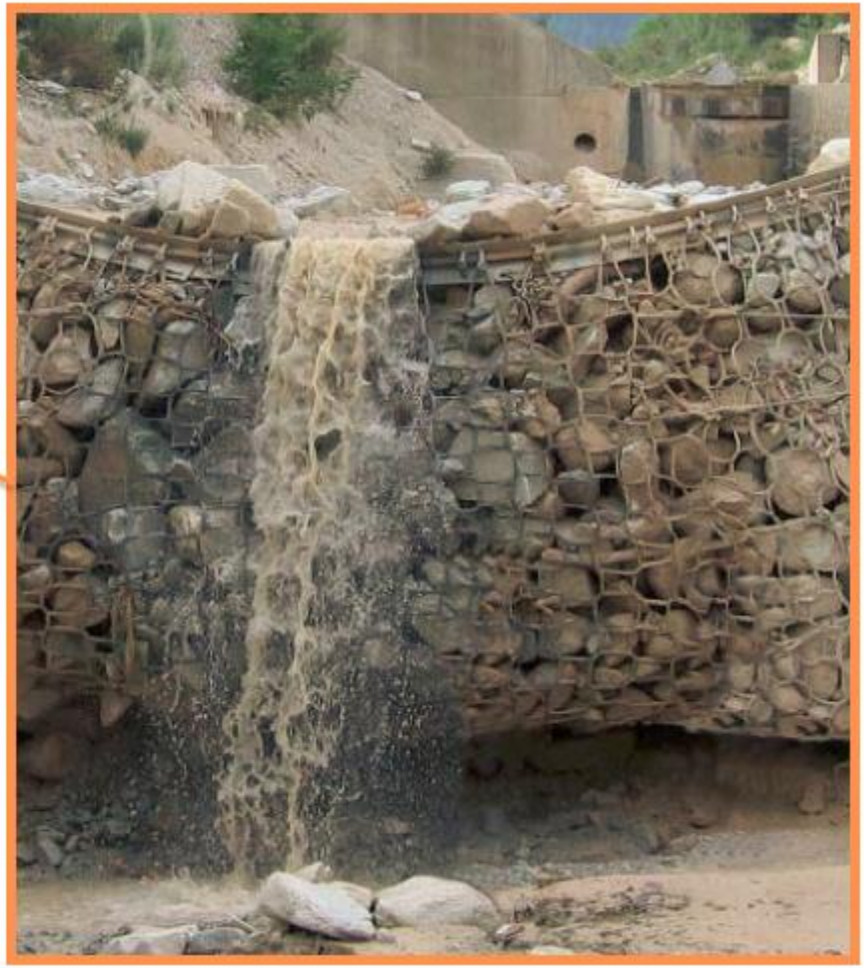
Peng, Ch, Wu, W, Yu, HS, Wang, CH, A SPH approach for large deformation analysis with hypoplastic constitutive model, *Acta Geotechnica*, **10**, 703-717, 2015

LBM-DEM-FEM

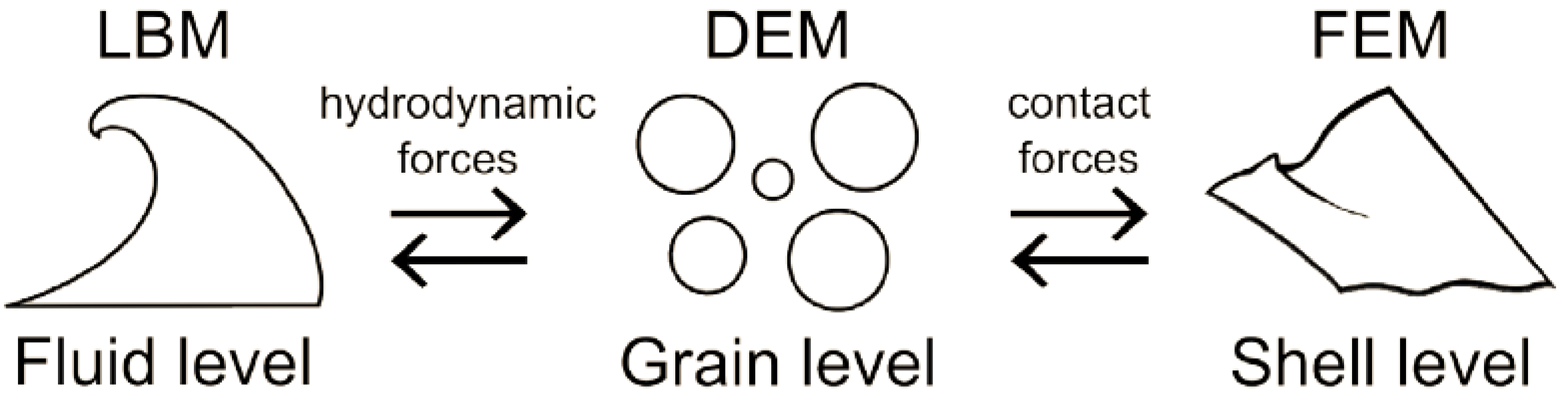
Images of the test conducted by Geobrugg AG



(Bardou, 2003)



Why these methods?



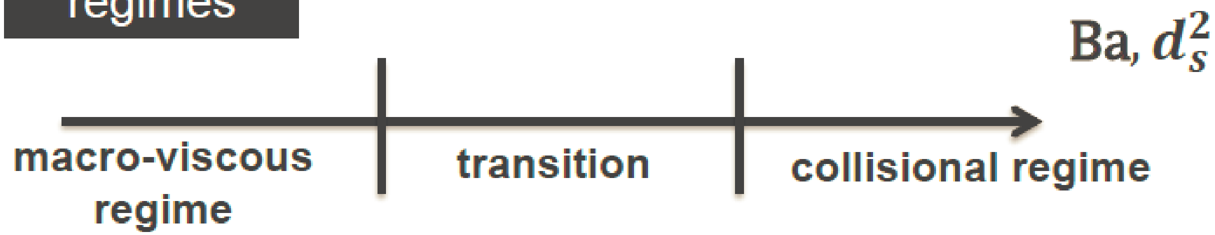
Rheology of debris material

$$\frac{\sigma_{coll}}{\sigma_{visc}} = \frac{\rho_s d_s^2 {\lambda_s}^{1/2} \dot{\gamma}}{\mu_f} = \mathrm{Ba}$$

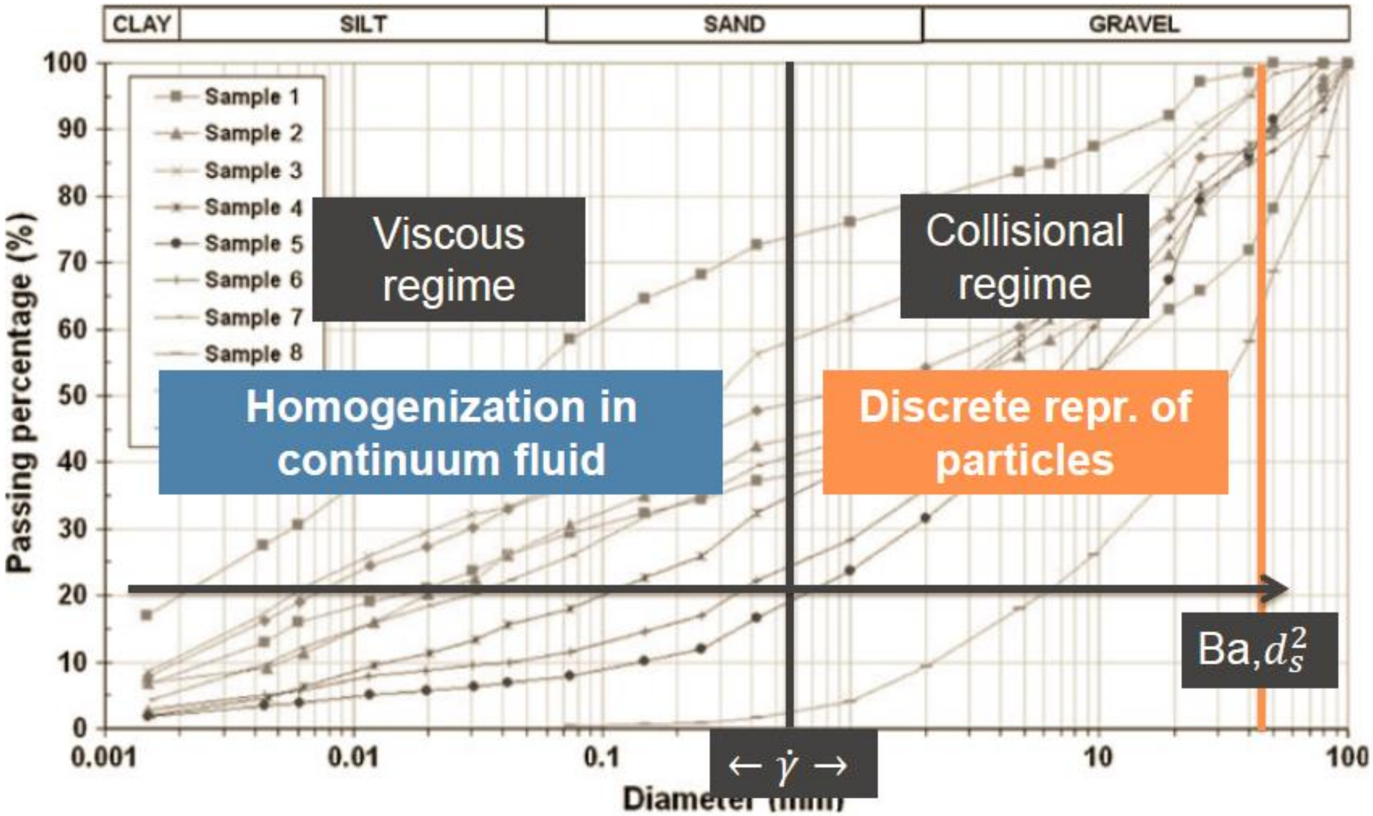
Bagnold Number

- ρ_s particle density
- d_s particle size
- λ_s particle concentration
- $\dot{\gamma}$ shear rate
- μ_f fluid dynamic viscosity

Mixture regimes

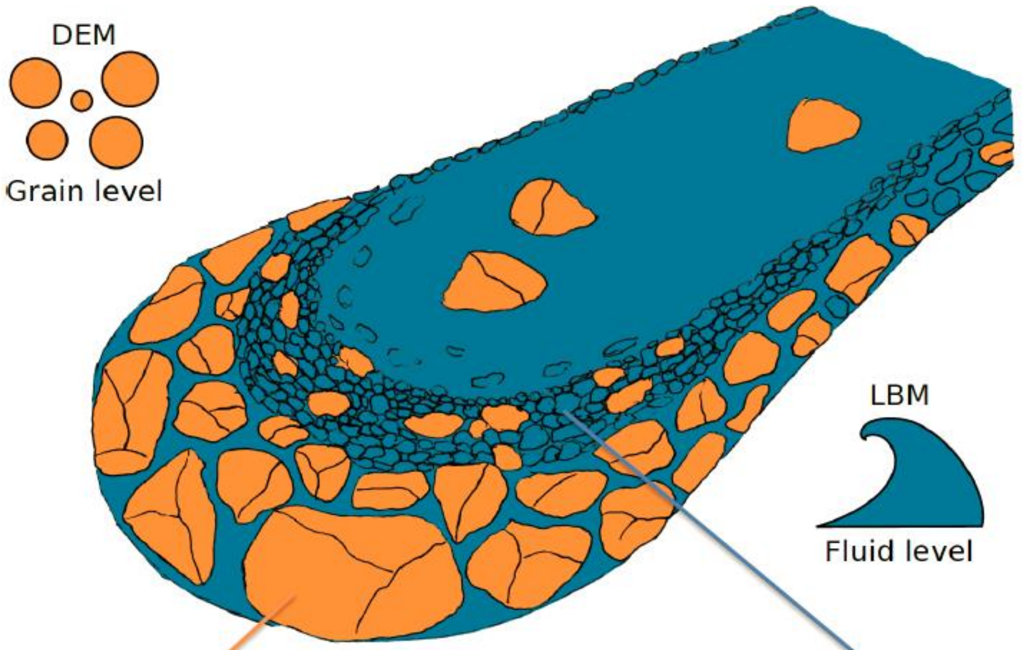


Fluid and solid



T. Bisantino, P. Fischer & F. Gentile, "Rheological characteristics of debris-flow material in South-Gargano watersheds", Natural Hazards, 54(2), 209-223, 2009

LBM-DEM coupling



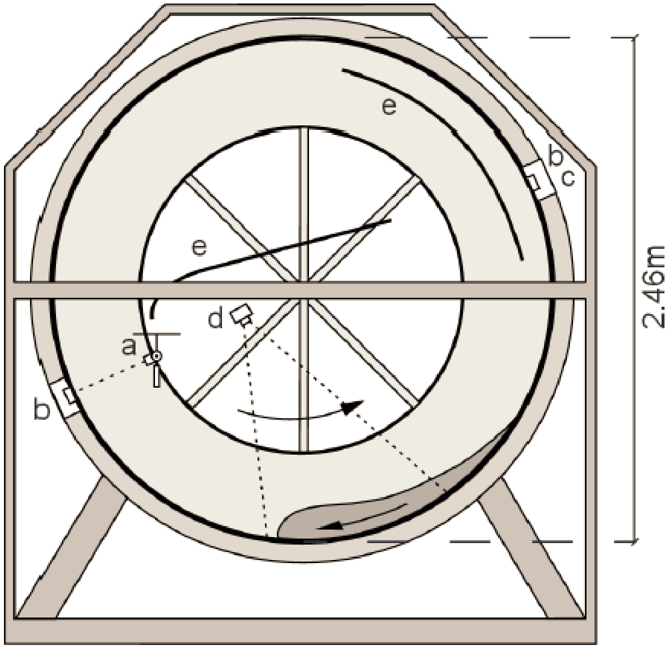
Particles: Discrete Element Method
- Collisional mechanism
- Captures segregation

Fluid: Lattice-Boltzmann Method
- Almost linear scaling
- Easy parallelization
- Can manage sophisticated boundary conditions

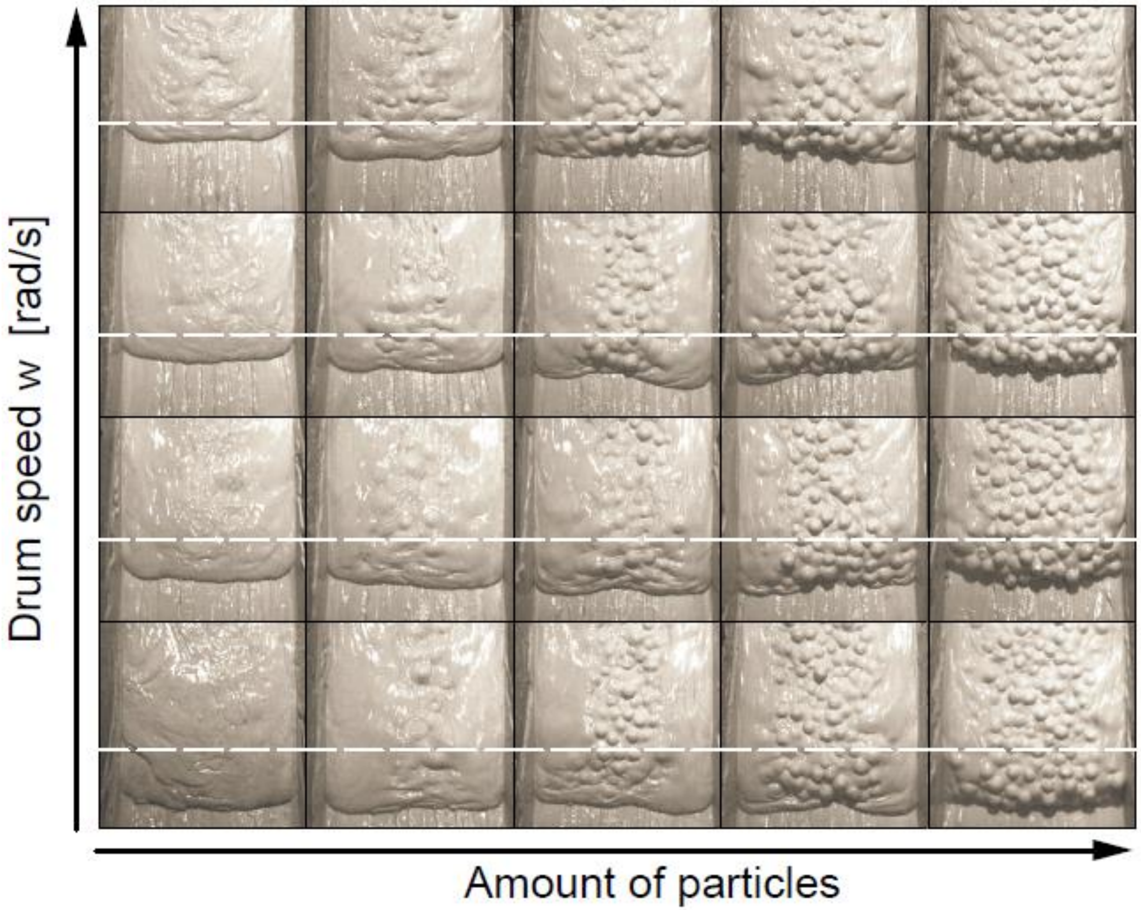
Debris material: viscosity measurement

Conventional

Rotating drum (cylinder)



Flow pattern (clay balls in kaolinite suspension)

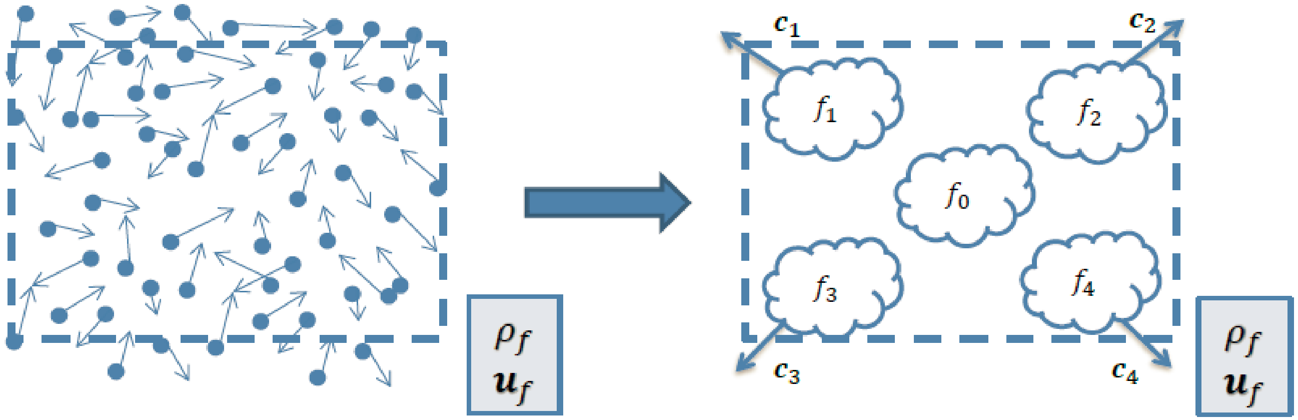


LBM basics

A mesoscopic approach to fluid dynamics

Probability density function «Population»

$f = f(\boldsymbol{x}, t, \boldsymbol{c})$

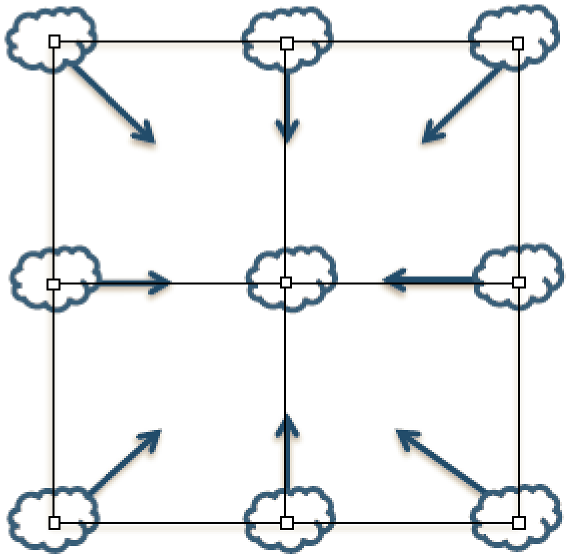


Density $\rho_f = \sum_i f_i$

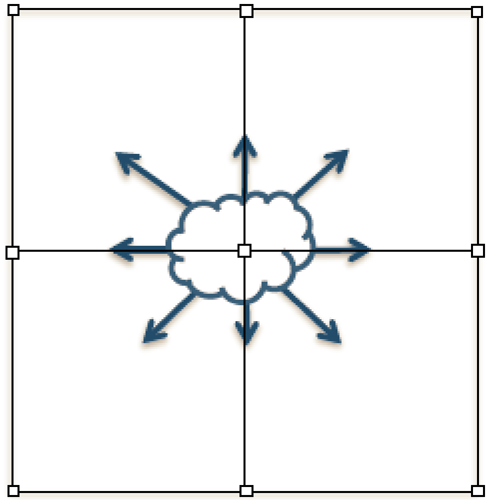
Velocity $\boldsymbol{u}_f = \sum_i f_i \boldsymbol{c}_i / \rho_f$

A two-step solution procedure:

Streaming Step



Collision Step



$$\underbrace{f_i(\boldsymbol{x} + \boldsymbol{c}_i, t + 1) = f_i(\boldsymbol{x}, t)}_{\text{Streaming}} + \underbrace{\Omega_i(\boldsymbol{x}, t)}_{\text{Collision}}$$

Rotating drum test

Mixture (40% particles), rotating drum with variable angular speed

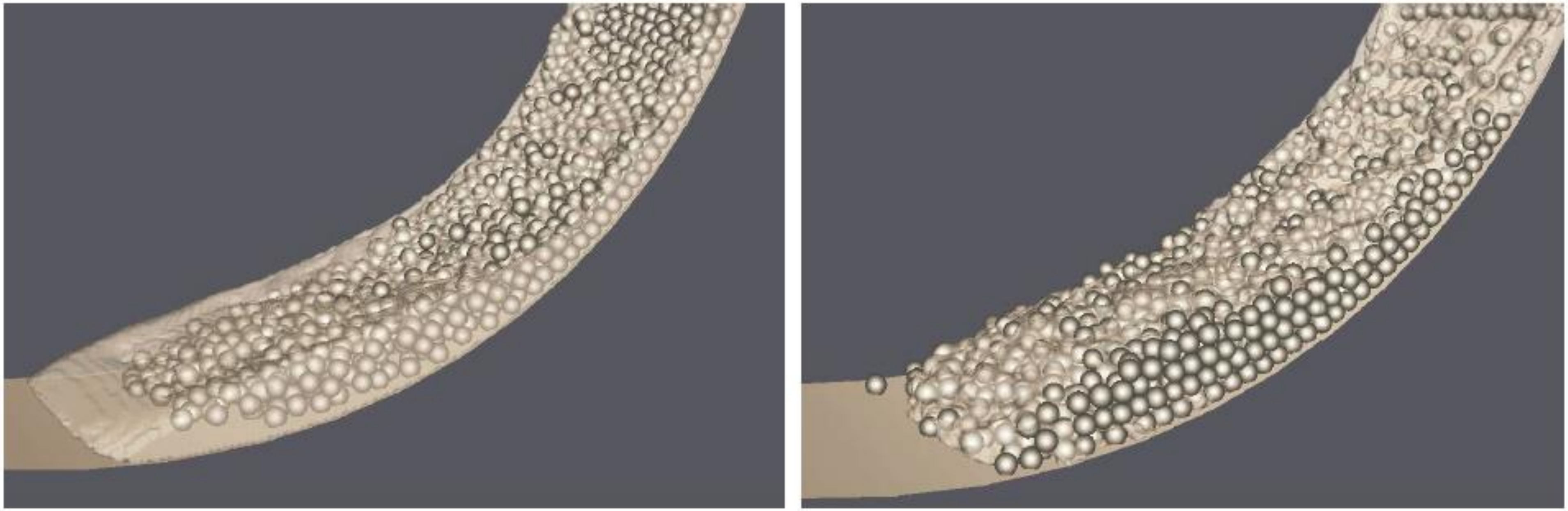
$0.3\ rad/s$

$0.7\ rad/s$

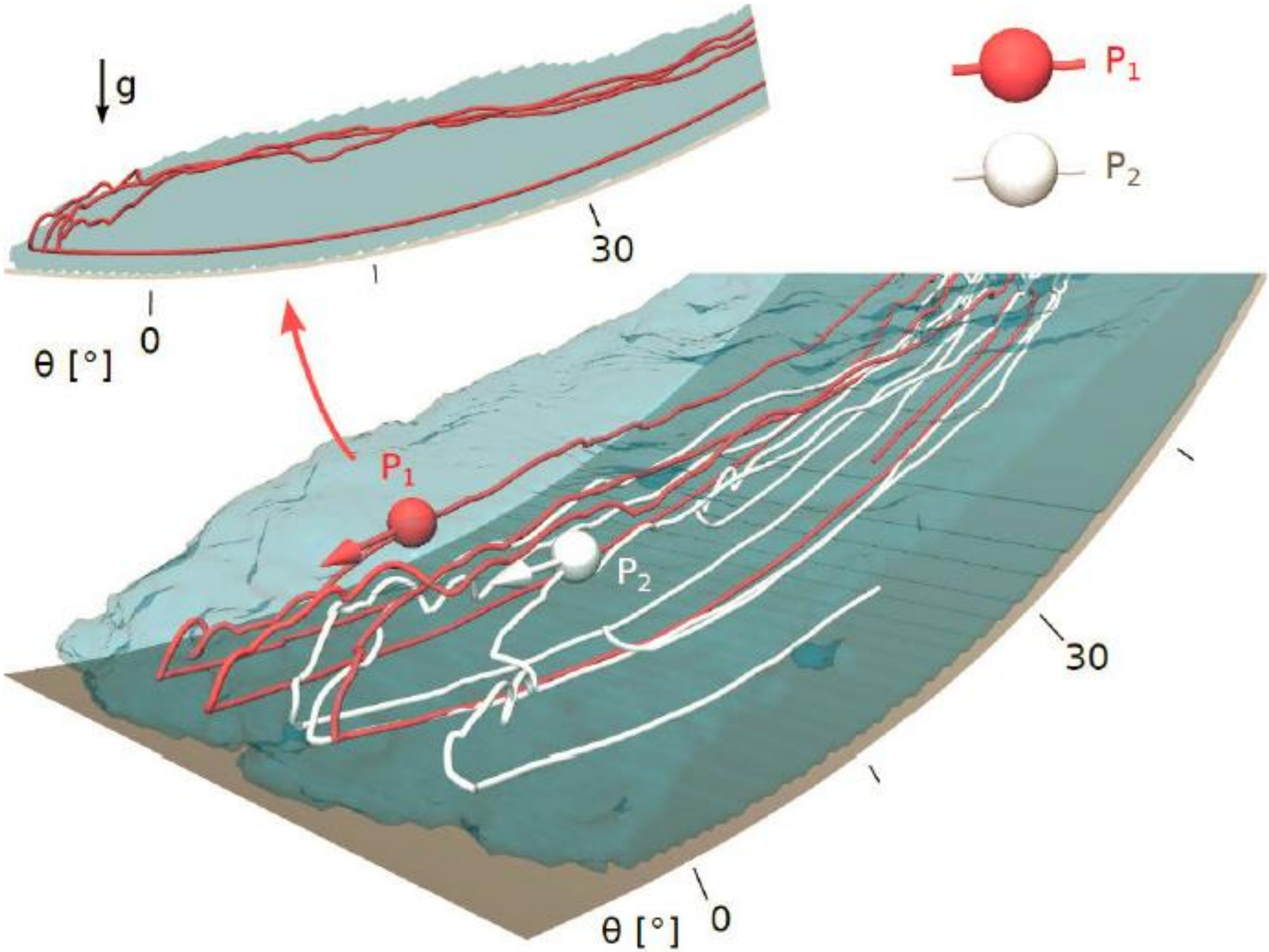
$0.9\ rad/s$

Numerical simulation

Mixture (40% particles)

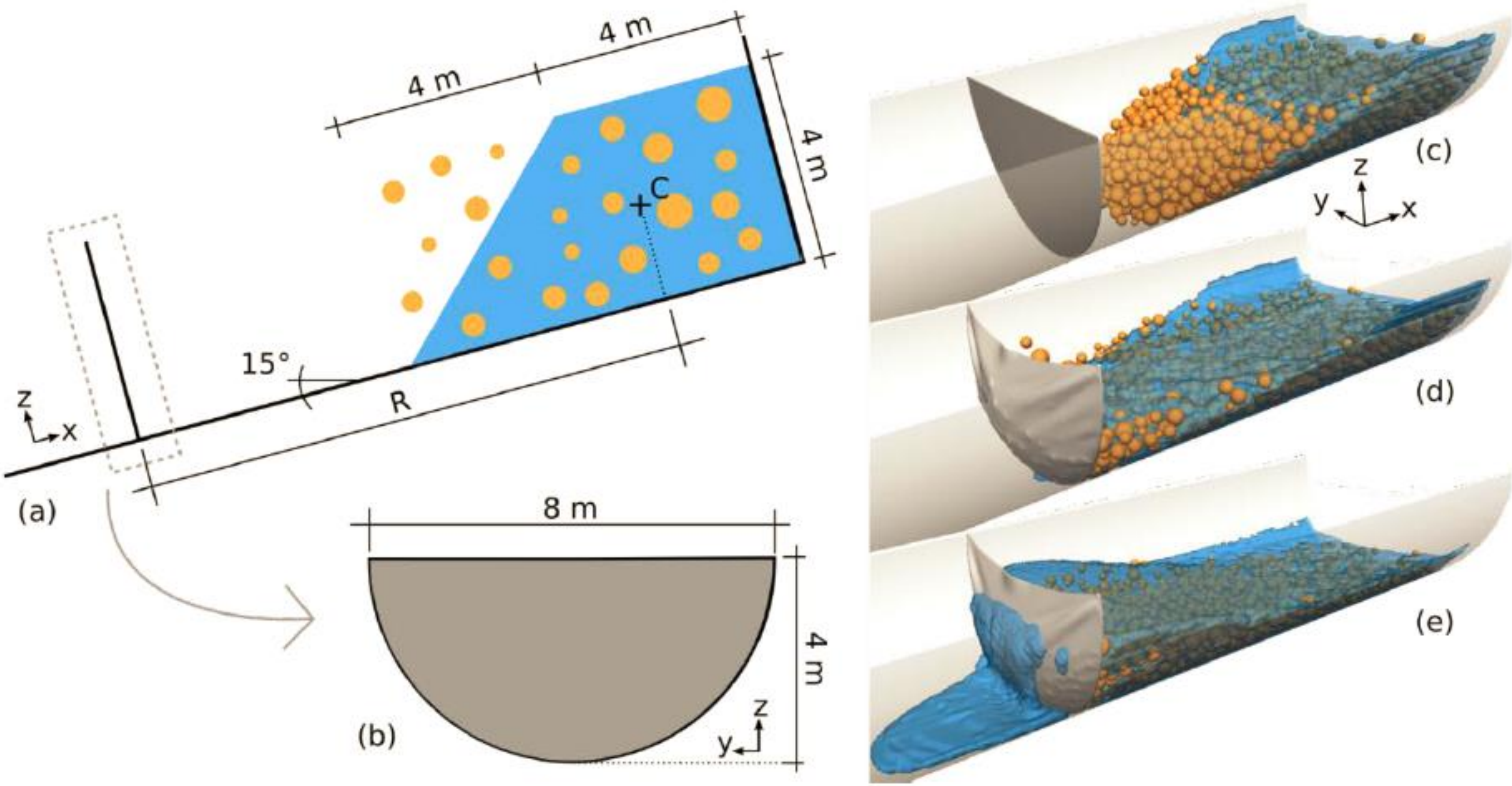


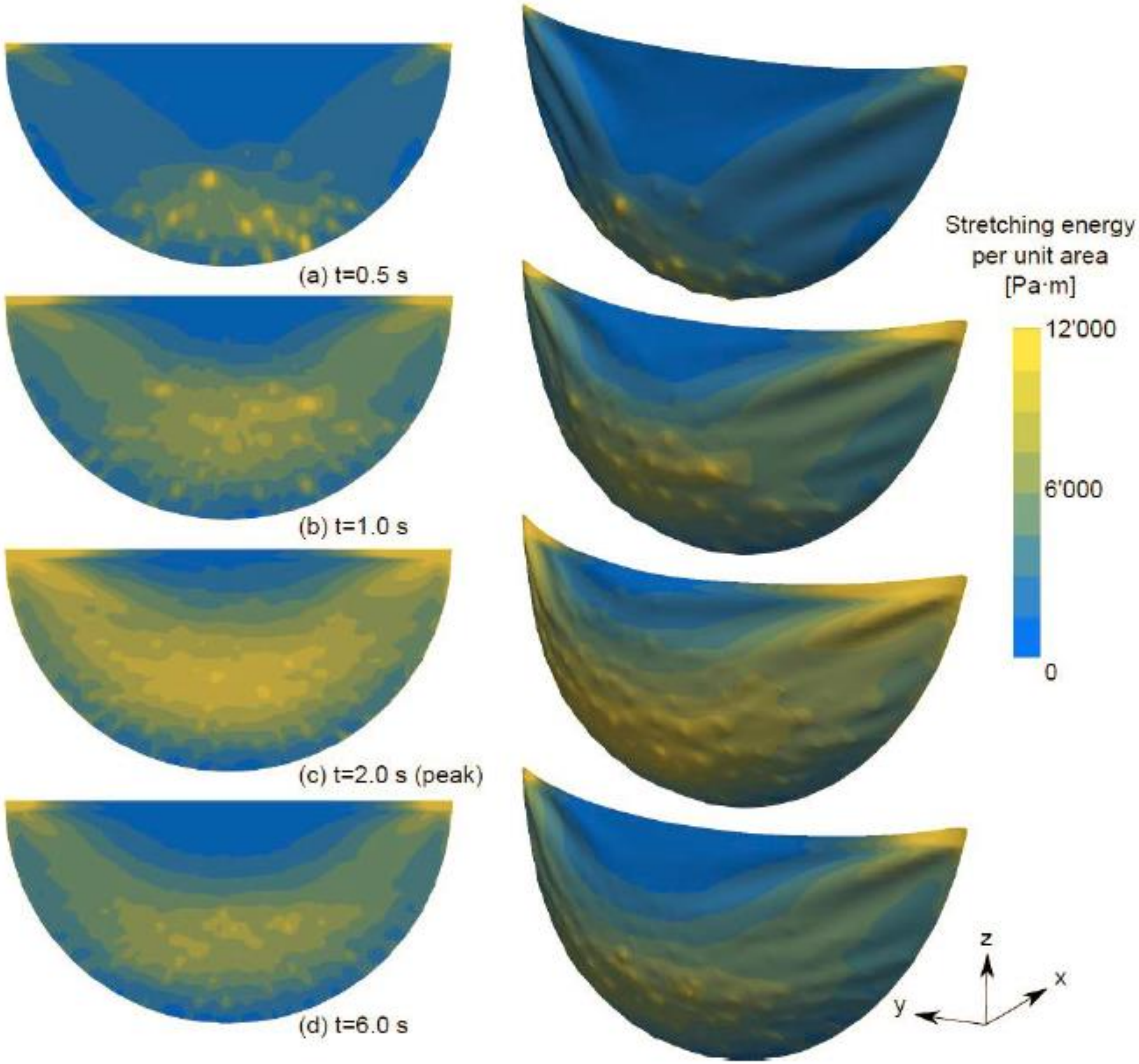
$0.3\ rad/s$ $0.9\ rad/s$



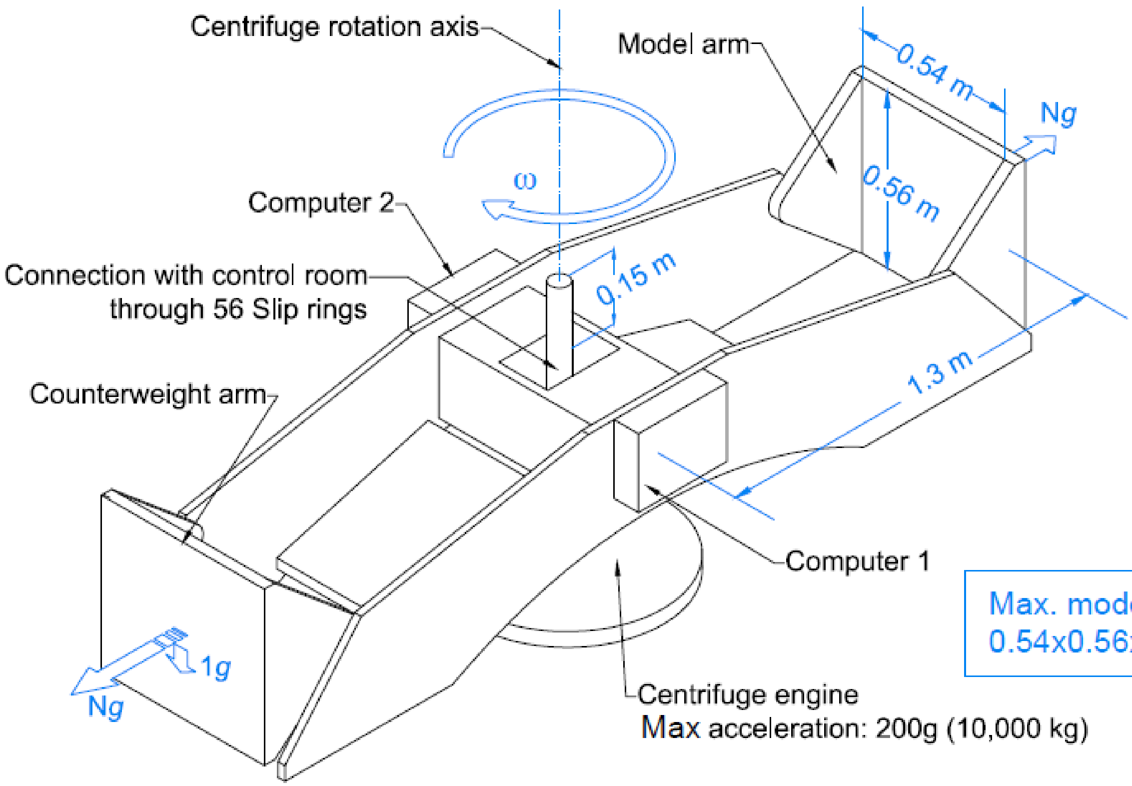
Leonardi, M. Cabrera, F. K. Wittel, R. Kaitna, M. Mendoza, W. Wu, H. J. Herrmann,, Granular front formation in free-surface flow of concentrated suspensions, *Physics Review*, E**92**, 052204, 2015

LBM-DEM-FEM: flexible barrier



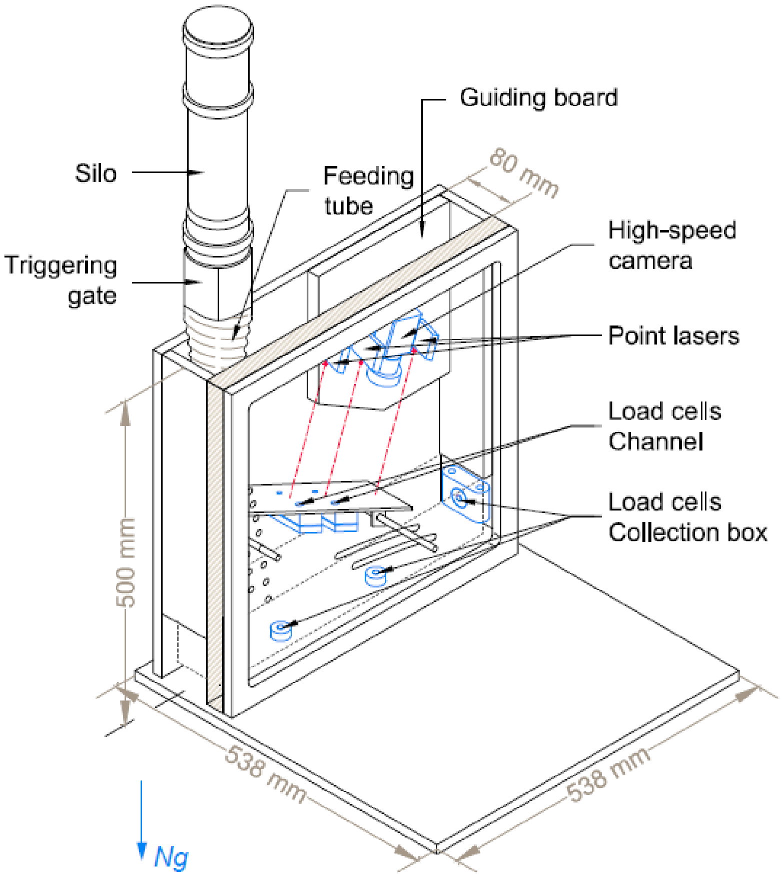

(a) t=0.5 s
(b) t=1.0 s
(c) t=2.0 s (peak)
(d) t=6.0 s
Stretching energy per unit area [Pa·m]
12'000
6'000
0
z
x
y

Centrifuge in Vienna

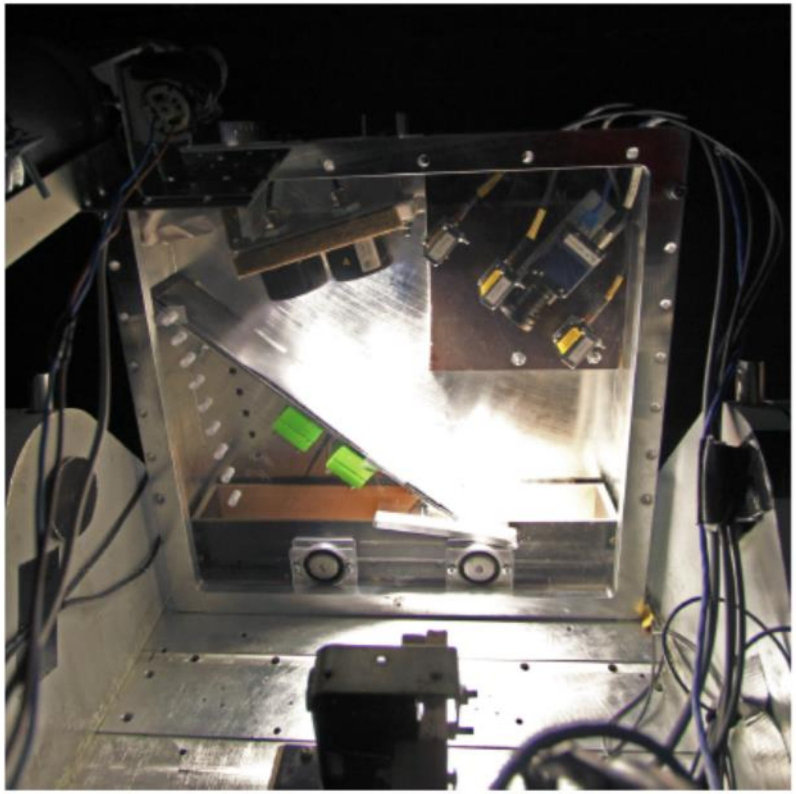


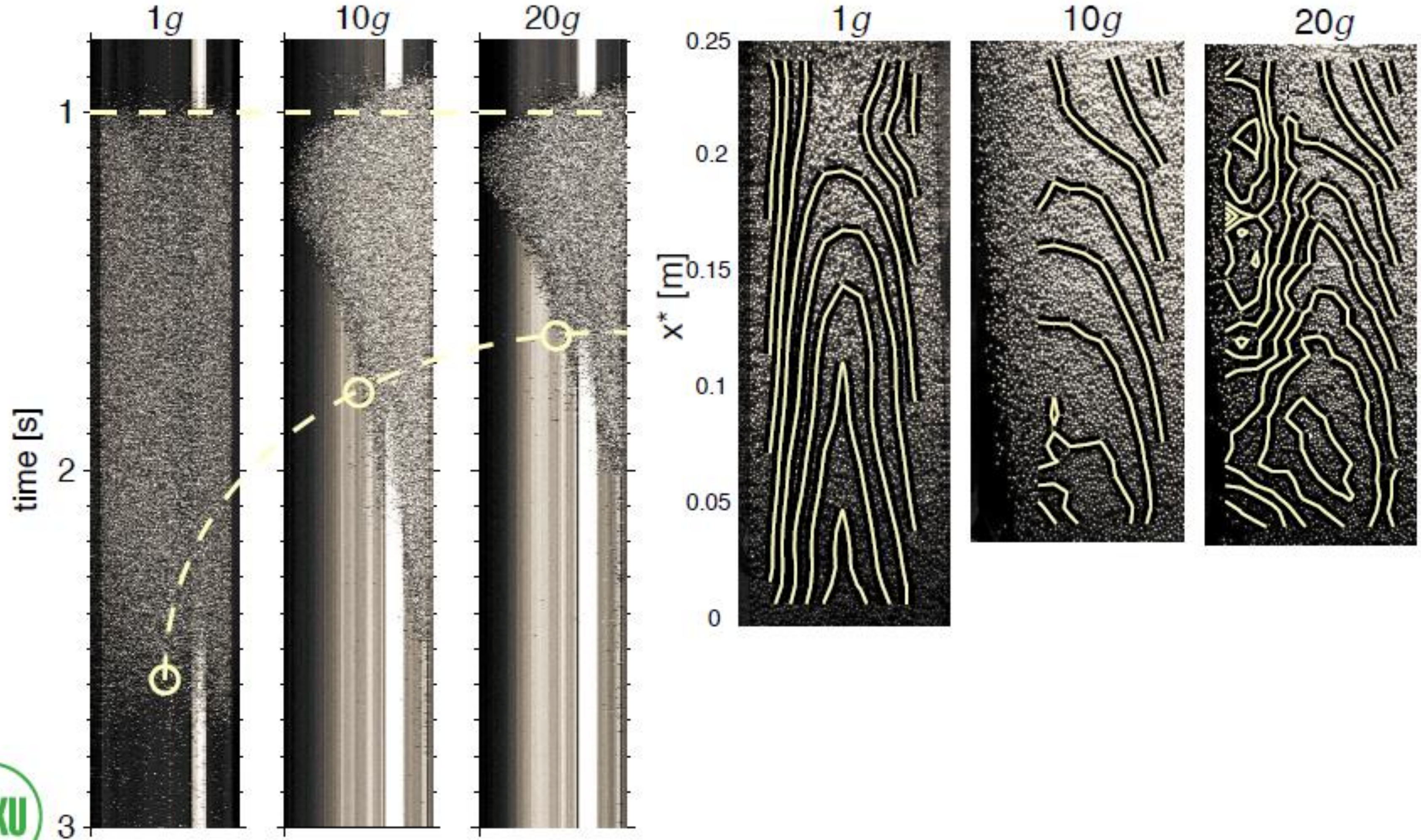
Max. model dimensions:
0.54x0.56x0.56 m

Model box



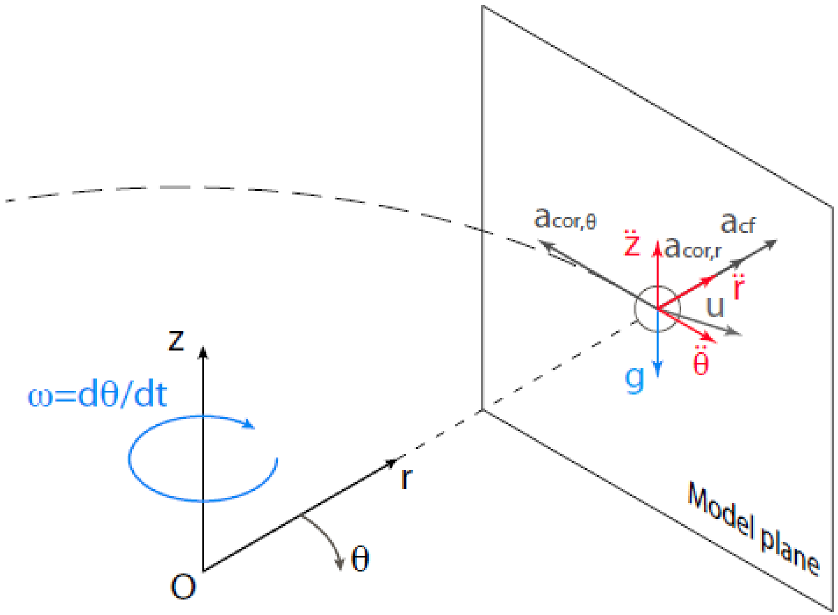
Guiding board
Silo
Feeding tube
80 mm
High-speed camera
Triggering gate
Point lasers
Load cells Channel
Load cells Collection box
500 mm
538 mm
538 mm
Ng





1g
10g
20g
time [s]
1
2
3
1g
10g
20g
x* [m]
0.25
0.2
0.15
0.1
0.05
0

Centrifuge and Corioli force



- Centrifugal acceleration:
  $\mathbf{a}_{cf} = \mathbf{r}\omega^2\hat{r} \equiv Ng$
- Coriolis acceleration:
  $\mathbf{a}_{cor} = -(2\dot{\mathbf{r}} \times \omega)\hat{\theta} + (2\dot{\theta} \times \omega)\hat{r}$
- Net-inertial acceleration:
  $\mathbf{a}_0 = \ddot{\mathbf{r}}\hat{r} + \ddot{\theta}\hat{\theta} + \ddot{\mathbf{z}}\hat{z} - g\hat{z}$

7th Framework of European Commission

MUltiscale MOdeling of LAndslides and DEbris flows

MUMOLADE

Ch. Peng, A. Leonardi, M. Cabrera, F. Wittel, H.J. Herrmann